UNIVERSITY OF NAIROBI

DEPARTMENT OF MECHANICAL & MANUFACTURING ENGINEERING

FME 301: SOLID & STRUCTURAL MECHANICS III

TUTORIAL SHEET NO. 2: ANALYSIS OF STRESS AND STRAIN

1. The following stress tensors represent states of stress at a point with reference to Ox, Oy and Oz axes through the point O. Make sketches of properly oriented elements around the point. The numerical values are in MN/m^2. Commment on the state of stress in each case (whether uniaxial, biaxial, triaxial, pure shear, etc.)

a) $\begin{vmatrix} 150 & -50 & 0 \\ -50 & -70 & 0 \\ 0 & 0 & 0 \end{vmatrix}$ b) $\begin{vmatrix} 30 & 0 & 15 \\ 0 & 0 & 0 \\ 15 & 0 & 40 \end{vmatrix}$ c) $\begin{vmatrix} 60 & 30 & 0 \\ 30 & 100 & 0 \\ 0 & 0 & 0 \end{vmatrix}$

d) $\begin{vmatrix} 0 & -5 & 0 \\ -5 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$ e) $\begin{vmatrix} 55 & 25 & 30 \\ 25 & -20 & 10 \\ 30 & 10 & 40 \end{vmatrix}$ f) $\begin{vmatrix} 0 & 0 & 0 \\ 0 & 65 & 40 \\ 0 & 40 & -35 \end{vmatrix}$

2. The stress system at a point O can be represented by the following stress tensor with reference to Ox and Oy axes (units in MN/m^2). Determine:

$$\begin{vmatrix} -83 & 55 & 0 \\ 55 & 100 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

   a) The principal stresses and their directions,
   b) The maximum and minimum shearing stresses and the directions of the planes on which they occur, and
   c) The normal and shearing stresses on the plane inclined at 45° to the x- axis.

(ANS. a) $\sigma_1 = 115.3$, $\sigma_2 = -98.3$, $\theta p = 15.5°$ or $74.5°$: b) $\tau_{max} = 106.8$, $\theta s = 29.5°$ or $119.5°$; $\sigma_{45} = 63.5$, $\tau_{45} = 91.5$)

3. The figure Q.3 represents the state of stress at a point:
   a) Represent the state of stress at a point using the stress tensor notation.
   b) Determine the normal and shearing stresses on the inclined plane m-m through the point shown in the figure.
   c) Determine the magnitudes of the principal stresses and their directions relative to the x-axis.
   d) Determine the magnitudes of the maximum and minimum shearing stresses and their directions relative to the x-axis.

You may use either analytical or graphical methods for your solution.
(ANS. b) $\sigma_n = 50.25\ MN/m^2$, $\tau = 5.93\ MN/m^2$ c) $\sigma_1 = 127.19\ MN/m^2$, $\sigma_2 = 49.8\ MN/m^2$, $\theta_1 = 32.38°$, $\theta_2 = 122.38°$, d) $\tau_{max} = 38.69\ MN/m^2$, $\theta_{s1} = -12.62°$ and $\theta_{s2} = 77.38°$)

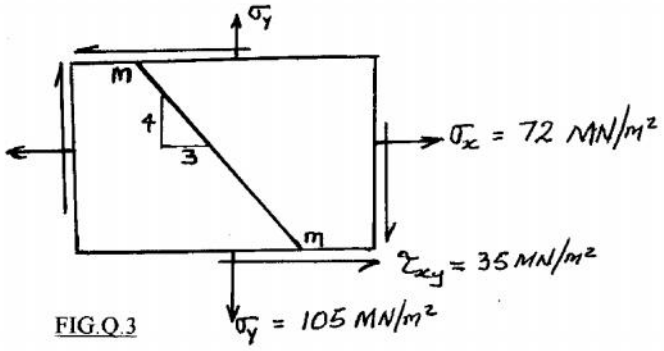

FIG.Q.3

4. At a point on a thin plate, which is loaded vertically, the normal stress on a vertical section is 210 MN/m^2 and the shear stress is 35 MN/m^2. The maximum shearing stress in the plane of the plate at the point is 91 MN/m^2. By sketching the Mohr's circles of stress for the point, calculate:
   a) The magnitudes of the principal stresses at the point, and
   b) The normal stress on the horizontal section through the point.

   (ANS. a) σ_1 = 217 MN/m^2, σ_2 = 33 MN/m^2 b) σ_n = 41 MN/m^2)

5. For the following rosette data in a) and b) below, determine the values and direction of the principal strains. Calculate also the values of the principal stresses. The arrangement of the strain gauges of the rosettes a) and b) are 0°-45°-90° and 0°-60°-120° respectively. Take the material on which the strain gauges are fixed to have Young's modulus and Poisson's ratio of 207 GN/m^2 and 0.33 respectively.
   a) $\varepsilon_o = 250 \times 10^{-6}$, $\varepsilon_{45} = -160 \times 10^{-6}$ and $\varepsilon_{90} = -50 \times 10^{-6}$
   b) $\varepsilon_o = -100 \times 10^{-6}$, $\varepsilon_{60} = 72 \times 10^{-6}$ and $\varepsilon_{120} = -273 \times 10^{-6}$

   (ANS. a) $\varepsilon_1 = 400 \times 10^{-6}$, $\varepsilon_2 = -200 \times 10^{-6}$ and $\varepsilon_{120} = -273 \times 10^{-6}$ at 30° measured clockwise from 0° axis, b) $\varepsilon_1 = 98.52 \times 10^{-6}$, $\varepsilon_2 = -299.52 \times 10^{-6}$ which is inclined at 19.5° clockwise from 0° axis. σ_1 = 45.4 MN/m^2, σ_2 = 36.2 MN/m^2)

6. A 120°-strain gauge rosette is used to measure strains at a point on the surface of a loaded structure. The strains measured in the directions 0°, 120° and 240° are $+10.8 \times 10^{-4}$, $+6.4 \times 10^{-4}$ and $+9.0 \times 10^{-4}$ respectively. Determine analytically or otherwise:
   a) The magnitudes of the principal strains and their inclinations with reference to the 0° strain gauge, and
   b) The magnitudes of the principal stresses.

   Take the material constants for the structure material to be Young's modulus E = 210 GN/m^2 and Poisson's ratio υ = 0.3.

   (ANS. a) $\varepsilon_1 = 11.29 \times 10^{-4}$ at $\theta_1 = 18°$ and $\varepsilon_2 = 6.18\ 10^{-4}$ at $\theta_2 = 178°$;
   b) σ_1 = 303 MN/m^2 and σ^2 = 221 MN/m^2)