COMPUTER SIMULATION OF DETERMINATE AND INDETERMINATE TRUSSES BY FINITE ELEMENT METHODS IN MATLAB PROGRAMMING LANGUAGE

PROJECT REPORT BY:

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DECLARATION

This research project is my original work and has not been presented for any award in any institution whatsoever.

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This research project has been submitted for examination with my knowledge as the department supervisor.

NAME: .....

SIGNATURE: .....

DATE: .....

DEDICATION

This research project is dedicated to my late dad, mum and two sisters. They have all been inspirational in my life and gave me the courage and determination to come this far.

ACKNOWLEDGEMENT

I realize the profound truth that He who created all things inert as well as live is GOD. But many things in this world are created through knowledge of some living beings and these living beings are groomed by their teachers and through their own effort of self study and practice.

Those who are gifted by God are exceptions and those who are gifted by their teachers are lucky. I am thankful to God as He has been grateful to me much more than I deserve and to my all teachers from school level to University heights as I am product of their efforts and guidance and hence this study.

I'm greatly indebted to the people and institution that made the writing of this project successful. Special thanks go to my supervisor, Dr. Hussein Jama for his invaluable guidance, continuous assistance and encouragement throughout the writing of this project report. I also acknowledge my colleague Henry Orare for the sacrifice he made in offering Matlab software and necessary training.

ABSTRACT

For any physical problem in engineering and science, there are traditionally two ways to deal with it by either theoretical approach or experiments. The theoretical approach in terms of mathematical modeling is an idealization and simplification of the real problems and the theoretical models often extract the essential or major characteristics of the problem. The mathematical equations obtained even for such over-simplified system are usually very difficult for mathematical analysis. On the other hand, the experimental approach is usually expensive if not impractical. It is now widely acknowledged that computational modeling and computer simulations serve as a cost-effective alternative, bridging the gap or complementing the traditional theoretical and experimental approaches to problem solving (Yang, 2006). This was the justification for the research work.

The objective as highlighted in chapter one was to perform a computer simulation of determinate and indeterminate trusses by finite element methods in Matlab programming language. Further, this chapter contains the research questions that guided the research work, significance of the study, scope of the study, limitations of the study, assumptions made and definition of terms used. The literature review which summarizes the information from other researchers who have carried out their research in the same field of study discusses the classical methods of analysis trusses and details various aspects of finite element methods.

Matlab (Matrix laboratory) is an interactive software system for numerical computations and graphics. Further discussions on Matlab are outlined in Chapter three including an overview of Matlab programming language, Matlab system and advantages of programming using Matlab. The Pseudo code followed in programming is also outlined within this chapter. Code validation is outlined in chapter four using four examples using classical methods and finite element methods.

Finally in conclusion it was possible to verify that in deed computer simulation of determinate and indeterminate could be conducted using finite element analysis. The results were also comparable with classical methods to accuracies of 99%.

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CHAPTER ONE

1.0 INTRODUCTION

1.1 Background of the Study

For any physical problem in engineering and science, there are traditionally two ways to deal with it by either theoretical approach or experiments. The theoretical approach in terms of mathematical modeling is an idealization and simplification of the real problems and the theoretical models often extract the essential or major characteristics of the problem. The mathematical equations obtained even for such over-simplified system are usually very difficult for mathematical analysis. On the other hand, the experimental approach is usually expensive if not impractical. Apart from financial limitation and limitation by physical scale, other constraining factors include the range of physical parameters and time for carrying out various experiments. As the computing speed and power have increased dramatically in the last few decades, a practical third way or approach is emerging, which is the computational modeling and numerical experimentation. It is now widely acknowledged that computational modeling and computer simulations serve as a cost-effective alternative, bridging the gap or complementing the traditional theoretical and experimental approaches to problem solving (Yang, 2006).

The essence of computational engineering, are the computational or numerical methods, (Yang, 2006). Numerical simulation of a process, means the solution of the governing equations (or mathematical model) of the process using a numerical method and a computer. While the derivation of the governing equations for most problems is not unduly difficult, their solution by exact methods of analysis is a formidable task. In such cases, numerical methods of analysis provide an alternative means of finding solutions (Reddy, 2004).

Where some computing has to be done and the objective would be to do simple simulations to get correct or reasonably acceptable results the following limitations need to be overcome. Limitations of computing time requiring use of efficient algorithms and high-speed computation either by parallel computing or supercomputers. Further, almost all computations involve some degree of approximation resulting in limitations as regards finite digit precision. This implies

results are only correct within a certain limit. In overcoming these limitations more versatile and efficient methods are highly needed. In fact, according to (Yang, 2006) the finite element method is one class of the most successful methods in computational engineering that has a wide range of applications.

According to (Jin & Riley, 2009) the finite element method is a numerical procedure used to obtain approximate solutions to boundary-value problems of mathematical physics with the aid of an electronic computer. (Bhavikatti, 2004) describes finite element analysis as a numerical technique where all the complexities of the problems, like varying shape, boundary conditions and loads are maintained as they are but the solutions obtained are approximate. Because of its diversity and flexibility as an analysis tool, it is receiving much attention in engineering. The vast improvements in computer hardware technology and reduction of cost of computers have boosted this method, since the computer is the basic need for the application of this method.

The Finite Element Method (FEM) is a product of the computer age and its application to solve practical problems requires use of computer programs for analysis. Nowadays there are several commercial finite element packages available that can solve varieties of problems. Some of these packages are ANSYS, ALGOR, NISA, ABAQUS, NASTRAN etc. having pre and postprocessors that give graphical interfaces of the structure before and after loading (Rao, 2007).

As a general rule FORTRAN codes are faster but more difficult to modify and are unable to handle graphical operations, whereas Matlab allows for easy graphical interfaces. In addition, Matlab's graphical part and the computational part are kept completely separate, so as to allow a more flexible utilization and avoid possible compatibility issues with different Matlab versions (Pelosi, Cocciolli, & Selleri, 2009).

Skeletal structures can be analyzed by a variety of hand-oriented methods of structural analysis taught in beginning Mechanics of Materials courses: the Displacement and Force methods. They can also be analyzed by the computer-oriented FEM. That versatility makes those structures a good choice to illustrate the transition from the hand-calculation methods taught in

undergraduate courses, to the fully automated finite element analysis procedures available in commercial programs (Carlos, 2004).

1.2 Objectives of the study

The objectives of this study were:

1.2.1 General Objective

The general objective was to become familiar with finite element methods by use of Matlab programming language.

1.2.2 Specific Objectives

The study was guided by the following specific objectives:

i) To conduct finite element analysis of two dimensional determinate and indeterminate trusses
ii) To compare finite element analysis results for two dimensional determinate and indeterminate trusses with classical methods
iii) To investigate whether finite element analysis of two dimensional determinate and indeterminate trusses could be modeled in Matlab programming language
iv) To evaluate the efficiency and effectiveness of finite element analysis of two dimensional determinate and indeterminate trusses modeled in Matlab Language

1.3 Research Questions

The research strived to answer the following questions:-

i. How do finite element methods analyze two dimensional determinate and indeterminate trusses?
ii. Do results from finite element analysis of two dimensional determinate and indeterminate trusses compare with classical methods?

iii. Is it possible to model finite element analysis of two dimensional determinate and indeterminate trusses in Matlab programming language?

iv. How efficient and effective is computer modeling in finite element analysis of two dimensional determinate and indeterminate trusses?

1.4 Significance of the study

The study is beneficial to engineers, as the study may be employed by, structural engineers in assessing safety of structures. Due to increasing demand for safety assessment outside the conventional code formats. Such assessment may be necessary for design purposes for critical space structures or for the assessment of existing space structures for remaining life, particularly where the safety or economic considerations are (Melchers & Hough, 2007). Hence, an efficient and effective computerized structural analysis of trusses would aid in their assessment purpose.

The study would also benefit present students undertaking solid and structural mechanics as a course as they would utilize the program to assess their understanding of the subject during their private study. Hence it would act as a self assessment tool.

The study would benefit solid and structural mechanics lecturers as they would utilize the program to analyze assessment questions set to students. The resulting program would save them time to prepare for lectures and perform other responsibilities required of them.

1.5 Scope of the study

The main focus of this study was finite element analysis of two dimensional determinate and indeterminate trusses through computer modeling in Matlab environment.

1.6 Limitations of the study

The study was limited to analysis of determinate and indeterminate trusses by finite element methods. The computer program was limited to Matlab which employed numerical programming. Further, the study was limited to duration of 30 weeks, where during this period the researcher defined the research problem in the project proposal, prepared an algorithm in

Matlab Language for the two dimensional finite element analysis of determinate trusses and an analysis of results obtained.

1.7 Assumptions of the study

The study made several assumptions. Specifically in this idealization, truss members carry only axial loads, have no bending resistance, and are connected by frictionless pins (Da Silva, 2006).

1.8 Definition of Terms

Matrix: (Matloff, 2011) describes a matrix as a vector with two additional attributes: the number of rows and the number of columns. Since matrices are vectors, they also have modes, such as numeric and character.

Direct force structures: (Matloff, 2011) describes them as those structures where only one internal force or stress resultant that is axial force may arise. Loads can be applied directly on the members also but they are replaced by equivalent nodal loads. In the loaded members additional internal forces such as bending moments, axial forces and shears are produced. Examples include pin jointed plane frames and ball jointed space frames which are loaded and supported at the nodes.

Plane frames: (Kalani, 1957) describes them as frames which all the members and applied forces lie in same plane. The joints between members are generally rigid. The stress resultants are axial force, bending moment and corresponding shear force.

Mathematical model: This is an idealization in which geometry, material properties, loads and/or boundary conditions are simplified based on the analyst's understanding of what features are important or unimportant in obtaining the results required (Narasaiah, 2008).

CHAPTER TWO

2.0 Literature Review

2.1 Introduction

This chapter summarizes the information from other researchers who have carried out their research in the same field of study. The literature— that is, prior research that informs and shapes the research question (Mohrman, 2011)— plays a crucial role in helping to keep these elements working together for research purposes. The literature is an integrating force, helping to shape research to make its best contribution. Familiarity with what has come before in a given field that relates to a research question makes sure prior findings are integrated, elaborated, or refuted in the current work (Mohrman, 2011). More specifically, with respect to understanding a problem, finding out what others have done to understand that problem lowers the risk of reinventing the wheel. Prior research in a field of inquiry informs and shapes one's research question, (Mohrman, 2011). In this regard the specific areas covered here are classical theories of truss analysis, Finite Element Analysis of trusses and Matlab programming Language.

Theoretical Review

2.2 Trusses

(Da Silva, 2006) indicated that in a bar under pure axial loading the stress state may be considered as uniform, irrespective of how the forces are applied, provided that the material points under consideration are not close to the region of the member where the forces are applied (Saint-Venant's principle). In axially loaded slender members these regions are generally a small part of the member, so that a uniform distribution of the stresses may be accepted when the elongation of the bar is computed.

$$\Delta l = l^{l} - l = \varepsilon l = (\sigma / E) l = \frac{Nl}{E\Omega} \qquad (2.1)$$

Where $\Delta l = Change\ in\ length$

ε = strain

l = length

σ = stress

E = Youngs Modulus

N = Normal Force

Ω = Area

2.3 Statically determinate Vs Statistically Indeterminate Structures

(Da Silva, 2006) indicates that a statically determinate structure only has internal force if external forces are applied, which means that they are insensitive to temperature change, material retraction, or any other actions that alter the dimensions of structural elements. Statically indeterminate structures, on the other hand, may have internal forces in the absence of external forces.

Statistically determinate structures are freely deformable, in the sense that their supports and internal connections do not restrict the deformations. This means that a small change in the geometry or size of the structural elements does not change the distribution of internal forces.

(Kalani, 1957) states that if skeletal structure is subjected to gradually increasing loads, without distorting the initial geometry of structure, that is, causing small displacements, the structure is said to be stable. If for the stable structure it is possible to find the internal forces in all the members constituting the structure and supporting reactions at all the supports provided from statical equations of equilibrium only, the structure is said to be determinate. If it is possible to determine all the support reactions from equations of equilibrium alone the structure is said to be externally determinate else externally indeterminate. If structure is externally determinate but it is not possible to determine all internal forces then structure is said to be internally indeterminate. Therefore according to (Kalani, 1957) a structural system may be:

i. Externally indeterminate but internally determinate

ii. Externally determinate but internally indeterminate
iii. Externally and internally indeterminate
iv. Externally and internally determinate

A system which is externally and internally determinate is said to be determinate system. A system which is externally indeterminate, internally indeterminate or externally and internally indeterminate is said to be indeterminate system.

If the total number of unknown internal and support reactions = to total number of independent statical equations of equilibrium then the system is determinate. Else it is indeterminate.

2.3.1 Statical Indeterminacy

(Kalani, 1957) defines it as the difference of the unknown forces (internal forces plus external reactions) and the equations of equilibrium.

2.3.2 Kinematic Indeterminacy

It is the number of possible relative displacements of the nodes in the directions of stress resultants (Kalani, 1957).

2.3.3 Analysis of Determinate Trusses

The joints of the trusses are idealized for the purpose of analysis. In case of plane trusses the joints are assumed to be hinged or pin connected. In case of space trusses ball and socket joint is assumed which is called universal joint. If members are connected to a hinge in a plane or universal joint in space, the system is equivalent to m members rigidly connected at the node with hinges or socketed balls in (m-1) number of members at the nodes.

The plane truss requires supports equivalent of three reactions and determinate space truss requires supports equivalent of six reactions in such a manner that supporting system is stable and should not turn into a mechanism. For this it is essential that reactions should not be concurrent and parallel so that system will not rotate and move. As regards loads they are

assumed to act on the joints or points of concurrency of members. If load is acting on member it is replaced with equivalent loads applied to joints to which it is connected. Here the member discharges two functions that is function of direct force member in truss and flexural member to transmit its load to joints. For this member the two effects are combined to obtain final internal stress resultants in this member.

The truss is said to be just rigid or determinate if removal of any one member destroys its rigidity and turns it into a mechanism. It is said to be over rigid or indeterminate if removal of member does not destroy its rigidity.

2.3.4 Relation between number of members and joints for just rigid truss

Let m = Number of members and j = Number of joints

Space truss

According to (Kalani, 1957) the number of equivalent links or members or reactive forces to constrain the truss in space is 6 corresponding to equations of equilibrium in space (Σ Fx = 0, Σ Fy = 0, Σ FZ = 0, ΣMx = 0, ΣMy = 0, ΣMz = 0). For ball and socket (universal) joint the minimum number of links or force components for support or constraint of joint in space is 3 corresponding to equations of equilibrium of concurrent system of forces in space (Σ Fx = 0, Σ Fy = 0, Σ FZ = 0). Each member is equivalent to one link or force.

Total number of links or members or forces which support j number of joints in space truss is (m + 6). Thus total number of unknown member forces and reactions is (m + 6). The equations of equilibrium corresponding to j number of joints is 3j. Therefore for determinate space truss system: (m + 6) = 3j. For a determinate plane truss system the relation is given as 2j-3 = m

2.3.5 Method of analysis of determinate trusses

There are two methods of analysis for determining axial forces in members of truss under point loads acting at joints (Kalani, 1957) The forces in members are tensile or compressive. The first step in each method is to compute reactions. Now we have system of members connected at

nodes and subjected to external nodal forces. The member forces can be determined by following methods.

i. Method of joints
ii. Method of sections

i) Method of joints

The method of joints is used when forces in all the members are required. A particular joint is cut out and its free body diagram is prepared by showing unknown member forces. Now by applying equations of equilibrium the forces in the members meeting at this joint are computed. Proceeding from this joint to next joint and thus applying equations of equilibrium to all joints the forces in all members are computed. In case of space truss the number of unknown member forces at a joint should not be more than three. For plane case number of unknowns should not be more than two.

Equations for space ball and socket joint equilibrium: $\Sigma Fx = 0$, $\Sigma Fy = 0$, $\Sigma FZ = 0$

Equations for xy plane pin joint equilibrium: $\Sigma Fx = 0$, $\Sigma Fy = 0$

ii) Method of sections

This method is used when internal forces in some members are required. A section is passed to cut the truss in two parts exposing unknown forces in required members. The unknowns are then determined using equations of equilibrium. In plane truss not more than 3 unknowns should be exposed and in case of space truss not more than six unknowns should be exposed.

Equations of equilibrium for space truss $\Sigma Fx = 0$, $\Sigma Fy = 0$, $\Sigma FZ = 0$ using method of sections: $\Sigma Mx = 0$, $\Sigma My = 0$, $\Sigma MZ = 0$

Equations of equilibrium for xy-plane $\Sigma Fx = 0$, $\Sigma Fy = 0$, $\Sigma MZ = 0$ truss using method of sections:

2.5 Finite Element Methods

2.5.1 Introduction

According to (Przemieniecki, 2009) the finite element method (FEM) for the analysis and design of structures and mechanical components is based on the concept of replacing the actual continuous structure by a mathematical model made up of elements of finite size (referred to as finite elements) having known elastic and inertia properties that can be expressed in matrix form. For this reason, early finite element methods were described as matrix methods of analysis, but today FEM has become the accepted terminology. The matrices representing these properties are considered as building blocks, which, when assembled together according to a set of rules derived from the theory of elasticity, provide the static and dynamic properties of the actual system.

Furthermore, the finite element analysis is a numerical technique. In this method all the complexities of the problems, like varying shape, boundary conditions and loads are maintained as they are but the solutions obtained are approximate. Because of its diversity and flexibility as an analysis tool, it is receiving much attention in engineering. The fast improvements in computer hardware technology and slashing of cost of computers have boosted this method, since the computer is the basic need for the application of this method. A number of popular brand of finite element analysis packages are now available commercially. Some of the popular packages are STAAD-PRO, GT-STRUDEL, NASTRAN, NISA and ANSYS. Using these packages one can analyse several complex structures (Bhavikatti, 2004).

The problems of structural mechanics such as deformation, trusses, stress analysis of automotive aircraft, building and bridge structures, magnetic flux, seepage etc. have been reduced to a system of linear simultaneous equations. Solution of these equations gives us the approximate behaviour of the continuum. These facts suggest that we need to keep pace with the developments by understanding the basic theory, modeling techniques, and computational aspects of the finite element method. Applications range from problems relating to heat transfer, fluid flow, lubrication, soil mechanics, electric and magnetic fields, structural engineering and discussions related to structural analysis problems (Rao, 2007).

In order to use FEM packages properly, the user must know the following points clearly:

i) How to discretise the domain to get good results.
ii) Identification of variables.
iii) Which elements are to be used for solving the problems in hand?
iv) Incorporation of boundary conditions.
v) Solution of simultaneous equations.
vi) Choice of approximating functions.
vii) Identification of variables.
viii) How the element properties are developed and what are their limitations?

2.5.2 A Brief History of FEM

Engineers, physicists and mathematicians have developed finite element method independently. In 1941, Hrenikoff found a solution of elasticity problems using the "framework method (Rao, 2007). In 1943 Courant R. made an effort to use piecewise continuous functions defined over triangular domain (Bhavikatti, 2004).

In 1956, Turner et al. derived stiffness matrices for truss, beam, and other elements and presented their new findings (Rao, 2007). Still in the fifties renewed interest in this field was shown by Polya G, Hersh J and Weinberger H.F. Argyris and Kelsey who introduced the concept of applying energy principles to the formation of structural analysis problems in 1960. In the same year Clough R. W introduced the word 'Finite Element Method' (Bhavikatti, 2004).

The first book on finite elements by O.C. Zienkiewicz and Cheng was published in 1967. The finite element method was applied to the problems which are non-linear and large deformations in nature, appeared in the late 1960's and early 1970's. A book on nonlinear continua appeared in 1972. It is curious to note that the mathematicians continue to put the finite element method on sound theoretical ground whereas the engineers continue to find interesting extensions in various branches of engineering. In 1959, Greenstadt utilised this technique to discretise cell. He defined the unknowns through a series of functions for each cell, proper variational principle for them

and satisfied continuity requirements to tie together the cells giving fundamentalists of finite element technique (Rao, 2007).

In sixties convergence aspect of the finite element method was pursued more rigorously. One such study by Melosh R.J led to the formulation of the finite element method based on the principles of minimum potential energy. Soon after that de Veubeke introduced equilibrium elements based on the principles of minimum potential energy, Pian T. H. H introduced the concept of hybrid element using the duel principle of minimum potential energy and minimum complementary energy (Bhavikatti, 2004).

In Late 1960's and 1970's, considerable progress was made in the field of finite element analysis. The improvements in the speed and memory capacity of computers largely contributed to the progress and success of this method. In the field of solid mechanics from the initial attention focused on the elastic analysis of plane stress and plane strain problems, the method has been successfully extended to the cases of the analysis of three dimensional problems, stability and vibration problems, non-linear analysis. A number of books [10 – 20] have appeared and made this field interesting (Bhavikatti, 2004).

In summary, mathematicians and engineers have simultaneously been developing the approximation technique and some of the noted researchers in the field of FEM are listed below:

	Researchers	**Period**		**Researchers**	**Period**
1	Rayleigh	1870	12	Mchenry	1943
2	Ritz	1909	13	Courant	1943
3	Richardson	1910	14	Prager and Synge	1947
4	Galerkin	1915	15	Newmark	1949
5	Liebman	1918	16	Morsh and Feshback	1953
6	Biezenokoch	1923	17	Mchahon	1953
7	Southwell	1940	18	Argyris	1955
8	Hrenikoff	1941	19	Turner and his group	1956
9	Clough, Martin and Topp	1956	20	Besseling	1963

10	Greenstadt	1959	21	Melosh	1963
11	Varga	1962	22	Zienkiewicz	1965
Source: Rao H.S.G. 2007, (p 13-14)					

Table 1: FEM researchers

2.5.3 General Description of FEM

In engineering problems there are some basic unknowns. If they are found, the behaviour of the entire structure can be predicted. The basic unknowns or the Field variables which are encountered in the engineering problems are displacements in solid mechanics, velocities in fluid mechanics, electric and magnetic potentials in electrical engineering and temperatures in heat flow problems. In a continuum, these unknowns are infinite. The finite element procedure reduces such unknowns to a finite number by dividing the solution region into small parts called elements and by expressing the unknown field variables in terms of assumed approximating functions (Interpolating functions/Shape functions) within each element. The approximating functions are defined in terms of field variables of specified points called nodes or nodal points. Thus in the finite element analysis the unknowns are the field variables of the nodal points. Once these are found the field variables at any point can be found by using interpolation functions (Bhavikatti, 2004).

In FEM, the entire structure is analysed without using assumptions about the degree of fixity at the joints of members and hence better estimation of stresses in the members was possible. This method generates a large set of simultaneous equations, representing load-displacement relationships. Matrix notation is ideally suited for computerising various relations in this method. Development of numerical methods and availability of computers, therefore, helped growth of matrix method. Sound knowledge of strength of materials, theory of elasticity and matrix algebra are essential pre-requisites for understanding this subject (Narasaiah, 2008).

2.5.4 Principle of FEM

FEM approach, which is based on minimum potential energy theorem, converges to the correct solution from a higher value as the number of elements in the model increases. The number of elements used in a model is selected by the engineer, based on the required accuracy of solution as well as the availability of computer with sufficient memory. FEM has become popular as it ensures usefulness of the results obtained even with lesser number of elements (Narasaiah, 2008).

Finite Element Analysis (FEA) based on FEM is a simulation, not reality, applied to the mathematical model. Even very accurate FEA may not be good enough, if the mathematical model is inappropriate or inadequate. A mathematical model is an idealisation in which geometry, material properties, loads and/or boundary conditions are simplified based on the analyst's understanding of what features are important or unimportant in obtaining the results required. The error in solution can result from three different sources.

i. Modelling error - associated with the approximations made to the real problem.
ii. Discretisation error - associated with type, size and shape of finite elements used to represent the mathematical model; can be reduced by modifying mesh.
iii. Numerical error - based on the algorithm used and the finite precision of numbers used to represent data in the computer; most softwares use double precision for reducing numerical error.

It is entirely possible for an unprepared software user to misunderstand the problem, prepare the wrong mathematical model, discretise it inappropriately, fail to check computed output and yet accept nonsensical results. FEA is a solution technique that removes many limitations of classical solution techniques; but does not bypass the underlying theory or the need to devise a satisfactory model. Thus, the accuracy of FEA depends on the knowledge of the analyst in modelling the problem correctly (Narasaiah, 2008).

2.5.5 Limitations of FEM

The traditional method in finite element analysis is based on the assumed displacement field within each element. These fields satisfy equation of compatibility of strains, but in general they violate the equations of stress equilibrium within the element. Only the simple three-node triangular plate element from among the family of isoparametric elements does not violate the equations of stress equilibrium. Another method of determining the properties of finite elements is to use stress fields that satisfy equations of stress equilibrium, but this approach is limited only to some simple elements. For elements for which no stress fields are available, the displacement field can be augmented by an additional displacement field vanishing on the element boundaries and its magnitude determined from the principle of minimum potential energy in the element (Bhavikatti, 2004).

2.5.6 Advantages of Numerical Simulation

(Jin & Riley, 2009) outlines the following advantages of numerical simulation

i. Because of their high predictive power and capability of dealing with complex structures, numerical simulation tools can support a wide variety of engineering applications, such as designing antennas analytically and predicting the impact of platforms on antenna performance, and address more complex applications, including calibration of antenna systems, estimating co-site interference of multiple-antenna systems on a platform, and predicting scattering from low-observable antenna installations.

Numerical simulation has four more distinctive advantages over traditional design by experiment.

ii. The first advantage is low cost. When an item can be designed, analyzed, and optimized on a computer, its design cost is reduced significantly compared to that of constructing a prototype physically and measuring it.

iii. The second advantage is the short design cycle. It typically takes far less time to simulate a structure on a computer than to actually build one and measure it in a laboratory.

iv. The third advantage is the full exploration of the design space. Because of the low cost and short design cycle, the designer can evaluate a large variety of design parameters systematically to come up with an optimal design through numerical simulation, which is simply impossible with laboratory experiments.

v. The last but not the least advantage of numerical simulation is the enormous amount of physical insight it provides. With a numerical solution to Maxwell's equations, the designer can now use a computer visualization tool to "see" the current flow on an antenna and field distributions around the antenna.

2.5.7 Disadvantages of Numerical Simulation

According to (Jin & Riley, 2009) the great advantages of numerical simulation are also accompanied by a series of challenges. The main challenge is due to improper use of a numerical simulation, such as insufficient discretization and use of a method outside its bounds. Such improper use would yield either a poor or a completely erroneous design while wasting time and resources. Therefore, it is very important to understand the basic principles, solution technologies, and applicability and capabilities of numerical methods behind the numerical simulation tools. Such knowledge can not only reduce the possibility of improper use of a method, but also help in choosing from a suite of tools the technique best suited for a specific problem, thus increasing the designer's productivity.

2.5.8 Distinction between finite element methods and classical methods

In classical theory, the deformational behavior of continous media is considered on a macroscopic scale without regard to size and shape of the elements within the prescribed boundary of the structure. In finite element methods, elements are of finite size and have a specified shape. These elements are specified arbitrarily in the process of defining the mathematical model of the continuous structure. The properties of each element are calculated using the theory of continuous elastic media, and the analysis of the entire structure is carried out

for the assembly of the individual elements. When the size of the elements is decreased, the deformational behavior of the mathematical model converges to that of the continuous structure (Przemieniecki, 2009).

(Bhavikatti, 2004) summarises the differences between finite element method vs classical methods as follows

a) In classical methods exact equations are formed and exact solutions are obtained where as in finite element analysis exact equations are formed but approximate solutions are obtained.
b) Solutions have been obtained for few standard cases by classical methods, where as solutions can be obtained for all problems by finite element analysis.
c) Whenever the following complexities are faced, classical method makes the drastic assumptions' and looks for the solutions:
    i. Shape
    ii. Boundary conditions
    iii. Loading

    To get the solution in the above cases, rectangular shapes, same boundary condition along a side and regular equivalent loads are to be assumed. In FEM no such assumptions are made. The problem is treated as it is.

d) When material property is not isotropic, solutions for the problems become very difficult in classical method. Only few simple cases have been tried successfully by researchers. FEM can handle structures with anisotropic properties also without any difficulty.
e) Problems with material and geometric non-linearities cannot be handled by classical methods. There is no difficulty in FEM.

Hence FEM is superior to the classical methods only for the problems involving a number of complexities which cannot be handled by classical methods without making drastic assumptions. For all regular problems, the solutions by classical methods are the best solutions. Infact, to

check the validity of the FEM programs developed, the FEM solutions are compared with the solutions by classical methods for standard problems (Bhavikatti, 2004).

2.5.9 Classification of FEM

The basic problem in any engineering design is to evaluate displacements, stresses and strains in any given structure under different loads and boundary conditions. Several approaches of Finite Element Analysis have been developed to meet the needs of specific applications. According to (Narasaiah, 2008) the common methods are:

Displacement method - Here the structure is subjected to applied loads and/or specified displacements. The primary unknowns are displacements, obtained by inversion of the stiffness matrix, and the derived unknowns are stresses and strains. Stiffness matrix for any element can be obtained by variational principle, based on minimum potential energy of any stable structure and, hence, this is the most commonly used method.

Force method - Here the structure is subjected to applied loads and/or specified displacements. The primary unknowns are member forces, obtained by inversion of the flexibility matrix, and the derived unknowns are stresses and strains. Calculation of flexibility matrix is possible only for discrete structural elements (such as trusses, beams and piping) and hence, this method is limited in the early analyses of discrete structures and in piping analysis

Mixed method - Here the structure is subjected to applied loads and/or specified displacements. The method deals with large stiffness coefficients as well as very small flexibility coefficients in the same matrix. Analysis by this method leads to numerical errors and is not possible except in some very special cases.

Hybrid method - Here the structure is subjected to applied loads and stress boundary conditions. This deals with special cases, such as airplane dour frame which should be designed for stress-free boundary, so that the door can be opened during flight, in cases of emergencies.

Displacement method is the most common method and is suitable for solving most of the engineering problems.

2.5.10 Mathematical Models

One of the most important thing engineers and scientists do is to model natural phenomena. They develop conceptual and mathematical models to simulate physical events, whether they are aerospace, biological, chemical, geological, or mechanical. The mathematical models are developed using laws of physics and they are often described in terms of algebraic, differential, and/or integral equations relating various quantities of interest. A mathematical model can be broadly defined as a set of relationships among variables that express the essential features of a physical system or process in analytical terms (Reddy, 2004).

2.5.11 Design and Analysis of a component

(Narasaiah, 2008) Mechanical design is the design of a component for optimum size, shape, etc., against failure under the application of operational loads. A good design should also minimise the cost of material and cost of production. Failures that are commonly associated with mechanical components are broadly classified as:

a) Failure by breaking of brittle materials and subjected to repetitive loads) of ductile materials.
b) Failure by yielding of ductile loads.
c) Failure by elastic deformation. materials, fatigue subjected to failure (when non-repetitive

The last two modes cause change of shape or size of the component rendering it useless and, therefore, refer to functional or operational failure. Most of the design problems refer to one of these two types of failures. Designing, thus, involves estimation of stresses and deformations of the components at different critical points of a component for the specified loads and boundary conditions, so as to satisfy operational constraints.

Design is associated with the calculation of dimensions of a component to withstand the applied loads and perform the desired function. **Analysis** is associated with the estimation of displacements or stresses in a component of assumed dimensions so that adequacy of assumed dimensions is validated. **Optimum design** is obtained by many iterations of modifying dimensions of the component based on the calculated values of displacements and/or stresses vis-a-vis permitted values and re-analysis.

An analytic method is applied to a model problem rather than to an actual physical problem. Even many laboratory experiments use models. A geometric model for analysis can be devised after the physical nature of the problem has been understood. A model excludes superfluous details such as bolts, nuts, rivets, but includes all essential features, so that analysis of the model is not unnecessarily complicated and yet provides results that describe the actual problem with sufficient accuracy. A geometric model becomes a mathematical "model" when its behaviour is described or approximated by incorporating restrictions such as homogeneity, isotropy, constancy of material properties and mathematical simplifications applicable for small magnitudes of strains and rotations.

Several methods, such as method of joints for trusses, simple theory of bending, simple theory of torsion, analyses of cylinders and spheres for axisymmetric pressure load etc., are available for designing/analysing simple\ components of a structure. These methods try to obtain exact solutions of second order partial differential equations and are based on several assumptions. on sizes of the components, loads, end conditions, material properties, likely deformation pattern etc. Also, these methods are not amenable for generalisation and effective utilisation of the computer for repetitive jobs.

Strength of materials approach deals with a single beam member for different loads and end conditions (free, simply supported and fixed). In a space -,. frame involving many such beam members, each member is analysed independently by an assumed distribution of loads and end conditions (Narasaiah, 2008).

2.5.12 Types of Analysis

According to (Narasaiah, 2008) mechanical engineers deal with two basic types of analyses for discrete and continuum structures, excluding other applicatiion areas like fluid flow, electromagnetics. FEM helps in modelling the component once and perform both the types of analysis using the same model.

a) **Thermal analysis** - Deals with steady state or transient heat transfer by conduction and convection, both being linear operations while radiation is a non-linear operation, and estimation of temperature distribution in the component. This result can form one load condition for the structural analysis

b) **Structural analysis** – Deals with estimation of stresses and displacements in discrete as well as continuum structures under various types of loads such as gravity, wind, pressure and temperature. Dynamic loads may also be considered.

2.5.13 Approximate method versus Exact method

An analytical solution is a mathematical expression that gives the values of the desired unknown quantity at any location of a body and hence is valid for an infinite number of points in the component. However, it is not possible to obtain analytical mathematical solutions for many engineering problems.

For problems involving complex material properties and boundary conditions, numerical approximate but acceptable solutions (with reasonable accuracy) for the unknown quantities - only at discrete or finite number of points in the component.-Approximation is carried out in two stages (Narasaiah, 2008):

a) In the formulation of the mathematical model, with respect to the physical behaviour of the component. Example : Approximation of joint with multiple rivets at the junction of any two members of a truss as a pin joint, assumption that the joint between a column and a beam behaves like a simple support for the beam,.... The results are reasonably accurate far away from the joint.

b) In obtaining numerical solution to the simplified mathematical model. The methods usually involve approximation of a functional (such as Potential energy) in terms of unknown functions (such as displacements) at finite number of points. There are two broad categories

i. Weighted residual methods such as Galerkin Collocation method, Least squares method, etc. method,

ii. Variational method (Rayleigh-Ritz method, FEM). FEM is an improvement of Rayleigh-Ritz method by choosing a variational function valid over a small element and not on the entire component, which will be discussed in detail later. These methods also use the principle of minimum potential energy.

Principle of minimum potential energy

Among all possible kinematically admissible displacement fields (satisfying compatibility and boundary conditions) of a conservative system, the one corresponding to stable equilibrium state has minimum potential energy. For a component in static equilibrium, this principle helps in the evaluation of unknown displacements of deformable solids (continuum structures).

Weighted Residual Methods

Most structural problems end up with differential equations. Closed form solutions are not feasible in many of these problems. Different approaches are suggested to obtain approximate solutions. One such category is the weighted residual technique.

Different methods are proposed based on how the residual is used in obtaining the best (approximate) solution.

(a) called Galerkin Method

b) Collocation Method

c) Least Squares Method

In this method, integral of the residual over the entire component is minimised i.e

$$\frac{dI}{dx} = 0 \qquad (2.2)$$

$$I = \int [R(\{a\}, x)]^2 \, . \, dx \qquad (2.3)$$

This method results in n algebraic simultaneous equations in unknown coefficients, which can be easily evaluated.

2.5.14 Structural Analysis

Structural analysis is a most exciting field of activity, but it is clearly only a support activity in the larger field of structural design. Analysis is a main part of the formulation and the solution of any design problem, and it often must be repeated many times during the design process. The analysis process helps to identify improved designs with respect to performance and cost (Kirsch, 2002).

Referring to behavior under working loads, the objective of the analysis of a given structure is to determine the internal forces, stresses and displacements under application of the given loading conditions. In order to evaluate the response of the structure it is necessary to establish an analytical model, which represents the structural behavior under application of the loadings. An acceptable model must describe the physical behavior of the structure adequately, and yet be simple to analyze. That is, the basic assumptions of the analysis will ensure that the model represents the problem under consideration and that the idealizations and approximations used result in a simplified solution. This latter property is essential particularly in the design of complex or large systems. Two categories of mathematical models are often considered:

A. Lumped-parameter (discrete-system) models.
B. Continuum-mechanics-based (continuous-system) models

The solution of discrete analysis models usually involves the following steps:

i. Idealization of the system into a form that can be solved. The actual structure is idealized as an assemblage of elements that are interconnected at the joints.
ii. Formulation of the mathematical model. The equilibrium requirements of each element are first established in terms of unknown displacements, and the element interconnection requirements are then used to establish the set of simultaneous analysis equations for the unknown displacements.
iii. Solution of the model. The response is calculated by solving the simultaneous equations for the unknown displacements; the internal forces and stresses of each element are calculated by using the element equilibrium requirements.

The overall effectiveness of an analysis depends to a large degree on the numerical procedures used for the solution of the equilibrium equations. The accuracy of the analysis can, in general, be improved if a more refined model is used. In practice, there is a tendency to employ more and more refined models to approximate the actual structure. This means that the cost of an analysis and its practical feasibility depend to a considerable degree on the algorithms available for the solution of the resulting equations. Because of the requirement that large systems be solved, much research effort has been invested in equation solution algorithms. The time required for solving the equilibrium equations can be a high percentage of the total solution time, particularly in nonlinear analysis or in dynamic analysis, when the solution must be repeated many times. An analysis may not be possible if the solution procedures are too costly or unstable.

2.5.15 Plane Trusses

The truss elements are part of a truss structure linked together by pinned joints that transmit only axial force to the elements. Unlike an axial element, a truss element is two-dimensional, i.e. it can displace both in x and y directions. Thus, we have two degrees of freedom at each node and totally four degrees of freedom for a truss element (Rao, 2007). The geometry of the structure is referred to a common Cartesian coordinate system $\{x, y\}$, which is called the *global coordinate system*. Other names for it in the literature are *structure coordinate system* and *overall coordinate system*. The key ingredients of the stiffness method of analysis are the *forces* and *displacements* at the joints.

In a idealized pin-jointed truss, externally applied forces as well as reactions *can act only at the joints*. All member axial forces can be characterized by the *x* and *y* components of these forces, denoted by *fx* and *fy* , respectively. The components at joint *i* being identified as *fxi* and *fyi* , respectively.

2.5.16 Analysis of Trusses using FEM

The truss may be statically determinate or indeterminate. In the analysis all joints are assumed pin connected and all loads act at joints only. These assumptions result into no bending of any member. All members are subjected to only direct stresses– tensile or compressive (Bhavikatti, 2004).

The following steps illustrate how finite element analysis is conducted

Step 1: Selection of the Reference Origin for the Global Coordinate System

The reference origin is used for the coordinates of the element nodes and applied loads or constraints on the structure (Bhavikatti, 2004).

Step 2: Selection of a Rigid Frame of Reference:

A rigid frame of reference is needed to calculate the node displacements. This is accomplished by defining displacements u x and u y at all the nodes. This establishes a rigid frame of reference with respect to which all other node displacements are calculated. This means that rigid-body translations and rotations of the whole structure are eliminated. This is accomplished simply by eliminating all rows and columns in the assembled stiffness, thermal stiffness, and/or mass matrices corresponding to zero displacements defining the rigid frame of reference (Bhavikatti, 2004).

Step 3: Field Variables and Elements

Joint displacements are selected as basic field variables. Since there is no bending of the members, we have to ensure only displacement continuity (C -continuity). Hence we select two

node bar elements for the analysis of trusses. Since the members are subjected to only axial forces, the displacements are only in the axial directions of the members. Therefore the nodal variable vector for the typical bar element shown in Figure1 is

$$\{\delta'\} = \begin{Bmatrix} \delta'_1 \\ \delta'_1 \end{Bmatrix} = \begin{bmatrix} \delta'_{x1} \\ \delta'_{y1} \\ \delta'_{x2} \\ \delta'_{y2} \end{bmatrix} \qquad (2.4)$$

Where δ_1' and δ_2' are in the axial directions of the element. But the axial direction is not same for all members. If we select x-y as global coordinate system, there are two displacement components at every node. Hence the nodal variable vector for a typical element is,

$\{\delta\} = [\delta_1\, \delta_2\, \delta_3\, \delta_4]$

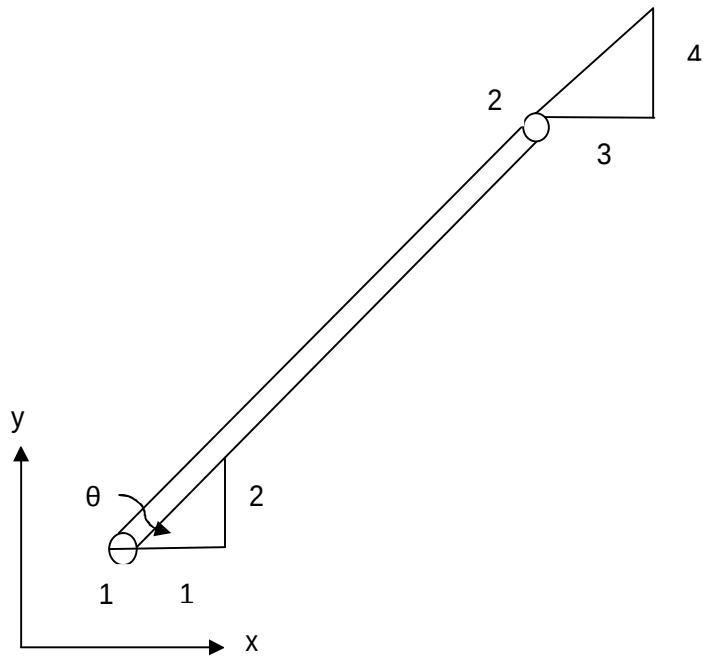


Figure 1: Truss Element

As shown in figure 1 above

$$\delta_{x1}' = \delta_1 \cos\theta + \delta_2 \sin\theta \; : \; \delta_{y1}' = -\delta_1 \sin\theta + \delta_2 \cos\theta \tag{2.5}$$

$$\delta_{x2}' = \delta_3 \cos\theta + \delta_4 \cos\theta \; : \delta_{y2}' = -\delta_3 \sin\theta + \delta_4 \cos\theta \tag{2.6}$$

If c and s are the direction cosines

$c = \cos\theta$, $s = \sin\theta$

$$\delta_{x1}' = \delta_1 c + \delta_2 s \; : \; \delta_{y1}' = -\delta_1 s + \delta_2 c$$

$$\delta_{x2}' = \delta_3 c + \delta_4 s \; : \delta_{y2}' = -\delta_3 s + \delta_4 c$$

i.e

$$\{\delta'\} = \begin{Bmatrix} \delta'_1 \\ \delta'_1 \end{Bmatrix} = \begin{bmatrix} \delta'_{x1} \\ \delta'_{y1} \\ \delta'_{x2} \\ \delta'_{y2} \end{bmatrix} = \begin{bmatrix} c & s & 0 & 0 \\ -s & c & 0 & 0 \\ 0 & 0 & c & s \\ 0 & 0 & -s & c \end{bmatrix}$$

$$\{\delta'\} = [T]\{\delta\}$$

Where

$$[T] = \begin{bmatrix} c & s & 0 & 0 \\ -s & c & 0 & 0 \\ 0 & 0 & c & s \\ 0 & 0 & -s & c \end{bmatrix}$$

And [T] is called displacement transformation (or rotation) matrix. If the coordinates (x_1,y_1) and (x_1,y_2) of node 1 and 2 of the elements are known, we can find

$$c = \frac{x_2 - x_1}{l_e}, s = \frac{y_2 - y_1}{l_e}$$

$$l_e = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The node forces transform as $f_{x1} = f'_{x1}c - f'_{y1}s$

Which in matrix form become

$$\begin{bmatrix} f_{x1} \\ f_{y1} \\ f_{x2} \\ f_{y2} \end{bmatrix} = \begin{bmatrix} c & -s & 0 & 0 \\ s & c & 0 & 0 \\ 0 & 0 & c & -s \\ 0 & 0 & s & c \end{bmatrix} \begin{bmatrix} f'_{x1} \\ f'_{y1} \\ f'_{x2} \\ f'_{y2} \end{bmatrix} \quad (2.7)$$

The 4 × 4 matrix that appears above is called a *force transformation matrix*. A comparison of (T) and (T^T) reveals that the force transformation matrix is the *transpose* $\mathbf{T^T}$ of the displacement transformation matrix **T.**

Where $T^T = \begin{bmatrix} c & -s & 0 & 0 \\ s & c & 0 & 0 \\ 0 & 0 & c & -s \\ 0 & 0 & s & c \end{bmatrix}$

Discritising: Where a member may be taken as an element conveniently. Number of nodal points for the truss structure is determined. The total degrees of freedom for the truss structure are also determined by (Total number of nodes X degrees of freedom of each node). Finally, the direction cosines for the elements are established.

Step 4: Element Properties in Local Coordinates

Since bar elements are used

$$[f] = [K]\{\delta\}$$

$$[u'] = [T]\{\delta\}$$

$$[f] = [T]^T\{f'\}$$

Inserting these matrix expressions into $\{f'\} = [K'][u']$ and comparing with $[f] = [K]\{\delta\}$ this reveals that [K] = $[T]^T[K'][T]$ (2.8)

Carrying out the matrix multiplication in closed form we get

$$K = \frac{E\,A}{L}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

Where K = element stiffness matrix

Step 5: Global Properties

It may be noted that for member i j

$$\delta_1 = \delta_{2i-1}, \delta_2 = \delta_{2i}$$

$$\delta_3 = \delta_{2j-1}, \delta_2 = \delta_{2j}$$

Hence this will give us the position of elements of [K'] in global stiffness matrix. Then for element number 1, element matrix is generated and they are added to the existing values of elements of global matrix at approximate places. Then next element stiffness matrix is generated and placed in appropriate positions in global matrix. The process is continued till all elements are handles in the truss structure (Bhavikatti, 2004).

Step 6: Boundary Conditions

The boundary conditions are then imposed using the penalty approach, (Bhavikatti, 2004).

Step 7: Solution for the Global Node Displacements

Once the properties of Individual elements in the global coordinate system are calculated, they are then assembled into the complete structure using the global coordinate system. This is accomplished by a simple addition of the properties in the directions (i, j) to form the static load displacement equation for the whole structure (Bhavikatti, 2004).

Assembly of the globalised matrices follows the relation that $f = \sum f_i = (\sum K)\, u = Ku$

The resulting stiffness matrix is referred to as the Master stiffness matrix

Step 8: Computation of the Node Displacements

K is a singular matrix and therefore cannot be inverted to obtain a solution for U. Therefore in this step all rows and columns corresponding to the rigid frame of reference, and any other rows and columns corresponding to additional constraints (zero displacements) are eliminated from K.

In practice, however, in order to preserve the original numbering system the rows and columns in K are not eliminated but are simply replaced with zeros except for the diagonal terms, which are replaced with ones (Bhavikatti, 2004).

Step 9: Additional Calculations Analysts are interested in finding stresses and forces in the members of the truss (Bhavikatti, 2004).

We know $\sigma = E_e \varepsilon$ (2.9)

But $\frac{\text{Change in Length}}{Original\ length} = \varepsilon$

$$\frac{\delta' - \delta''}{l_e}$$

CHAPTER THREE

3.0 Matlab Programming

3.1 Introduction

Matlab (Matrix laboratory) is an interactive software system for numerical computations and graphics. As the name suggests, Matlab is especially designed for matrix computations: solving systems of linear equations, computing eigenvalues and eigenvectors, factoring matrices. In addition, it has a variety of graphical capabilities, and can be extended through programs written in its own programming language. Many such programs come with the system; a number of these extend Matlab's capabilities to nonlinear problems, such as the solution of initial value problems for ordinary differential equations. (http://www.math.mtu.edu)

Matlab is designed to solve problems numerically, that is, in finite-precision arithmetic. Therefore it produces approximate rather than exact solutions, and should not be confused with a symbolic computation system (SCS) such as Mathematica or Maple. It should be understood that this does not make Matlab better or worse than an SCS; it is a tool designed for different tasks and is therefore not directly comparable. (http://www.math.mtu.edu)

3.2 Overview of the MATLAB Environment

The MATLAB high-performance language for technical computing integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation. Typical uses include

i. Math and computation
ii. Algorithm development
iii. Data acquisition
iv. Modeling, simulation, and prototyping
v. Data analysis, exploration, and visualization
vi. Scientific and engineering graphics

vii. Application development, including graphical user interface building

MATLAB is an interactive system whose basic data element is an array that does not require dimensioning. It allows an individual to solve many technical computing problems, especially those with matrix and vector formulations, in a fraction of the time it would take to write a program in a scalar non interactive language such as C or FORTRAN (http://www.mathwork.com).

MATLAB was originally written to provide easy access to matrix software developed by the LINPACK and EISPACK projects. Today, MATLAB engines incorporate the LAPACK and BLAS libraries, embedding the state of the art in software for matrix computation.

MATLAB has evolved over a period of years with input from many users. In university environments, it is the standard instructional tool for introductory and advanced courses in mathematics, engineering, and science. In industry, MATLAB is the tool of choice for high-productivity research, development, and analysis Matlab Product help guide.

MATLAB features a family of add-on application-specific solutions called toolboxes. Very important to most users of MATLAB, toolboxes allow an individual to learn and apply specialized technology. Toolboxes are comprehensive collections of MATLAB functions (M-files) that extend the MATLAB environment to solve particular classes of problems. One can add on toolboxes for signal processing, control systems, neural networks, fuzzy logic, wavelets and simulation Matlab Product help guide.

3.3 The MATLAB System

According to the (The Mathwork, Inc., 2008) MATLAB system consists of these main parts:

Desktop Tools and Development Environment

This is the set of tools and facilities that help a person to use and become more productive with MATLAB functions and files. Many of these tools are graphical user interfaces. It includes the MATLAB desktop and Command Window, a command history, an editor and debugger, a code

analyzer and other reports, and browsers for viewing help, the workspace, files, and the search path.

Mathematical Function Library

This is a vast collection of computational algorithms ranging from elementary functions, like sum, sine, cosine, and complex arithmetic, to more sophisticated functions like matrix inverse, matrix eigen values, Bessel functions, and fast Fourier transforms.

The Language

This is a high-level matrix/array language with control flow statements, functions, data structures, input/output, and object-oriented programming features. It allows both "programming in the small" to rapidly create quick and dirty throw-away programs, and "programming in the large" to create large and complex application programs.

Graphics

MATLAB has extensive facilities for displaying vectors and matrices as graphs, as well as annotating and printing these graphs. It includes high-level functions for two-dimensional and three-dimensional data visualization, image processing, animation, and presentation graphics. It also includes low-level functions that allow a person to fully customize the appearance of graphics as well as to build complete graphical user interfaces on MATLAB applications.

External Interfaces

This is a library that allows a person to write C and FORTRAN programs that interact with MATLAB. It includes facilities for calling routines from MATLAB (dynamic linking), for calling MATLAB as a computational engine, and for reading and writing MAT-files.

3.4 Advantages of Matlab

As a general rule FORTRAN codes are faster but more difficult to modify and are unable to handle graphical operations, whereas Matlab allows for easy graphical interfaces. Indeed, also for Matlab, the graphical part and the computational part has been kept completely separate, so as to allow a more flexible utilization and avoid possible compatibility issues with different Matlab versions (Pelosi, Cocciolli, & Selleri, 2009).

3.5 How the Program works

The program executes by way of utilizing the following features:

3.5.1 M-File Functions

Functions are program routines, usually implemented in M-files that accept input arguments and return output arguments. They operate on variables within their own workspace. This workspace is separate from the workspace accessed at the MATLAB command prompt (The Mathwork, Inc., 2008).

Types of Functions

The **Primary M-File Functions** is the first function in an M-file and typically contains the main program.

In this program the primary function is referred to as

```
function varargout = Trial_guif(varargin)
use: This function identifies the file Trial_guif which contains the main
program
```

Subfunctions act as subroutines to the main function. Subfunction can be used to define multiple functions within a single M-file (The Mathwork, Inc., 2008). Examples of a subfunctions used in this code includes:

```
function edit2_Callback(hObject, eventdata, handles)
Use: for executing the edit 2 callback in the graphical user interface
```

and `function [Fr, st] = output(handles)`

```
Use: for executing the call function to the input in the graphical user
interface
```

3.5.2 Built-In Classes (Data Types)

Structures

Structures are MATLAB arrays with named "data containers" called fields. The fields of a structure can contain any kind of data. For example, one field might contain a text string representing a name, another might contain a scalar representing a billing amount, a third might hold a matrix of medical test results, and so on (The Mathwork, Inc., 2008). In this code, structure data type is employed in,

```
handles.Y for holding the variables input as member properties in table 1.

handles.Z for holding the variables input as nodal properties in table 2.
```

Cell array

A cell array provides a storage mechanism for dissimilar kinds of data. An individual can store arrays of different types and/or sizes within the cells of a cell array (The Mathwork, Inc., 2008). For example, in this code a cell array is used in

```
Kr{1,m} = [c s 0 0;...
        -s c 0 0;...
        0 0 c s;...

        0 0 -s c]; to store the displacement transformation matrix
```

3.5.3 A function handle

A function handle is a callable association to a MATLAB function. It contains an association to that function that enables a person to invoke the function regardless of where it's called from. This means that, even when outside the normal scope of a function, it is possible to still call it if its handle is used Matlab Product help guide (2008).

3.5.4 Matrices

The most basic MATLAB data structure is the *matrix*: a two-dimensional, rectangularly shaped data structure capable of storing multiple elements of data in an easily accessible format. These data elements can be numbers, characters, logical states of true or false, or even other MATLAB structure types (The Mathwork, Inc., 2008). Example here in is:

```
K =v*[c^2 c*s -c^2 -c*s;...
        s*c s^2 -c*s -s^2;...
        -c^2 -s*c c^2 c*s;...
        -c*s -s^2 c*s s^2];
```

For storing the element stiffness matrix properties

Concatenating Matrices

Matrix concatenation is the process of joining one or more matrices to make a new matrix.

In this code matrix concatenation is done at `W(d,f,m)= K(n,p);` for storing the globalised elements stiffness matrices.

Matrix Indexing

To reference a particular element in a matrix, specify its row and column number using the following syntax, A (row, column) where A is the matrix variable. Always specify the row first and column second (The Mathwork, Inc., 2008): This is as shown for instance in indexing of displacement boundary conditions as follows `A(1,2*m-1)`.

Dimensions of the Matrix

According to the (The Mathwork, Inc., 2008), **Numel** is a matrix dimension that returns the number of elements. It is employed in returning the number of zero displacement boundary conditions as follows `numel(find(A == 0))`

Reshaping a Matrix

The following function changes the shape of a matrix.

Transpose - Flip a matrix about its main diagonal, turning row vectors into column vectors and vice versa. This is particularly seen in the relation `Fr(:,2) = U.'` when transposing the displacement matrix.

3.5.6 Program Control Statements

Conditional Control — if, switch

This group of control statements enables a programmer to select at run-time which block of code is executed. To make this selection based on whether a condition is true or false, use the if statement (which may include else or elseif). To select from a number of possible options depending on the value of an expression, use the switch and case statements (which may include otherwise) (The Mathwork, Inc., 2008).

if, else, and elseif

If evaluates a logical expression and executes a group of statements based on the value of the expression (The Mathwork, Inc., 2008). For instance if is employed in this code as follows, when evaluating if the elements in the displacement matrix are 0,1 or others.

```
if A(1,i) == 0
        Fmod(1,i) = F(1,i);
    elseif A(1,i) == 1
        Fmod(1,i) = F(1,i);
    else
        Fmod(1,i) = A(1,i);
end
```

Switch, case, and otherwise

Switch executes certain statements based on the value of a variable or expression (The Mathwork, Inc., 2008). For instance, switch is employed in this code to evaluate whether the value of n is 1,2,3 or any other, so as to obtain the corresponding value of d.

```
switch n
```

```
            case 1
                d = n1-1;
            case 2
                d = n1;
            case 3
                d = n2-1;
            otherwise
                d =n2;
end
```

Loop Control — for, while, continue, break

With loop control statements, one can repeatedly execute a block of code, looping back through the block while keeping track of each iteration with an incrementing index variable. Use the for statement to loop a specific number of times. The, while statement, is more suitable for basing loop execution, on how long a condition continues to be true or false. The continue and break statements give a person more control on exiting the loop.

for

The for loop executes a statement or group of statements a predetermined number of times.

```
for j=1:w
                    D(i,j)=0;
                    D(j,i)=0;
                    D(i,i)=1;

                end
```

For example, in the case above, the for loop executes w times before stopping.

3.5.6 Types of Variables

A MATLAB variable is essentially a tag assigned to a value while that value remains in memory. The tag gives a way to reference the value in memory so that programs can read it, operate on it with other data, and save it back to memory.

MATLAB provides three basic types of variables:

i. Local Variables

ii. Global Variables

iii. Persistent Variables

Local Variables

Each MATLAB function has its own local variables. These are separate from those of other functions (except for nested functions), and from those of the base workspace. Variables defined in a function do not remain in memory from one function call to the next, unless they are defined as global or persistent (The Mathwork, Inc., 2008). Examples of local variables here include K which is an element stiffness matrix local variable in the sub function Output.

3.5.7 Graphical User Interface

GUIDE, is the MATLAB graphical user interface development environment. Guide provides a set of tools for creating graphical user interfaces (GUIs). These tools simplify the process of designing and building GUIs (The Mathwork, Inc., 2008). GUIDE tools are used to:

i) Lay out the GUI.

Using the GUIDE Layout Editor, it is possible to lay out a GUI by clicking and dragging GUI components—such as panels, buttons, text fields, sliders, menus, and so on—into the layout area. GUIDE stores the GUI layout in a FIG-file Matlab Product Help (2008).

ii) Program the GUI.

GUIDE automatically generates an M-file that controls how the GUI operates. The M-file initializes the GUI and contains a framework for the most commonly used callbacks for each component—the commands that execute when a user clicks a GUI component. Using the M-file editor, you can add code to the callbacks to perform the functions you want Matlab Product Help (2008).

For this program guide tools were utilized in designing the GUI as seen in the first part of the code in the appendix.

3.5.8 Pseudo code

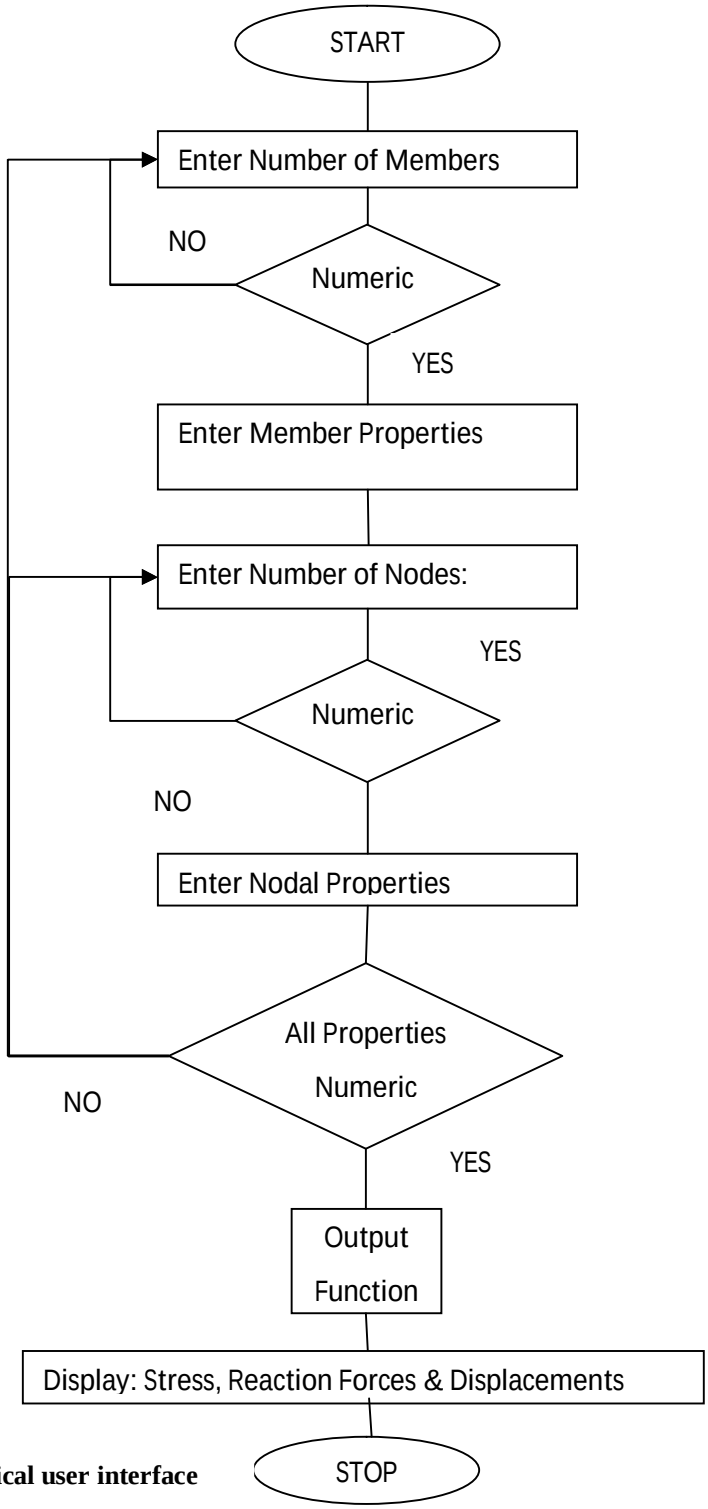


Figure 2: Graphical user interface

Explanation: Figure 1:

At the graphical user interface the user is required to enter numerical information on number of members, corresponding member properties, number of nodes and corresponding properties at the nodes. Where all the properties are entered correctly then they are passed by the checkbox to the Output function.

Output Function

Explanation: Figure 2:

Once the output function is called, the number of members is passed to 'for' loop to control the number of loops executed. The 'for' loop output's the element stiffness matrix (K). The element stiffness matrix is passed to the nested for loop which generates the globalised element stiffness matrix. This is done with the help of the conditional switch statement that selects the index for the global element stiffness matrix. The global element stiffness matrix is stored in the variable W in concatenated form. The stored matrix is used in preparing the master stiffness matrix found by summing all the global element stiffness matrices.

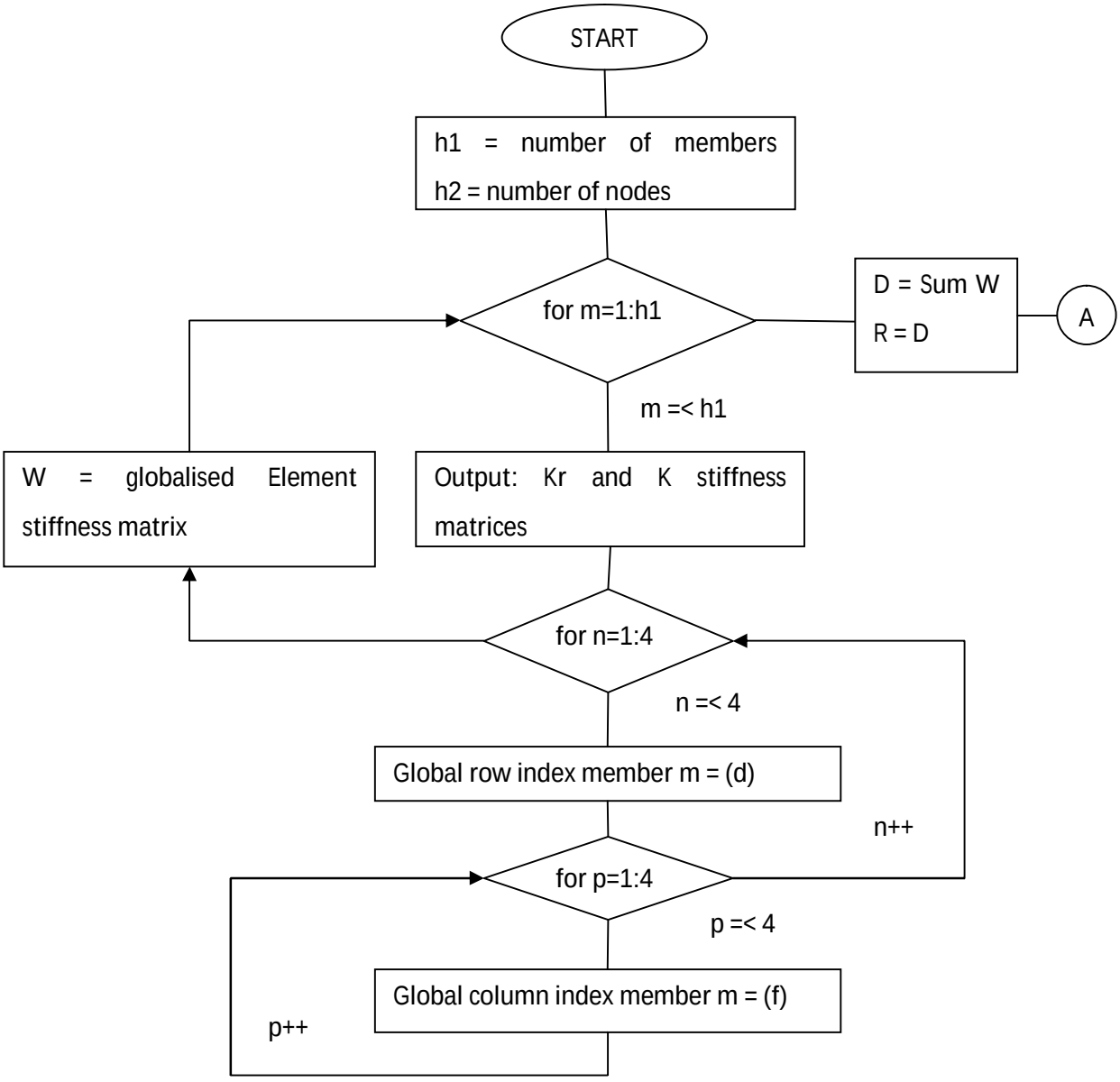


Figure 3: Ouput Sub function

Output Function Continued

Explanation Figure 3:

Once the master stiffness matrix is prepared, the nodal properties are evaluated by passing them to the for loop function. The for loop is meant to extract and separate the displacement boundary conditions (A) and Forces boundary conditions (F) from the user input. The variable b is meant to sum up values of rigid displacement boundary conditions and unbounded nodes.

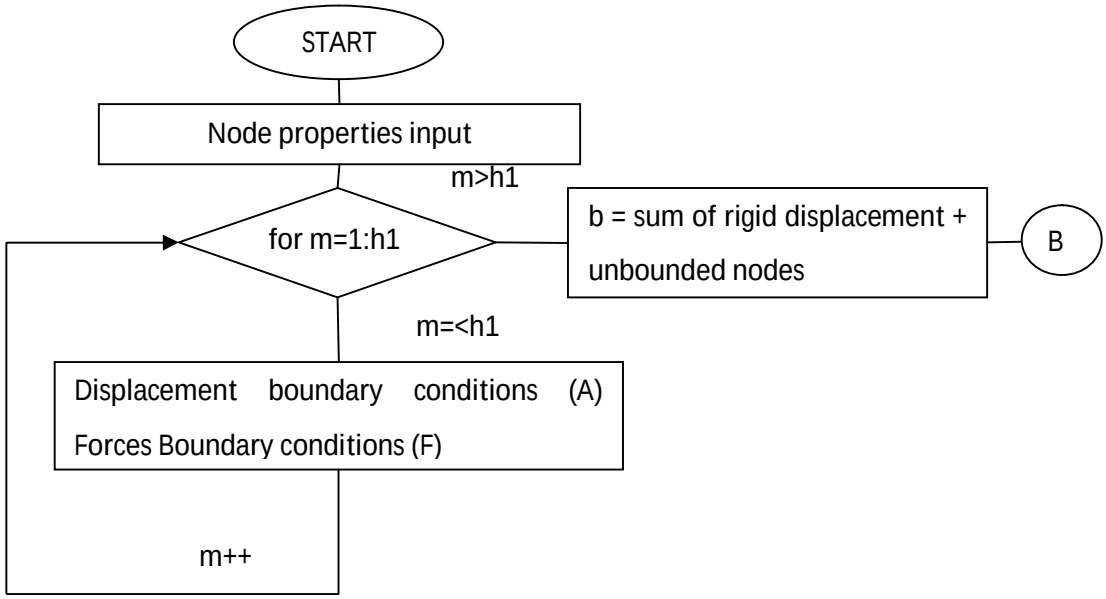


Figure 4: Output sub function

Output Function Continued

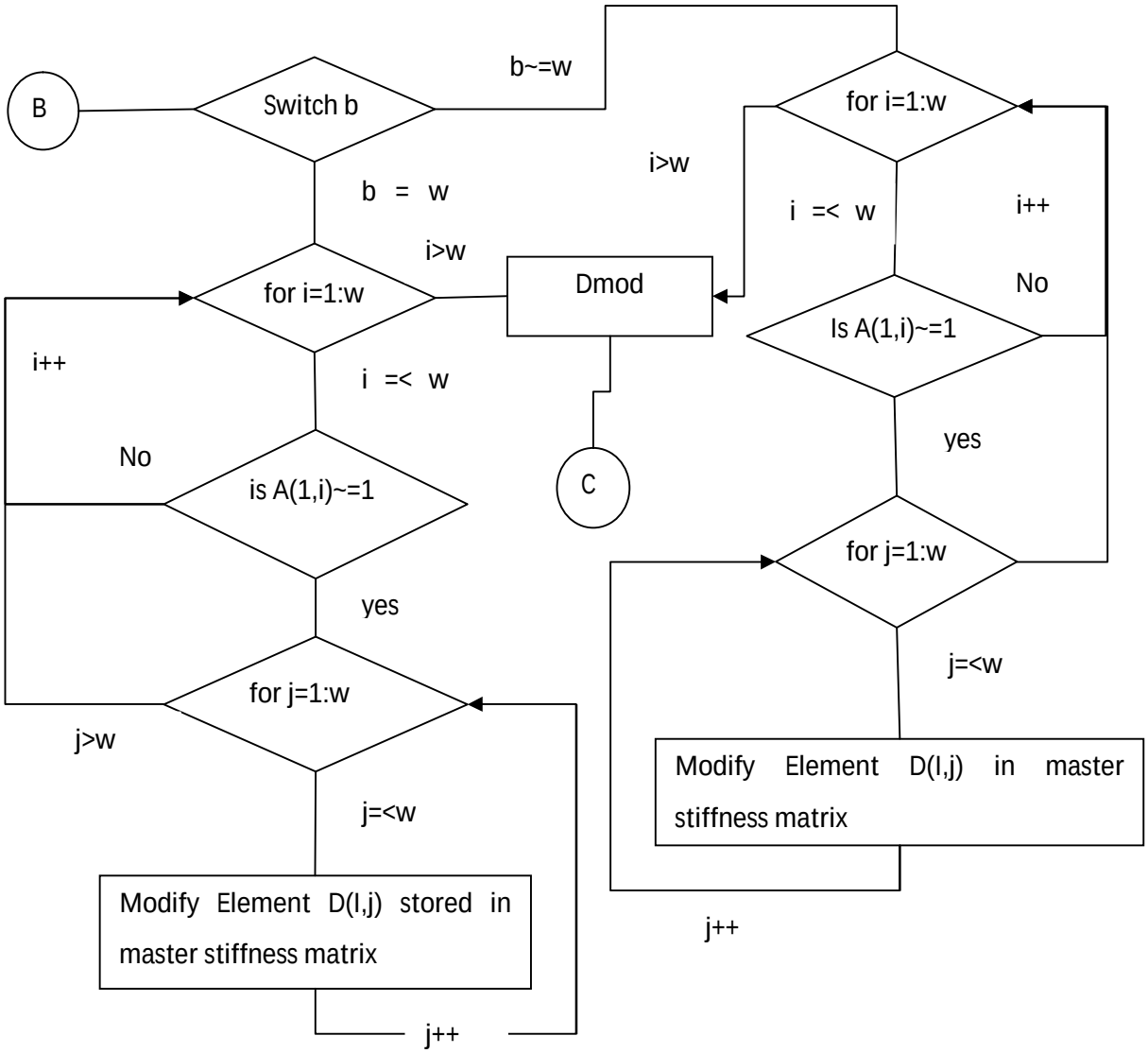


Figure 5: Output Function Continued

Explanation Figure 4 above:

Once the output variable b is obtained, it is passed to the conditional switch statement. The switch statement selects which function to execute when applying the displacement boundary

conditions on the master stiffness matrix (D) by modification or what is referred to in literature as penalty approach. The output from this is the modified master stiffness matrix (Dmod).

Output Function Continued.

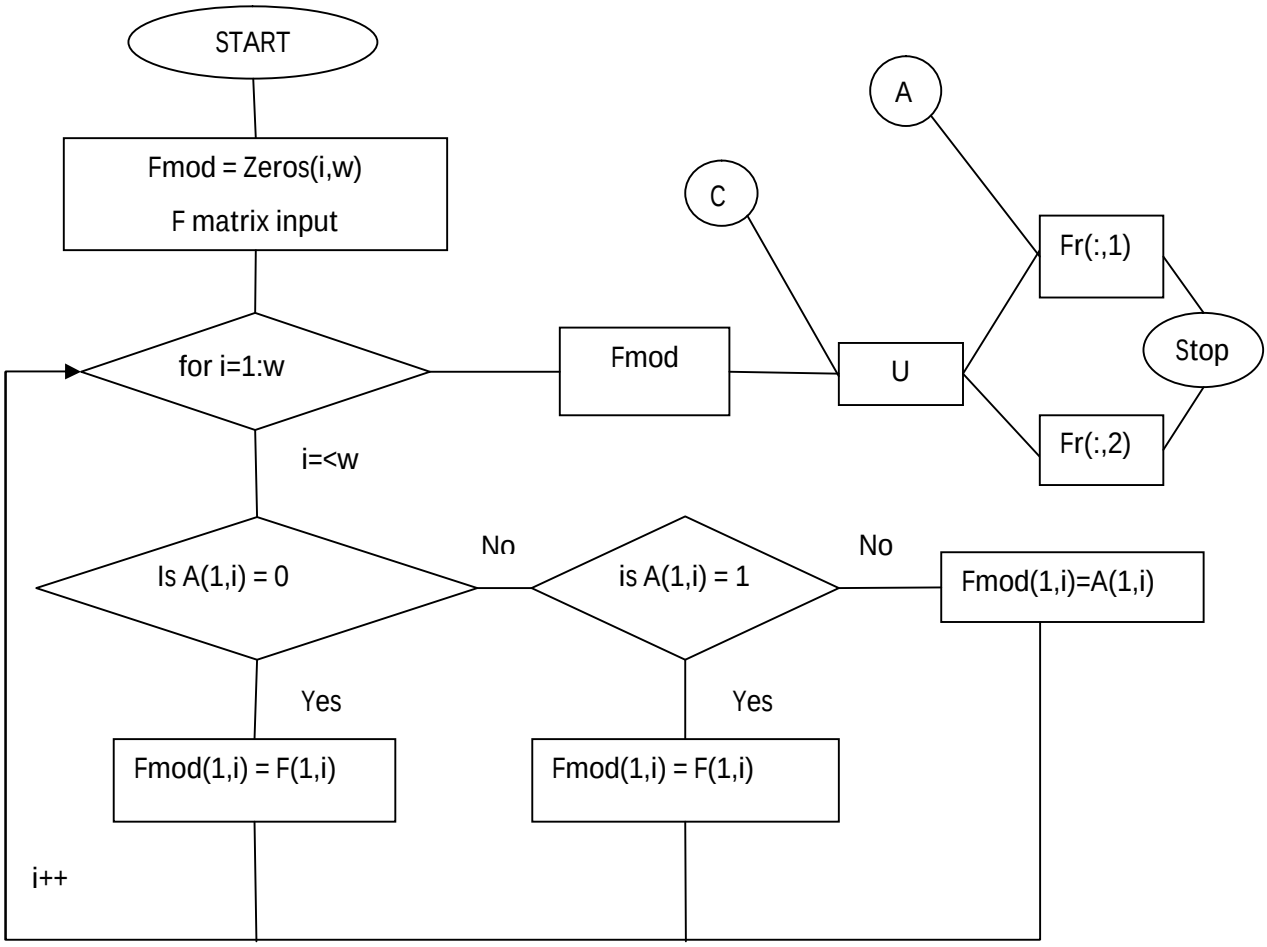


Figure 6: Output Function continued

Explanation Figure 5:

In this part of the program the modified force vector was initialized. Inputs to this part of the program were the force boundary conditions (F) separated as shown in figure 3 above. The force vector F is modified using the conditional if statement. The modified force vector is then output from this function. The resulting displacements (U) at the nodes are computed from product of

modified master stiffness matrix and the modified force matrix Fmod. The displacements are multiplied with the initial master stiffness matrix to generate the reaction forces at the nodes.

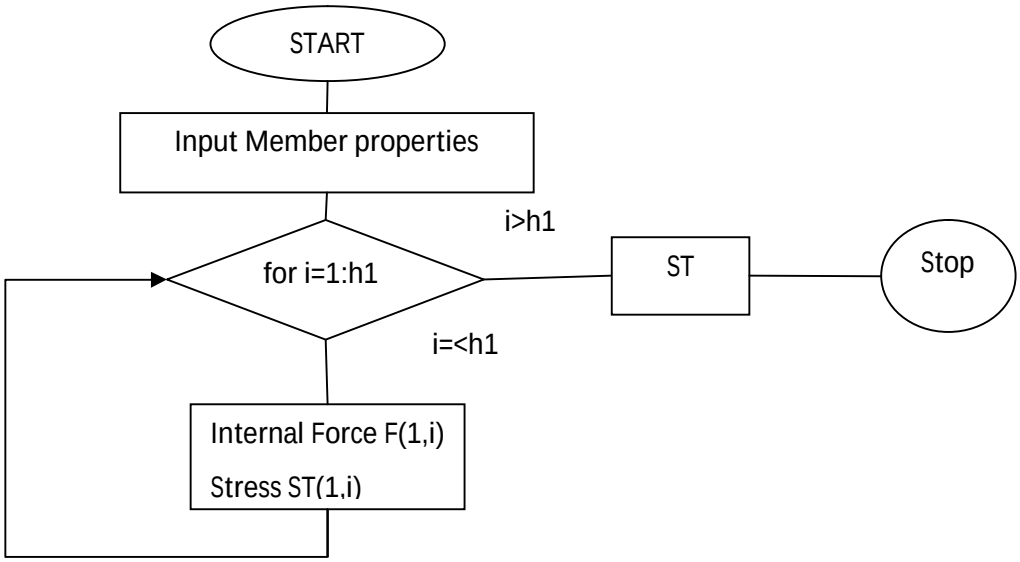


Figure 7: Output Function

Explanation Figure 6:

Once reaction forces are determined, member properties are input in this section to calculate the internal force (Fs) and internal stresses (st). Internal forces are computed by multiplying the displacement transformation matrix with each element displacement (ul) along the x direction. Internal stresses are computed by dividing the internal stresses with area.

CHAPTER FOUR

4.0 Code Validation

This involves verification of written code. Code validation was done using the following four illustrations:

4.1 Example 1

4.1.1 Classical Method of joints

Member Properties:

Young's Modulus 1000

Length L = 10m

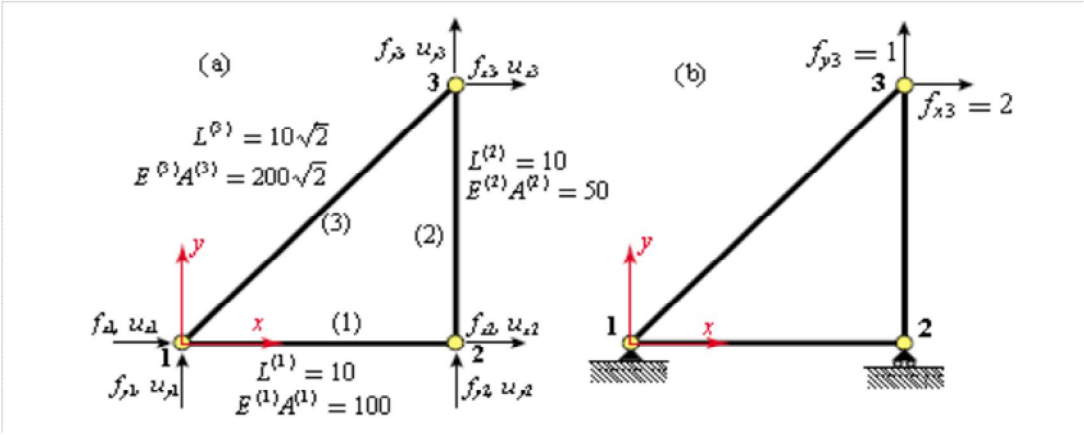


Figure 8: Pin-jointed idealization of example truss: (a) geometric and elastic properties, (b) support conditions and applied loads.

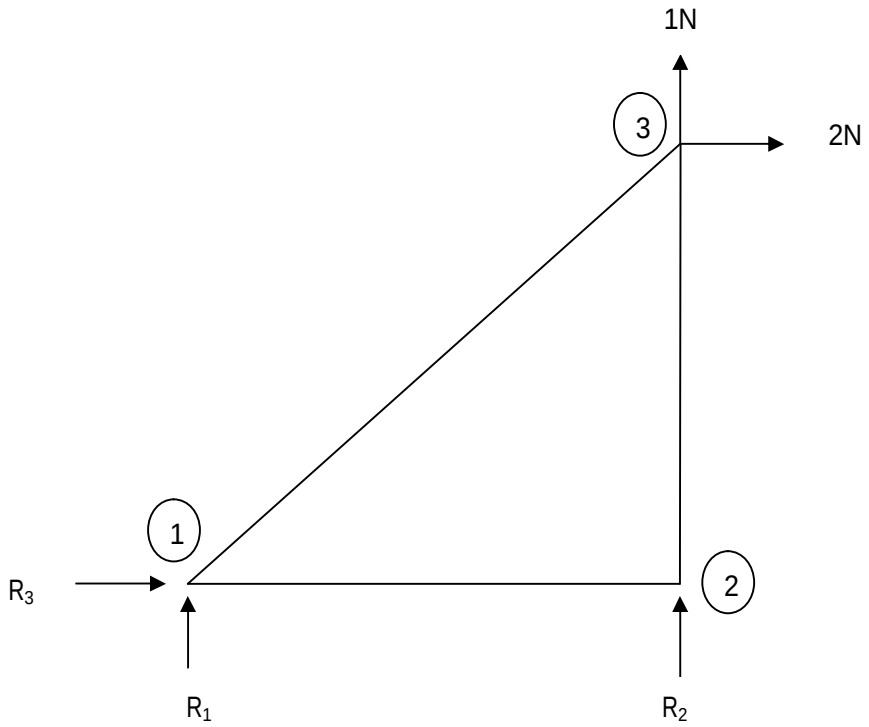


Figure 9: free body diagram Example 1

$$\sum F_x = 0; \quad gives: R_3 + 2 = 0; \quad \therefore R_3 = -2N$$

$$\sum F_y = 0; gives: R_1 + R_2 + 1 = 0; \therefore R_1 = -R_2 - 1N$$

$$\sum_1 M_Z = 0; gives: 10R_2 + 10(1) - 10(2) = 0; \therefore R_2 = \frac{10}{10} = 1N$$

$$\rightarrow R_1 = -R_2 - 1 = -1 - 1 = -2N$$

Joint 1

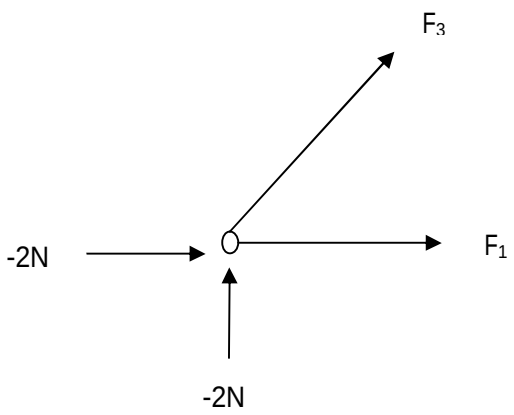


Figure 10: free body diagram joint 1

$$\sum_{1} F_X = 0 : F_1 + F_3 \cos 45^o - 2 = 0$$

$$\sum_{A} F_Y = 0 : F_3 \sin 45^o - 2 = 0, \qquad F_3 = \frac{2}{\sin 45} = 2\sqrt{2}N = 2.8284N,$$

$$F_1 = 2 - 2\sqrt{2} \times \frac{1}{\sqrt{2}} = 0$$

Joint 2

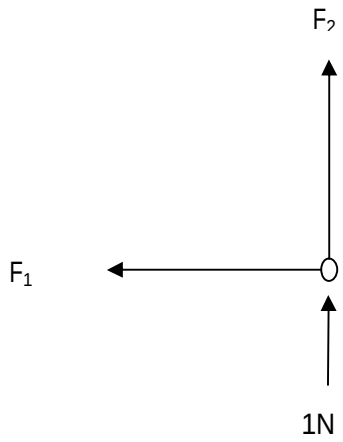


Figure 11: free body diagram joint 2

$$\sum_2 F_X = 0 \; gives, \qquad F_1 = 0$$

$$\sum_2 F_Y = 0, \qquad gives: \; F_2 + 1 = 0, \quad F_2 = -1$$

Internal Stresses

$$\sigma = \frac{F}{A}$$

Member 1 = $\frac{0}{0.1} = 0$

Member 2 = $\frac{-1}{0.05} = -20$

Member 3 = $\frac{2\sqrt{2}}{0.283} = 9.994$

4.1.2 Finite Element Methods

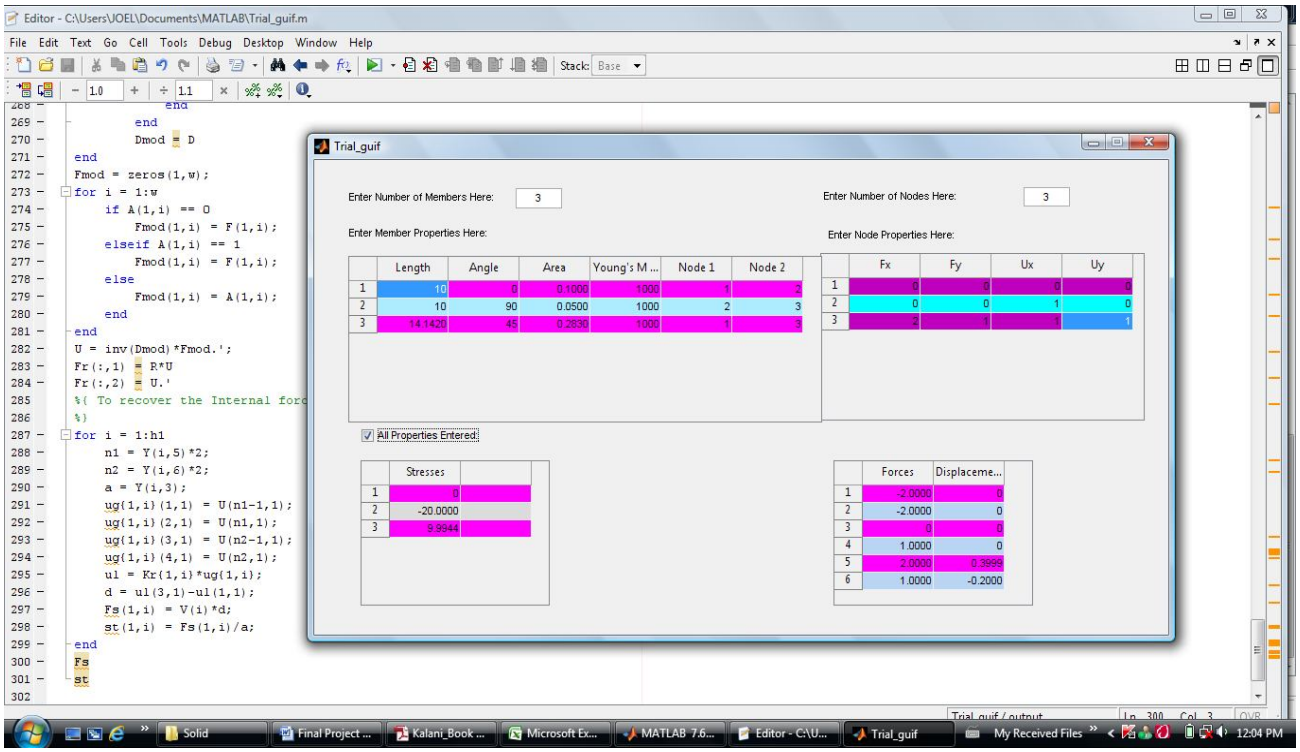


Figure 12: Graphical User Interface for Example 1

```
K =

    10     0   -10     0
     0     0     0     0
   -10     0    10     0
     0     0     0     0


K =

     0     0     0     0
     0     5     0    -5
     0     0     0     0
     0    -5     0     5


K =

   10.0057   10.0057  -10.0057  -10.0057
   10.0057   10.0057  -10.0057  -10.0057
  -10.0057  -10.0057   10.0057   10.0057
  -10.0057  -10.0057   10.0057   10.0057


Fr =

   -2.0000
   -2.0000
         0
    1.0000
    2.0000
    1.0000


Fs =

         0   -1.0000    2.8284


st =

         0  -20.0000    9.9944
```

Where

K- stiffness coefficient for element/member 1 to 3 sequentially

Fr- Nodal forces including reactions

Fs-Member Internal Forces

St-Member Internal Stresses

The reaction Forces are outlined hereunder:

Table 2: Reaction Forces Example 1:

Reaction Forces			
Reaction	**FEM(N)**	**Classical (N)**	**% difference**
1	-2	-2	0.00%
2	1	1	0.00%
3	-2	-2	0.00%

The internal stresses are outlined here under:

Table 3: Internal Stresses Example 1

Internal Stress			
Member	**FEM (N/m^2)**	**Classical (N/m^2)**	**% difference**
1	0	0	0%
2	-20	-20	0%
3	9.9944	9.994	0%

From the computer output it was found that the finite element stress results matched 100% with results from method of joints.

4.2 Example 2

4.2.1 Classical Method of Joints

Member Properties:

Young's Modulus 10,000

Cross-sectional Area .001m^2

Length a = 10m

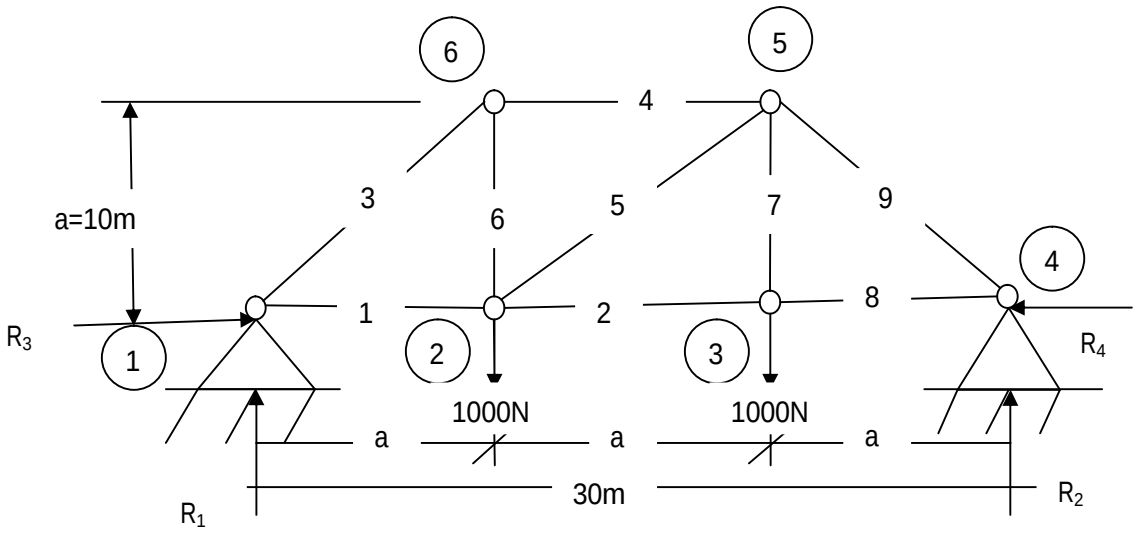


Figure 13: Free body diagram example 2

Setting R_4 as the redundant force

For equilibrium of external forces

$$\sum F_x = 0; \quad gives: R_3 = 0;$$

$$\sum F_y = 0; \; gives: R_1 + R_2 - 1000 - 1000 = 0; \; \therefore R_1 = -R_2 + 2000$$

$$\sum_1 M_Z = 0; \; gives: 30R_2 - 20(1000) - 10(1000) = 0; \; \therefore R_2 = \frac{30000}{30} = 1000KN$$

$$R_1 = -R_2 + 2000 = 1000$$

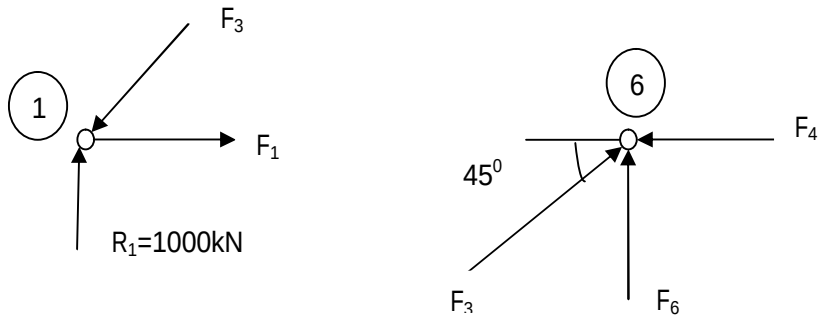


Figure 14: Free body diagram (a) Joint 1 (b) Joint 6

Joint 1

The free body diagram of joint 1 is shown in Figure 13 (a).

$$\sum_1 F_X = 0 : F_1 - F_3 \cos 45^o = 0$$

$$\sum_1 F_Y = 0 : -F_3 \sin 45^o + 1000 = 0, F_3 = 1000\sqrt{2} = 1414\, kN,$$

$$F_1 = 1000\sqrt{2} \times \frac{1}{\sqrt{2}} = 1000KN$$

Joint 6

The free body diagram of joint 6 is shown in Figure 13 (b).

$$\sum_6 F_X = 0\ gives, \quad F_3 \cos 45^0 - F_4 = 0, \quad F_4 = 1000\sqrt{2} \times \frac{1}{\sqrt{2}} = 1000kN$$

$$\sum_6 F_Y = 0, \quad gives : F_6 + F_3 \cos 45^0 = 0, \quad F_6 = -1000\sqrt{2} \times \frac{1}{\sqrt{2}} = 1000KN$$

Joint 2

The free body diagram for joint 2 is now prepared and is shown in figure 14

$$\sum_2 F_Y = 0\ gives, \quad F5 \cos 45^0 = 0, \quad F5 = 0$$

$$\sum_2 FX = 0\ gives, F2 = 1000kN$$

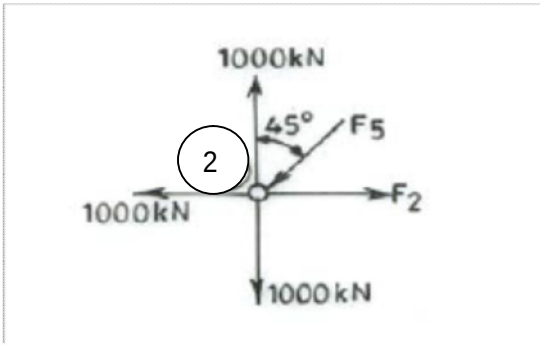


Figure 15: Free body diagram Joint 2

Setting R_4 as unit force acting then the equilibrium equations would yield

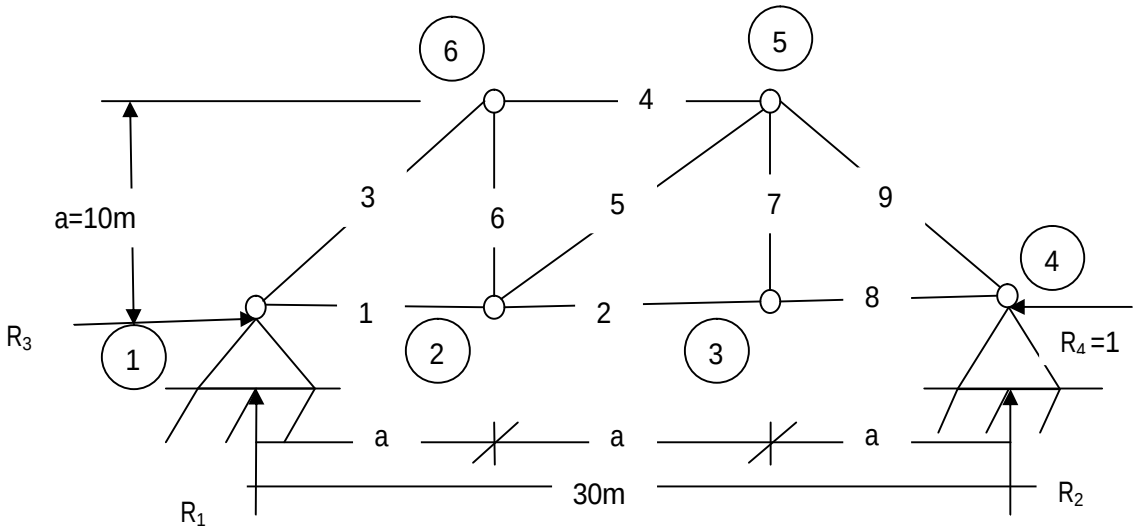


Figure 16: Free body diagram R_4 unit force

<u>For equilibrium of external forces</u>

$$\sum F_x = 0; \quad gives: R_3 = 1;$$

$$\sum F_y = 0; \; gives: R_1 + R_2 = 0;$$

$$\sum_1 M_Z = 0; \; gives: 30R_2 = 0; \; R_2 = 0$$

$$R_1 = 0$$

By inspection $F_8 = F_2 = F_1 = 1$

All other member forces being zero

Following the virtual work displacement method

$$\sum_{i=1}^{n} (\delta P)_i D_i = \sum_{j=1}^{m} \left(\delta F_p . \frac{F_D l}{E A} \right)$$

Table 4: Forces Analysis table

Member	Area	Length (l)	F0	F1	FoF1L/A	$F1^2L/A$	F(rs).R4	F(rs)0+F(rs).R1
1	0.001	10	1000	-1	-1000000	1000	-1000	0
2	0.001	10	1000	-1	-1000000	1000	-1000	0
3	0.001	14.142	-1414	0	0	0	0	-1414
4	0.001	10	1000	0	0	0	0	1000
5	0.001	14.142	0	0	0	0	0	0
6	0.001	10	1000	0	0	0	0	1000
7	0.001	10	1000	0	0	0	0	1000
8	0.001	10	1000	-1	-1000000	1000	-1000	0
9	0.001	14.142	-1414	0	0	0	0	-1414
Sum					-3000000	3000		

Table 5: Computing R_4 and Reactions

	δF_p	F_D	Sum	$(-\Delta_{10}/f_{11})$
Δ_{40}	F1	F0	-3000000	R_4 = 1000kN
f_{41}	F1	F1	3000	
Reactions				
	Rq0	**Rq1**	**Rq1.R1**	**Total Reactions**
1	1000	0	0	1000
2	1000	0	0	1000
3	0	1	1000	1000
4	0	-1	-1000	-1000

Internal Stresses

The internal stresses for figure 12 were as follows:

Table 6: Internal Stress Computation

Member	Area	F(rs)0+F(rs).R1	Internal stress (N/m^2)
1	0.001	0	0
2	0.001	0	0
3	0.001	-1414	-1414000
4	0.001	1000	1000000
5	0.001	0	0
6	0.001	1000	1000000
7	0.001	1000	1000000
8	0.001	0	0
9	0.001	-1414	-1414000

4.2.2 Finite Element methods

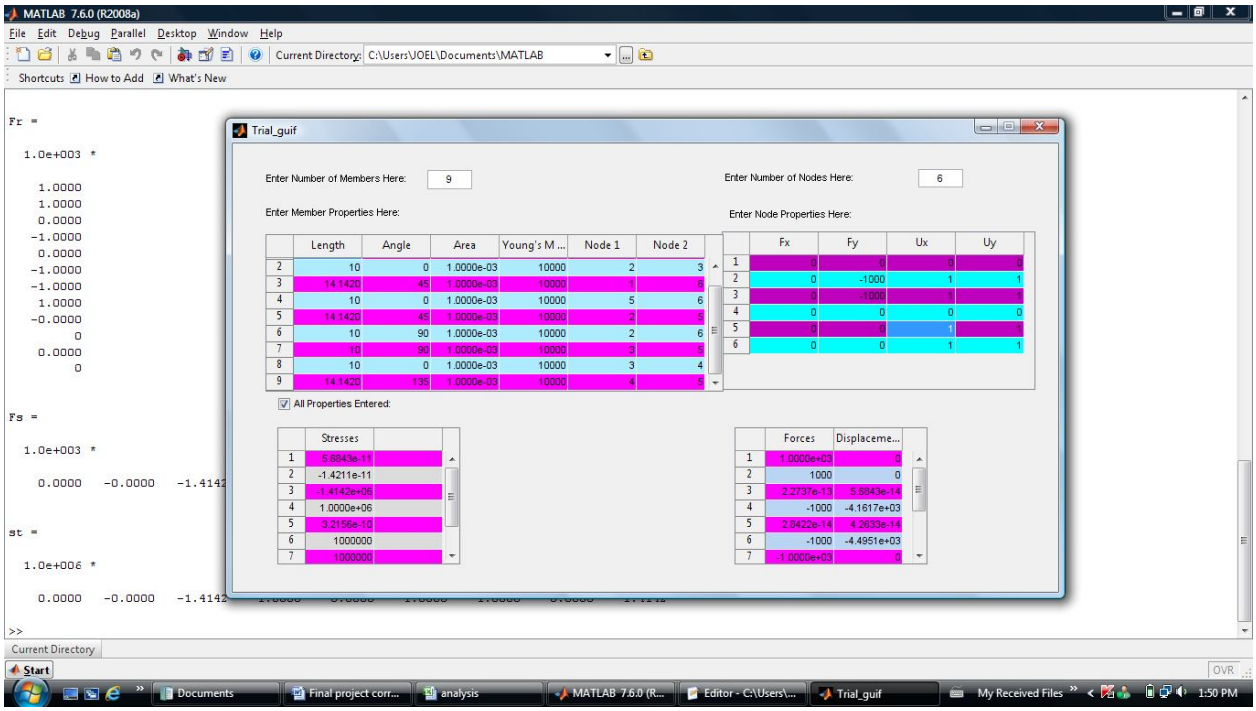


Figure 17: Graphical User Interface Example 2

K =

```
     1     0    -1     0
     0     0     0     0
    -1     0     1     0
     0     0     0     0

K =

     1     0    -1     0
     0     0     0     0
    -1     0     1     0
     0     0     0     0

K =

    0.3536    0.3536   -0.3536   -0.3536
    0.3536    0.3536   -0.3536   -0.3536
   -0.3536   -0.3536    0.3536    0.3536
   -0.3536   -0.3536    0.3536    0.3536

K =

     1     0    -1     0
     0     0     0     0
    -1     0     1     0
     0     0     0     0

K =

    0.3536    0.3536   -0.3536   -0.3536
    0.3536    0.3536   -0.3536   -0.3536
   -0.3536   -0.3536    0.3536    0.3536
   -0.3536   -0.3536    0.3536    0.3536

K =

     0     0     0     0
     0     1     0    -1
     0     0     0     0
     0    -1     0     1

K =

     0     0     0     0
     0     1     0    -1
     0     0     0     0
     0    -1     0     1

K =

     1     0    -1     0
     0     0     0     0
    -1     0     1     0
     0     0     0     0

K =

    0.3536   -0.3536   -0.3536    0.3536
   -0.3536    0.3536    0.3536   -0.3536
   -0.3536    0.3536    0.3536   -0.3536
    0.3536   -0.3536   -0.3536    0.3536
```

```
Fr =

  1.0e+003 *

    1.0000
    1.0000
    0.0000
   -1.0000
    0.0000
   -1.0000
   -1.0000
    1.0000
   -0.0000
         0
    0.0000
         0


Fs =

  1.0e+003 *

    0.0000   -0.0000   -1.4142    1.0000    0.0000    1.0000    1.0000   -0.0000   -1.4142


st =

  1.0e+006 *

    0.0000   -0.0000   -1.4142    1.0000    0.0000    1.0000    1.0000   -0.0000   -1.4142
```

Where

K- stiffness coefficient for element/member 1 to 9 sequentially

Fr- Nodal forces including reactions

Fs-Member Internal Forces

St-Member Internal Stresses

The reaction forces are outlined hereunder:

Table 7: Reaction Forces Example 2

Reaction Forces			
Reaction	**FEM(N)**	**Classical (N)**	**% difference**
1	1000	1000	0.00%
2	1000	1000	0.00%
4	1000	1000	0.00%
3	-1000	-1000	0.00%

The internal stresses are tabulated hereunder:

Table 8: Internal Stress Example 2

Internal Stress			
Member	**FEM(Mpa)**	**Classical (Mpa)**	**% difference**
1	0	0	0.00%
2	0	0	0.00%
3	-1.4142	-1.414	0.01%
4	1	1	0.00%
5	0	0	0.00%
6	1	1	0.00%
7	1	1	0.00%
8	0	0	0.00%
9	-1.4142	-1.414	0.01%

4.3 Example 3

4.3.1 Classical Method of joints

Member Properties:

Young's Modulus 200 X 10^6

Cross-sectional Area .0001m^2

Length of each member 10m

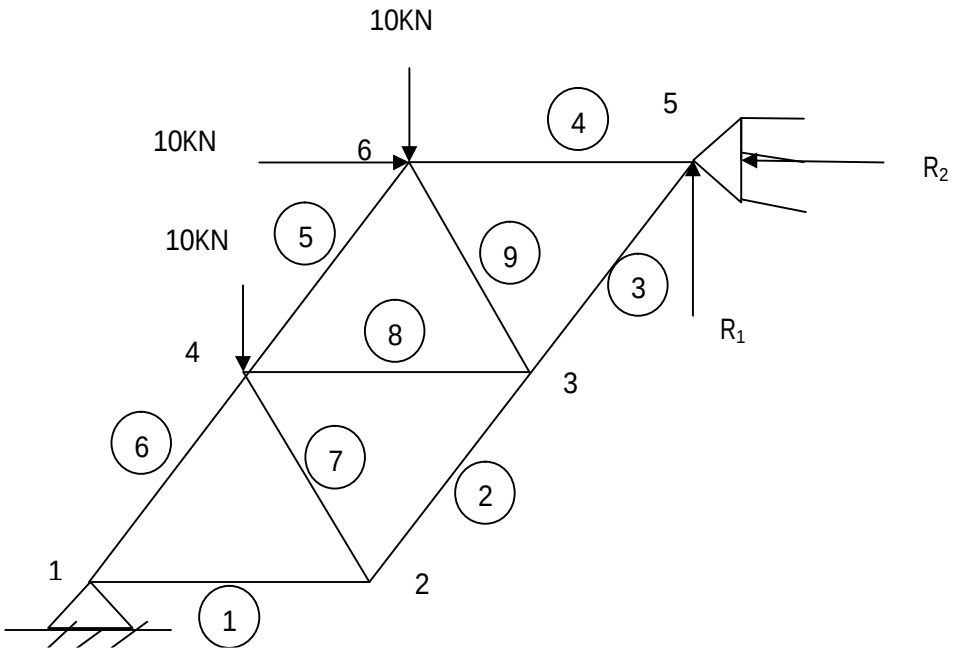


Figure 18: Example 3

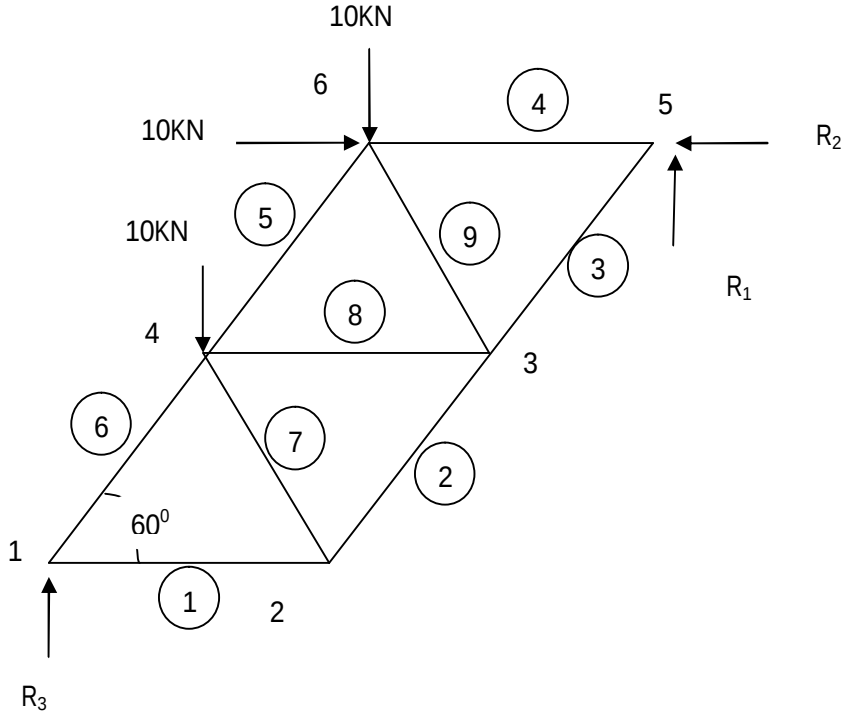


Figure 19: Free body diagram R_4 redundant

For equilibrium of external forces

$$\sum F_y = 0; \quad gives: R_3 + R_1 = 20; \; R_3 = 20 - R_1$$

$$\sum F_x = 0; \; gives: R_2 = 10KN;$$

$$\sum_5 M_Z = 0; \; gives: 20R_3 - 15(10) - 10(10) = 0;$$

$$20R_3 = 250$$

$$\therefore R_3 = \frac{250}{20} = 12.5KN$$

$$R_1 = 20 - (12.5) = 7.5KN$$

Joint 1

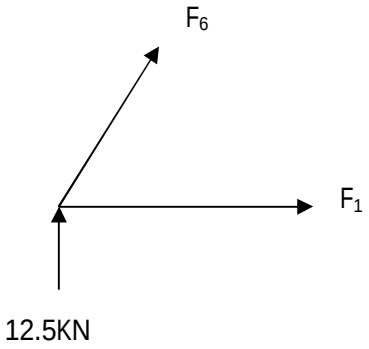


Figure 20: Free body diagram Joint 1

$$\sum_1 F_X = 0 : gives; \; F_6 \cos 60 + F_1 = 0$$

$$\sum_A F_Y = 0 : \; gives \; F_6 \sin 60^o + 12.5 = 0$$

$$F_6 = -14.4338\text{kN}$$

Substitute for F_6

$$F_1 = -F_6 \cos 60$$

$$F_1 = -(-14.4338)\, cos60 = 7.2169KN$$

Joint 5

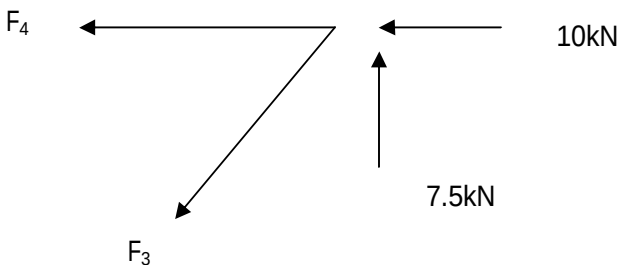


Figure 21: Free body diagram Joint 5

$$\sum_1 F_X = 0 : -F_4 - F_3\, cos\, 60^o - 10 = 0$$

$$\sum_A F_Y = 0 : -F_3\, sin\, 60^o + 7.5 = 0, \qquad F_3 = \frac{-7.5}{-\sin 60} = 8.6603kN,$$

$$F_4 = -10 - F_3\, cos\, 60^o$$

$$F_4 = -14.3301kN$$

Joint 2

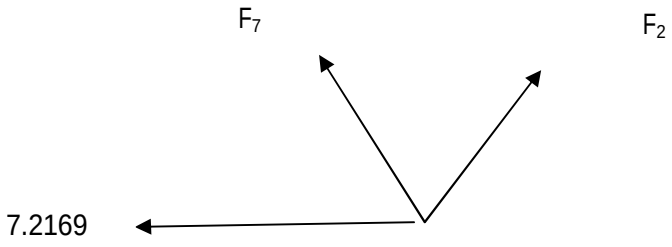


Figure 22: Free body diagram Joint 2

$$\sum_{1} F_X = 0 : gives; -F_7\, cos\, 60^o + F_2 \cos 60 - 7.2169 = 0$$

$$F_2 - F_7 = 14.4338KN$$

$$\sum_{A} F_Y = 0 : gives\ F_7\, sin\, 60^o + F_2 \sin 60 = 0,$$

$$F_7 + F_2 = 0$$

Solving simultaneously

F_7 = -7.2169KN

F_2 = 7.2169KN

Joint 3

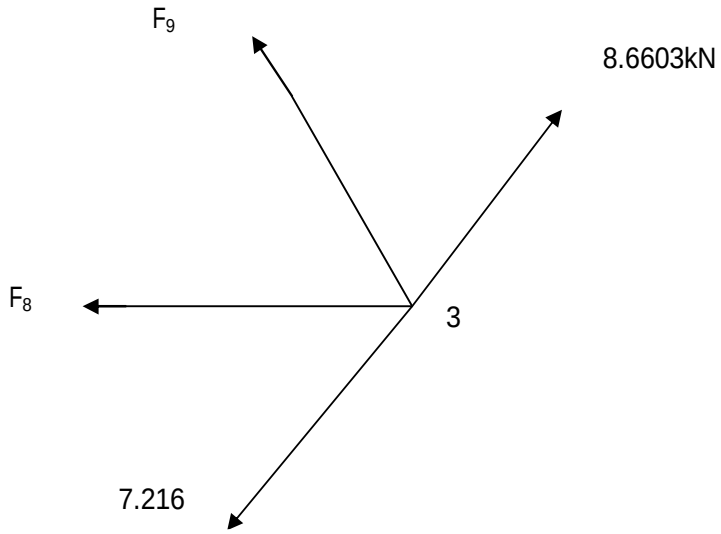


Figure 23: Free body diagram joint

$$\sum_{1} F_X = 0 : -F_9 \cos 60 - F_8 - 7.2169 \cos 60 + 8.6603 \cos 60 = 0$$

$$-0.5F_9 - F_8 = -0.7217 \ldots\ldots\ldots\ldots\ldots\ldots (i)$$

$$\sum_{A} F_Y = 0 : gives\ F_9 sin\, 60^o - 7.2169\, sin\, 60^o + 8.6603 \sin 60 = 0,$$

$$F_9 = -1.4434kN$$

$$-0.5F_9 + 0.7217 = F_8$$

$$-(-1.4434) + 0.7217 = F_8$$

$$F_8 = 1.4434kN$$

Joint 6

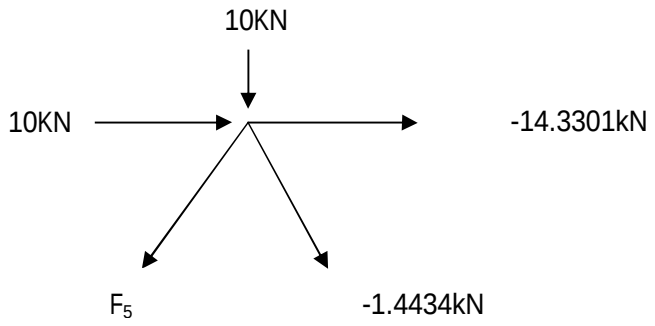


Figure 24: Free body diagram Joint 6

$$\sum_1 F_X = 0 : -F_5\, cos\, 60^o + 10 - 14.3301 - 1.4434 \cos 60 = 0$$

$$-0.5\, F_5 = 5.0518kN$$

$$F_5 = -10.1036kN$$

Setting R_4 as the redundant force

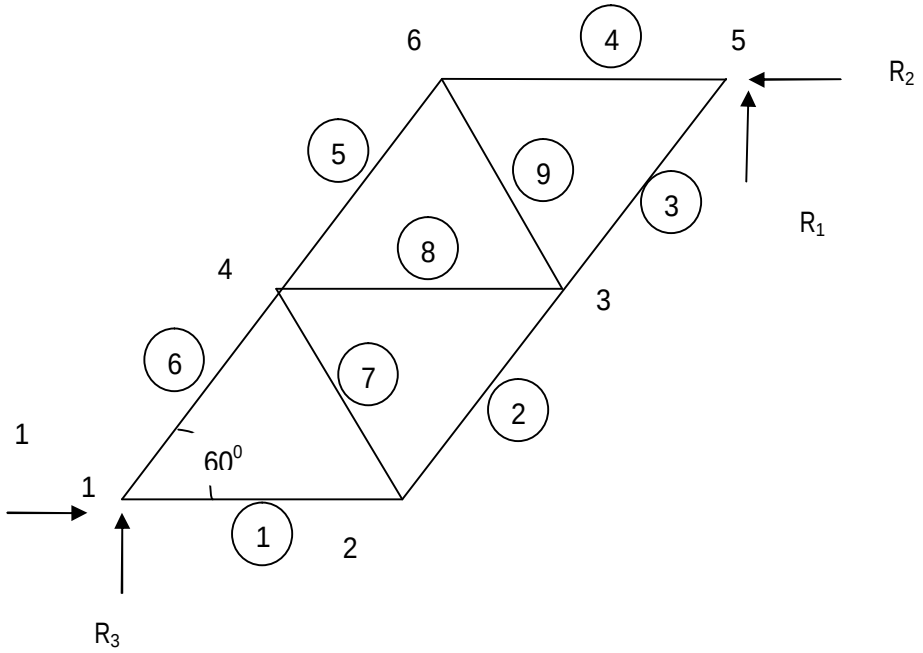


Figure 25: Free body diagram R_4 redundant

For equilibrium of external forces

$$\sum F_y = 0; \quad gives: R_3 + R_1 = 0;\ R_3 = -R_1$$

$$\sum F_x = 0;\ gives: R_2 = 1$$

$$\sum_5 M_Z = 0;\ gives: -17.3205(1) - 20(R_1) = 0;$$

$$-20R_1 = 17.3205$$

$$\therefore R_1 = \frac{17.3205}{-20} = -0.86603$$

$$R_3 = 0.86603$$

Joint 1

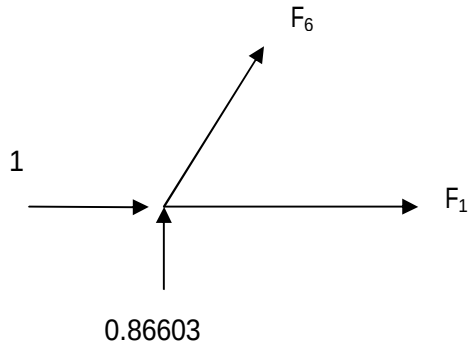


Figure 26: Free body diagram Joint 1

$$\sum_1 F_X = 0 : gives;\ F_6 \cos 60 + F_1 + 1 = 0$$

$$\sum_1 F_Y = 0 :\ gives;\ F_6\, sin\, 60^o + 0.866 = 0$$

$$F_6 = -1$$

Substitute for F_6

$$F_1 = -F_6 \cos 60 - 1$$

$$F_1 = -(-1)cos60 - 1 = -0.5$$

Joint 5

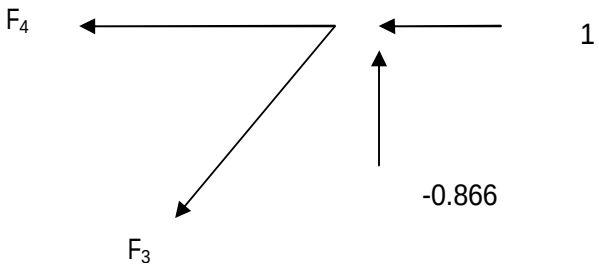


Figure 27: Free body diagram Joint 5

$$\sum_{1} F_X = 0 : -F_4 - F_3 \cos 60^o - 1 = 0$$

$$\sum_{A} F_Y = 0 : -F_3 \sin 60^o - 0.866 = 0, \qquad F_3 = \frac{0.866}{-\sin 60} = -1$$

$$F_4 = -1 - F_3 \cos 60^o$$

$$F_4 = -0.5$$

Joint 2

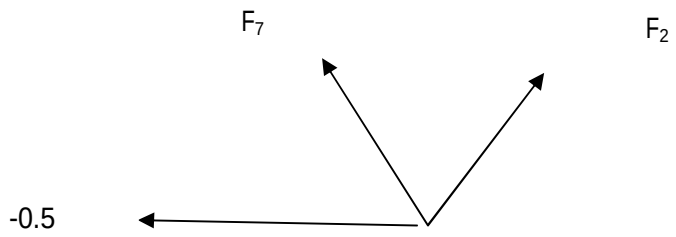


Figure 28: Free body diagram Joint 2

$$\sum_{1} F_X = 0 : gives; -F_7 \cos 60^o + F_2 \cos 60 + 0.5 = 0$$

$$F_2 - F_7 = -1$$

$$\sum_{A} F_Y = 0 : gives \; F_7 \sin 60^o + F_2 \sin 60 = 0,$$

$$F_7 + F_2 = 0$$

Solving simultaneously

$F_7 = 0.5$

$F_2 = -0.5$

Joint 6

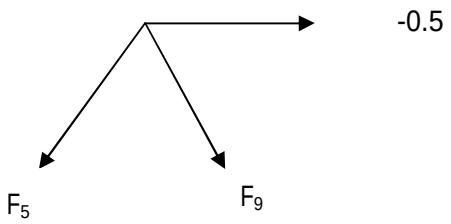


Figure 29: Free body diagram Joint 6

$$\sum_{1} F_X = 0 : -F_5\, cos\, 60^o - 0.5 + F_9 \cos 60 = 0$$

$$-F_5 + F_9 = 1$$

$$\sum_{A} F_Y = 0 : \; gives \; -F_9 sin\, 60^o - F_5\, sin\, 60^o = 0,$$

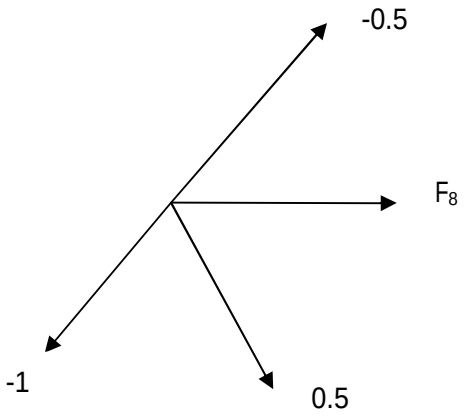
$$-F_9 - F_5 = 0$$

Solving simultaneously

F_5 = -0.5

F_9 = 0.5

Joint 4



$$\sum_{1} F_X = 0;\; gives;\; 1\cos 60 + F_8 - 0.5\cos 60 + 0.5\cos 60 = 0$$

$$F_8 = -0.5$$

Following the virtual work displacement method

$$\sum_{i=1}^{n} (\delta P)_i D_i = \sum_{j=1}^{m} \left(\delta F_p \cdot \frac{F_D l}{E A} \right)$$

Table 9: Analysis of Forces Example 3

Member	Area	Length (L)	F_0	F_1	F_0F_1L/A	F_1^2l/A	F(rs)*R4	$F(rs)_0$+F(rs).R4
1	0.0001	10	7216.9	-0.5	-36084500	2500	768.3467	7985.247
2	0.0001	10	7216.9	-0.5	-36084500	2500	768.3467	7985.247
3	0.0001	10	8660.3	-1	-86603000	10000	1536.693	10196.99
4	0.0001	10	-14403.1	-0.5	72015500	2500	768.3467	-13634.8
5	0.0001	10	-10103.6	-0.5	50518000	2500	768.3467	-9335.25
6	0.0001	10	-14433.8	-1	144338000	10000	1536.693	-12897.1
7	0.0001	10	-7216.9	0.5	-36084500	2500	-768.347	-7985.25
8	0.0001	10	1443.4	-0.5	-7217000	2500	768.3467	2211.747
9	0.0001	10	-1434.4	0.5	-7172000	2500	-768.347	-2202.75
Sum					**57626000**	**37500**		

Table 10: R_4 Computation and Reactions Forces

	δFp	F_D	Sum	$-\Delta 10/f_{11}$
Δ10	F1	F0	57626000	-1536.69
f_{11}	F1	F1	37500	
Reactions				
	Rq0	**Rq1**	**Rq1.R1**	**Total**
1	7500	-0.86603	1330.823	8830.823
2	10000	1	-1536.69	8463.307
3	12500	0.86603	-1330.82	11169.18
4		1	-1536.69	-1536.69

Table 11: Internal Stress Computation

Member	Area	F(rs)0+F(rs).R1	Internal stress (N/m^2)

1	0.0001	7985.247	79852466.7
2	0.0001	7985.247	79852466.7
3	0.0001	10196.99	101969933
4	0.0001	-13634.8	-136347533
5	0.0001	-9335.25	-93352533
6	0.0001	-12897.1	-128971067
7	0.0001	-7985.25	-79852467
8	0.0001	2211.747	22117466.7
9	0.0001	-2202.75	-22027467

4.3.2 Finite Element Method

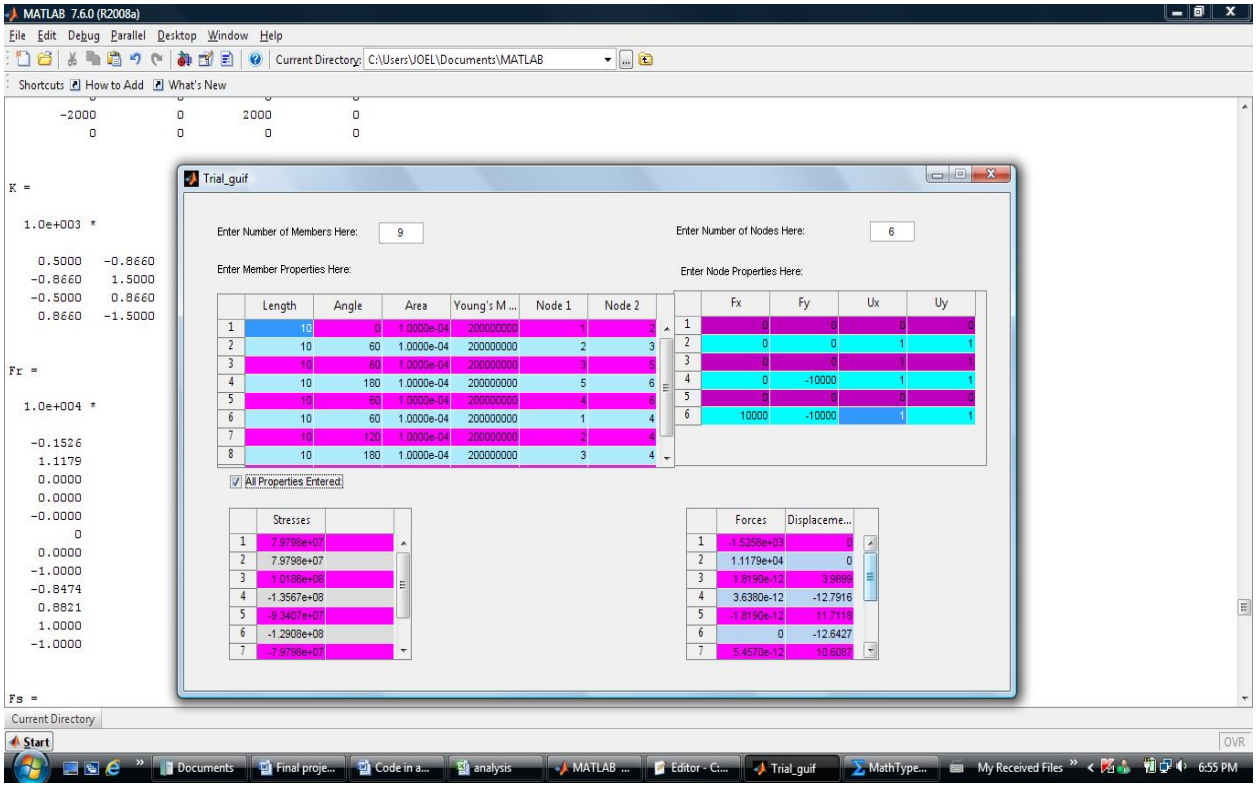


Figure 30: Graphical User Interface Example 3

```
K =

        2000           0       -2000           0
           0           0           0           0
       -2000           0        2000           0
           0           0           0           0
```

```
K =

  1.0e+003 *

    0.5000    0.8660   -0.5000   -0.8660
    0.8660    1.5000   -0.8660   -1.5000
   -0.5000   -0.8660    0.5000    0.8660
   -0.8660   -1.5000    0.8660    1.5000


K =

  1.0e+003 *

    0.5000    0.8660   -0.5000   -0.8660
    0.8660    1.5000   -0.8660   -1.5000
   -0.5000   -0.8660    0.5000    0.8660
   -0.8660   -1.5000    0.8660    1.5000


K =

        2000           0       -2000           0
           0           0           0           0
       -2000           0        2000           0
           0           0           0           0


K =

  1.0e+003 *

    0.5000    0.8660   -0.5000   -0.8660
    0.8660    1.5000   -0.8660   -1.5000
   -0.5000   -0.8660    0.5000    0.8660
   -0.8660   -1.5000    0.8660    1.5000


K =

  1.0e+003 *

    0.5000    0.8660   -0.5000   -0.8660
    0.8660    1.5000   -0.8660   -1.5000
   -0.5000   -0.8660    0.5000    0.8660
   -0.8660   -1.5000    0.8660    1.5000


K =

  1.0e+003 *

    0.5000   -0.8660   -0.5000    0.8660
   -0.8660    1.5000    0.8660   -1.5000
   -0.5000    0.8660    0.5000   -0.8660
    0.8660   -1.5000   -0.8660    1.5000


K =

        2000           0       -2000           0
           0           0           0           0
       -2000           0        2000           0
           0           0           0           0


K =

  1.0e+003 *
```

```
    0.5000   -0.8660   -0.5000    0.8660
   -0.8660    1.5000    0.8660   -1.5000
   -0.5000    0.8660    0.5000   -0.8660
    0.8660   -1.5000   -0.8660    1.5000

Fr =

  1.0e+004 *

   -0.1526
    1.1179
    0.0000
    0.0000
   -0.0000
         0
    0.0000
   -1.0000
   -0.8474
    0.8821
    1.0000
   -1.0000

Fs =

  1.0e+004 *

    0.7980    0.7980    1.0186   -1.3567   -0.9341   -1.2908   -0.7980    0.2206   -0.2206

st =

  1.0e+008 *

    0.7980    0.7980    1.0186   -1.3567   -0.9341   -1.2908   -0.7980    0.2206   -0.2206
```

Where

K- stiffness coefficient for element/member 1 to 9 sequentially

Fr- Nodal forces including reactions

Fs-Member Internal Forces

St-Member Internal Stresses

The Reaction Forces are outlined here under:

Table 12: Reaction Forces Example 3

Reaction Forces			
Reaction	**FEM(N)**	**Classical (N)**	**% difference**

1	8821	8830.823	-0.11%
2	-8474	-8463.31	0.13%
3	11179	11169.18	0.09%
4	-1526	-1536.69	-0.70%

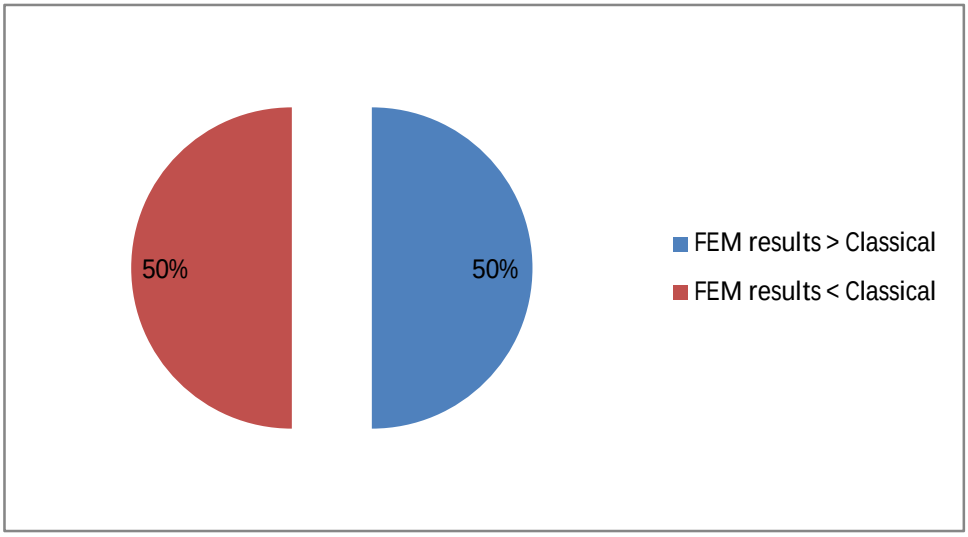


Figure 31: Comparison of Reaction Forces (FEM vs Classical)

The Internal Stresses are outlined here under:

Table 13: Internal Stress Example 3

Internal Stress			
Member	**FEM(Mpa)**	**Classical (Mpa)**	**% difference**
1	79.8	79.852	-0.07%
2	79.8	79.852	-0.07%
3	101.86	101.97	-0.11%
4	-135.67	-136.348	-0.50%
5	-93.41	-93.352	0.06%
6	-129.08	-128.971	0.08%
7	-79.8	-79.852	-0.07%
8	22.06	22.117	-0.26%
9	-22.06	-22.027	0.15%

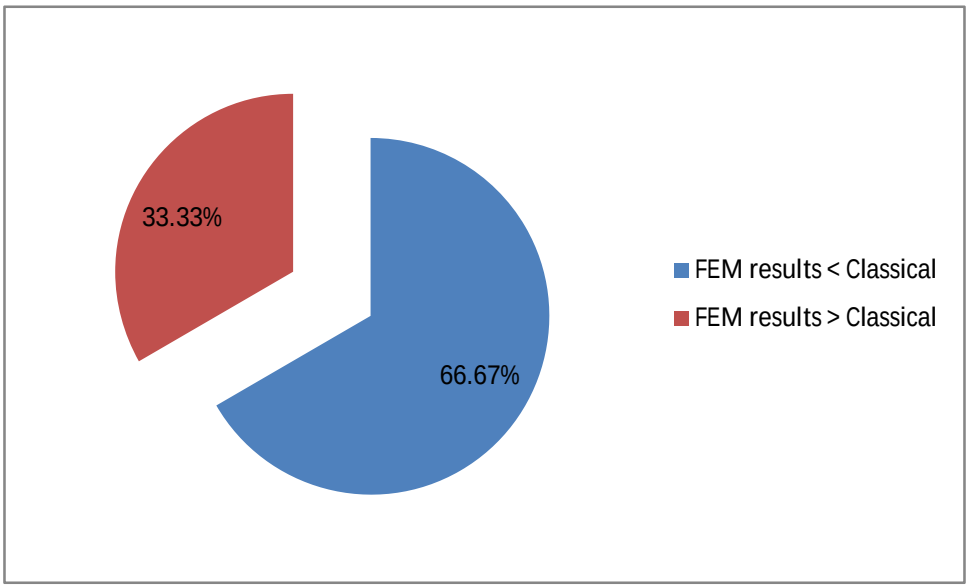


Figure 32: Internal Stresses Results Comparison

4.4 Example 4

4.4.1 Classical Strain Energy Method

There are three members meeting at joint 2 and two equations of equilibrium, hence, static indeterminacy is 1. Let 2-4 be redundant member. It is removed to obtain determinate system as shown in figure 4.24 (b):

Properties

Youngs Modulus (E) 200 X 10^6

Cross-sectional area A = 0.001m^2

Length L = 10m.

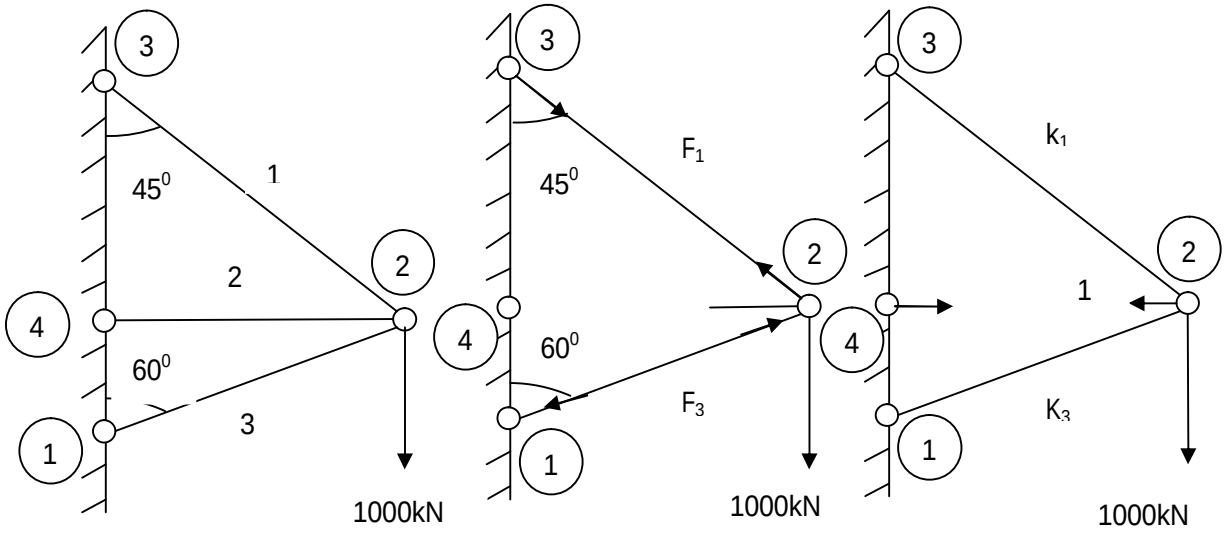


Figure 33: Example 4

Joint 2

$$F_1 Sin\,45 + F_3 Sin\,30 = 1000$$
$$F_1 Cos\,45 = F_1 Cos\,30$$
$$F_1 = \frac{\sqrt{3}\sqrt{2}}{2} F_3 = \sqrt{\frac{3}{2}}\, F_3$$
$$\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) F_3 = 1000$$
$$F_3 = -\frac{2000}{\left(\sqrt{3}+1\right)} = -732.051$$
$$F_1 = 896.576$$

Apply unit actions corresponding to unknown tensile force in 2-4 as shown in figure 13

Joint 2

$$k_1 Sin\,45 = k_3 Sin\,30$$
$$k_3 = 2\,k_1$$
$$k_1 Cos\,45 = k_3 Cos\,30 = 1$$
$$\frac{k_1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\sqrt{2}k_1 = 1$$
$$k_1\left(1+\sqrt{3}\right) = \sqrt{2}$$

$$k_1 = -\frac{\sqrt{2}}{(1+\sqrt{3})} = -\frac{1.4142}{2.7321} = -0.518$$

$$k_3 = -\frac{2}{(1+\sqrt{3})} = -0.732$$

$$X = -\frac{\sum \frac{FkL}{AE}}{\sum \frac{k^2 L}{AE} + \frac{L_0}{A_0 E}} = \frac{F_1 k_1 L_1 + F_1 k_1 L_1}{{k_1}^2 L_1 + {k_3}^2 L_3 + L_0}$$

$$X = -\frac{896.576(-0.518)\sqrt{2}L + (-732.051)(-0.732)\frac{2}{\sqrt{3}}L}{(-0.518)^2\sqrt{2}L + (-0.732)^2\frac{2}{\sqrt{3}}L + L} = \frac{38.039}{1.998} = 19.039kN$$

$$F_1{}' = F_1 + k_1 X = 896.576 - 0.518 * 19.039 = 886.714kN$$

$$F_2' = X = 19.039kN$$

$$F_3' = F_3 + k_3 X = -732.051 - 0.732 * 19.039 = -745.988kN$$

Reaction Forces

Joint 1

$R_{x1} = R_1 = -F_3 \cos 30 = -(-745.988)\cos 30 = 646.045$kN

$R_{y1} = R_2 = -F_3 \cos 60 = -(-745.988) \cos 60 = 372.994$kN

Joint 4

$R_{x4} = R_3 = F_3 = -19.039$kN

$R_{y4} = R_4 = 0$

Joint 3

$R_{x3} = R_5 = -F_1 \text{ Cos } 45 = -(886.714) \text{ Cos } 45 = -627.001kN$

$R_{y3} = R_6 = F_1 \sin 45 = 627.001kN$

Member Stresses;

Member 1 = $\frac{886714}{0.0001} = -8667.14\ Mpa$

Member 2 = $\frac{19039}{0.0001} = 190.39\ Mpa$

Member 3 = $\frac{-745988}{0.0001} = -7460 Mpa$

4.4.2 Finite element Methods

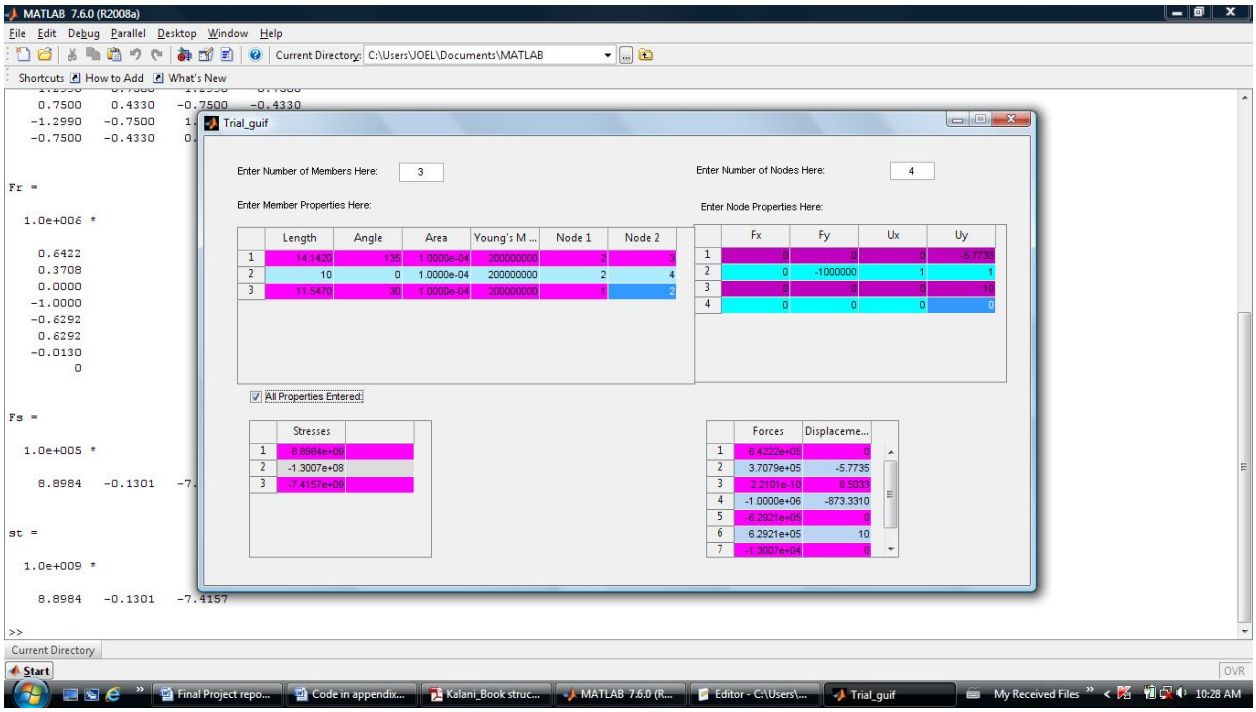


Figure 34: Graphical User Interface Example 4

```
K =

  707.1136 -707.1136 -707.1136  707.1136
 -707.1136  707.1136  707.1136 -707.1136
 -707.1136  707.1136  707.1136 -707.1136
  707.1136 -707.1136 -707.1136  707.1136
```

```
K =

        2000           0       -2000           0
           0           0           0           0
       -2000           0        2000           0
           0           0           0           0


K =

  1.0e+003 *

    1.2990    0.7500   -1.2990   -0.7500
    0.7500    0.4330   -0.7500   -0.4330
   -1.2990   -0.7500    1.2990    0.7500
   -0.7500   -0.4330    0.7500    0.4330


Fr =

  1.0e+006 *

    0.6422
    0.3708
    0.0000
   -1.0000
   -0.6292
    0.6292
   -0.0130
         0


Fs =

  1.0e+005 *

    8.8984   -0.1301   -7.4157


st =

  1.0e+009 *

    8.8984   -0.1301   -7.4157
```

Where

K- stiffness coefficient for element/member 1 to 3 sequentially

Fr- Nodal forces including reactions

Fs-Member Internal Forces in (N)

St-Member Internal Stresses (N)

The reaction forces are mentioned hereunder

Table 14: Reaction Forces Example 4

Reaction Forces			
	FEM(N)	**Classical (N)**	**% difference**
1	642200	646045	-0.60%
2	370800	372994	-0.59%
3	-13000	-19039	0.00%
4	0	0	0.00%
5	-629200	-627001	0.35%
6	629200	627001	0.35%

The internal stresses are mentioned hereunder:

Table 15: Internal Stress Example 4

Internal Stress			
Member	**FEM (Mpa)**	**Classical (Mpa)**	**% difference**
1	8898.4	8667.14	2.60%
2	-130.1	190.39	246.34%
3	-7415.7	-7460	-0.60%

4.5 Discussion of Results

The analysis of the examples was done using both the finite element methods and Classical method of joints and virtual work displacement method. Finite element method's analysis was conducted in the form of computer simulation in Matlab programming language. Further analysis of the reactions and internal stresses was done using excel software for tabulating the results.

For example 1 above it is important to note that the results from finite element methods were exactly the same as those obtained from classical method of joints for both reactions and internal forces as shown in table 2 and 3 above. This goes hand in hand with what theory of Finite element methods predicts. However it should also be noted that exact results are obtained only for the three truss members in a determinate system.

Example 2 above was a statically indeterminate truss structure. The study revealed that differences of about 0.01% existed between internal stress results from finite element methods and the classical method of joints and virtual work displacement method in the stress analysis as shown in table 8. This difference could have been brought about by rounding off of values during calculation. As regards external reactions the results were exact with no differences observed as shown in table 7 above.

For example 3 above the study revealed percentage difference lying between 0.15% and -0.5%. 66.67% of the internal stresses estimated using FEM were below values estimated using classical methods. 33.33% internal stress results being valued higher than those by classical methods as shown in table 13 above. The study also revealed differences in values of reaction forces estimated lying between -0.7% and 0.13%. 50% of the reaction forces were above values estimated using classical methods while 50% lay below. This stress and reaction results confirmed indeed that finite element method was an approximate method.

For example 4 above, which is a statically indeterminate system a significant difference of 246.34% existed in member 2. This was attributable to the orientation of internal forces in member 2 as shown in figure 32. Else the magnitudes were comparable and results reliable for design purposes. Other members' results were comparable between the finite element methods and classical method with a difference in stresses of magnitude below 2.6%. As regards external reaction forces, differences existed to the extent of 0.6% as shown in table 14.

CHAPTER FIVE

5.0 Conclusions and Recommendations

5.1 Conclusions

As pertains whether finite element analysis could be conducted on two dimensional determinate and indeterminate trusses, the study revealed that the finite element analysis for two dimensional determinate and indeterminate trusses could be performed yielding approximate results.

As regards how finite element analysis results compare with classical methods, the study indicated most differences being less than 1% magnitude. Hence the two methods were comparable. This confirmed that finite element method was an approximate method and not an exact method.

Furthermore, as pertains whether finite element analysis for two dimensional determinate and indeterminate trusses could be modeled in Matlab programming language; it was possible to confirm that Finite element analysis could be simulated using Matlab programming language. All the results mentioned here-in were obtained through finite element analysis based on the code attached here-in the appendix.

As regards the efficiency and effectiveness of finite element analysis of two dimensional determinate and indeterminate trusses, computer simulation of the finite element analysis of trusses revealed the following advantages:

i) High speed of executing the analysis task hence would save time for users. As compared to the classical methods.
ii) Results obtained were accurate hence it improves reliability in the results obtained especially when dealing with complex truss structures.
iii) Use of user friendly graphical interfaces would aid in minimizing errors in input and hence reliable results obtained.

iv) It would be possible to conduct sensitivity analysis of various proposed designs at minimal time and cost. Hence would be very useful to structural and mechanical engineers during their designs.

5.2 Recommendations

Based on the study conducted the following recommendations were made:

i) The project could be extended to three dimensional truss structures by other students willing to undertake the project using Matlab programming language or other numerical software such as octave and Mathematica.
ii) The project could be extended by willing students in investigating the possibility of integrating results directly from Drawing software such as AutoCad and Rhino to aid in analysis while designing.
iii) The program could be extended by including a database of various materials and corresponding yield stress values. Based on the output, the materials would then be suggested to users while satisfying predesigned factor of safety.
iv) The program could also be improved upon by investigating possible ways of optimization the program on computer hardware memory.
v) Following the fact that most users have access to mobile phones, willing students would extend the project by configuring it to a phone application for ease of access and convenience when carrying it.

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http://www.math.mtu.edu

Appendix

```
function varargout = Trial_guif(varargin)
% TRIAL_GUIF M-file for Trial_guif.fig
%      TRIAL_GUIF, by itself, creates a new TRIAL_GUIF or raises the existing
%      singleton*.
%
%      H = TRIAL_GUIF returns the handle to a new TRIAL_GUIF or the handle to
%      the existing singleton*.
%
%      TRIAL_GUIF('CALLBACK',hObject,eventData,handles,...) calls the local
%      function named CALLBACK in TRIAL_GUIF.M with the given input
arguments.
%
%      TRIAL_GUIF('Property','Value',...) creates a new TRIAL_GUIF or raises
the
%      existing singleton*.  Starting from the left, property value pairs are
%      applied to the GUI before Trial_guif_OpeningFcn gets called.  An
%      unrecognized property name or invalid value makes property application
%      stop.  All inputs are passed to Trial_guif_OpeningFcn via varargin.
%
%      *See GUI Options on GUIDE's Tools menu.  Choose "GUI allows only one
%      instance to run (singleton)".
%
% See also: GUIDE, GUIDATA, GUIHANDLES

% Edit the above text to modify the response to help Trial_guif

% Last Modified by GUIDE v2.5 06-May-2012 00:42:31

% Begin initialization code
gui_Singleton = 1;
gui_State = struct('gui_Name',       mfilename, ...
                   'gui_Singleton',  gui_Singleton, ...
                   'gui_OpeningFcn', @Trial_guif_OpeningFcn, ...
                   'gui_OutputFcn',  @Trial_guif_OutputFcn, ...
                   'gui_LayoutFcn',  [] , ...
                   'gui_Callback',   []);
if nargin && ischar(varargin{1})
    gui_State.gui_Callback = str2func(varargin{1});
end

if nargout
    [varargout{1:nargout}] = gui_mainfcn(gui_State, varargin{:});
else
    gui_mainfcn(gui_State, varargin{:});
end
% End initialization code -


% --- Executes just before Trial_guif is made visible.
function Trial_guif_OpeningFcn(hObject, eventdata, handles, varargin)
% This function has no output args, see OutputFcn.
```

```
% hObject    handle to figure
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)
% varargin   command line arguments to Trial_guif (see VARARGIN)

% Choose default command line output for Trial_guif
handles.output = hObject;

% Update handles structure
guidata(hObject, handles);

% UIWAIT makes Trial_guif wait for user response (see UIRESUME)
% uiwait(handles.figure1);


% --- Outputs from this function are returned to the command line.
function varargout = Trial_guif_OutputFcn(hObject, eventdata, handles)
% varargout  cell array for returning output args (see VARARGOUT);
% hObject    handle to figure
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Get default command line output from handles structure
varargout{1} = handles.output;

function edit1_Callback(hObject, eventdata, handles)
% hObject    handle to edit1 (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hints: get(hObject,'String') returns contents of edit1 as text
%        str2double(get(hObject,'String')) returns contents of edit1 as a
double
h = str2double(get(hObject,'String'));
if isnan(h)
    errordlg('You must enter a numeric value','Bad Input','modal')
    return
else
    handles.edit_text1 = h;
    A = zeros(h,6);
    set(handles.t1,'Data',A)
    set(handles.t1,'Enable','on')
end
guidata(hObject,handles)

% --- Executes during object creation, after setting all properties.
function edit1_CreateFcn(hObject, eventdata, handles)
% hObject    handle to edit1 (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    empty - handles not created until after all CreateFcns called

% Hint: edit controls usually have a white background on Windows.
%       See ISPC and COMPUTER.
```

```
if ispc && isequal(get(hObject,'BackgroundColor'),
get(0,'defaultUicontrolBackgroundColor'))
    set(hObject,'BackgroundColor','white');
end

% --- Executes on button press in Confirm_checkbox.
function Confirm_checkbox_Callback(hObject, eventdata, handles)
% hObject    handle to Confirm_checkbox (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hint: get(hObject,'Value') returns toggle state of Confirm_checkbox
if get(hObject,'Value') == get(hObject,'Max')
    set(handles.t1,'Enable','inactive')
    set(handles.t5,'Enable','inactive')
    handles.Y = get(handles.t1,'Data');
    handles.Z = get(handles.t5,'Data');
    [Fr,st] = output(handles);
    set(handles.t2,'Enable','on','Visible','on')
    set(handles.t3,'Enable','on','Visible','on')
    set(handles.t2,'Data',st.')
    set(handles.t3,'Data',Fr)
else
end
guidata(hObject,handles)

function edit2_Callback(hObject, eventdata, handles)
% hObject    handle to edit2 (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hints: get(hObject,'String') returns contents of edit2 as text
%        str2double(get(hObject,'String')) returns contents of edit2 as a
double
x = str2double(get(hObject,'String'));
if isnan(x)
    errordlg('You must enter a numeric value','Bad Input','modal')
    return
else
    handles.edit_text2 = x;
    dat = zeros(x,4);
    A = zeros(x,4);
    for i = 1:x
           for j =1:4
               if j>2
                 dat(i,j)= 1;
               else
                 dat(i,j)= A(i,j);
               end
           end
       end
    set(handles.t5,'Data',dat)
    set(handles.t5,'Enable','on')
end
```

```
guidata(hObject,handles)

% --- Executes during object creation, after setting all properties.
function edit2_CreateFcn(hObject, eventdata, handles)
% hObject    handle to edit2 (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    empty - handles not created until after all CreateFcns called

% Hint: edit controls usually have a white background on Windows.
%       See ISPC and COMPUTER.
if ispc && isequal(get(hObject,'BackgroundColor'),
get(0,'defaultUicontrolBackgroundColor'))
    set(hObject,'BackgroundColor','white');
end

function [Fr, st] = output(handles)
Y = handles.Y;                          % Stores  the matrix data from table 1
Z = handles.Z;                          % Stores matrix data from table 2
h2 = handles.edit_text2;                % Stores numeric data from edit_text2
h1 = handles.edit_text1;                % Stores numeric data from edit_text1
W = zeros(h2*2);                        % Initialises matrix W
%{
For evaluation of element local stiffness matrix and globalising the
stiffness matrix
%}
for m = 1:h1
    c = cosd(Y(m,2));                   % Stores the direction cosine of angle
    s = sind(Y(m,2));                   % Stores the sine of the input member's
angle
    e = Y(m,4);                         % Stores the Young's modulus for the
member
    a = Y(m,3);                         % Stores the area of the member
    l = Y(m,1);                         % Stores the length of the member
    n1 = Y(m,5)*2;                      % Stores the node coefficient Ux
    n2 = Y(m,6)*2;                      % Stores the node coefficient Uy
    v = a*e/l;                          % Stores the stiffness constant
    V(m)=v;                             % Stores the Stiffness constant in
array form
    % Stores the element stiffness matrix for each member
    K =v*[c^2 c*s -c^2 -c*s;...
        s*c s^2 -c*s -s^2;...
        -c^2 -s*c c^2 c*s;...
        -c*s -s^2 c*s s^2];
    % Stores the displacement transformation matrix for each element
    Kr{1,m} = [c s 0 0;...
        -s c 0 0;...
        0 0 c s;...
        0 0 -s c];
    %{
      For loop indexes the element stiffness coefficients to obtain the
globalised element stiffness matrix;
      W stores the master stiffness matrices;
    %}
    for n = 1:4
```

```
        switch n
            case 1
                d = n1-1;
            case 2
                d = n1;
            case 3
                d = n2-1;
            otherwise
                d =n2;
        end
        for p = 1:4
            switch p
                case 1
                    f = n1-1;
                case 2
                    f = n1;
                case 3
                    f = n2-1;
                otherwise
                    f =n2;
            end
            W(d,f,m)= K(n,p);
        end
    end
    W(:,:,m);                           % Concatenates the element global
stiffness matrices formed
end

D = sum(W,3);                           % Stores the master stiffness matrix
formed
R = D;
%{ Retrives the Displacement boundary conditions and stores them in matrix
form
%}
for m = 1:h2
    ux = Z(m,3);
    uy = Z(m,4);
    A(1,2*m-1) = ux;
    A(1,2*m) = uy;
    F(1,2*m-1) = Z(m,1);
    F(1,2*m) = Z(m,2);
end
Dmod = zeros(h2*2);                      % Initialise the modified master
stiffness matrix
b = numel(find(A == 0))+ numel(find(A == 1));
w = h2*2;
% Applying dispacement boundary conditions by modification
switch b
    case w
        for i = 1:w
            if A(1,i)~= 1
                for j=1:w
                    D(i,j)=0;
                    D(j,i)=0;
                    D(i,i)=1;
```

```
                end
            end
        end
        Dmod = D;
    otherwise
        for i = 1:w
            if A(1,i) ~= 1
                for j = 1:w
                    D(i,j)=0;
                    D(i,i)=1;
                end
            end
        end
        Dmod = D;
end
Fmod = zeros(1,w);                      % Initialize the Modified forces
matrix
for i = 1:w
    if A(1,i) == 0
        Fmod(1,i) = F(1,i);
    elseif A(1,i) == 1
        Fmod(1,i) = F(1,i);
    else
        Fmod(1,i) = A(1,i);
    end
end
U = inv(Dmod)*Fmod.';                   % resulting displacements
Fr(:,1) = R*U;                          % recovery of reactions
Fr(:,2) = U.';
%{ To recover the Internal forces Fs and Stresses ST
%}
for i = 1:h1
    n1 = Y(i,5)*2;
    n2 = Y(i,6)*2;
    a = Y(i,3);
    ug{1,i}(1,1) = U(n1-1,1);
    ug{1,i}(2,1) = U(n1,1);
    ug{1,i}(3,1) = U(n2-1,1);
    ug{1,i}(4,1) = U(n2,1);
    ul = Kr{1,i}*ug{1,i};
    d = ul(3,1)-ul(1,1);
    Fs(1,i) = V(i)*d;
    st(1,i) = Fs(1,i)/a;
end
```