DECLARATION

The work presented in this project is the original work, which to the best of our knowledge has never been produced and presented elsewhere for academic purposes.

……………………………………………..   ……………………………………………

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ACKNOWLEDGEMENT

We are grateful for the following persons for their endless support they gave us while working on this project.

First we thank our supervisor Prof.S.M Mutuli for his guidance in the fabrication and design and his availability whenever we needed him for consultations.

Secondly we highly appreciated the mechanical engineering workshop staff particularly Mr. Kimani for this assistance in welding together different parts of the rig.

We are deeply grateful for our families and friends for their moral support and encouragement throughout the project.

GOD BLESS YOU ALL.

DEDICATION

We dedicate this project to our families and all who made our five years of study success

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ABSTRACT

The Objectives of our project was to design fabricate and test a universal rig that will measure deflections on all types of beams subjected to any type of loading. This idea was born out of the limitations of the existing rig in the University of Nairobi mechanical laboratories which can only measure deflections on simply supported beam subjected to point loads only.

To achieve these objectives, research into the development of the concept of elastic deflections and its theory was done. The elastic deflection experienced by a beam under transverse loads was found to be dependent on the type of load, material and sectional properties of the beam. A general expression combining all these variables was developed for analysis for theoretical deflections subject to end and support conditions of the beam.

In order to fabricate the rig, design of different parts was done and recorded in form of drawings to scale. Different factors for selecting material for fabrication of the rig were considered. Material that was readily available, easily machinable, with suitable strength to meet these factors with least cost was selected. Mild steel was found to meet these conditions. Also to reduce weight with high stiffness, mild steel square tubes were selected for fabrication of rig structure and specimens beams.

The rig was fabricated from the different parts. A final rig assembly was then welded into a base mounting for its durability and stability. The rig was then tested by performing experimental measurement of deflections from each type of beam under point and couple loads. Corresponding theoretical deflections were analyzed for each end and loading condition using equations developed in theory section of the project. The two deflections were then compared. It was found that the experimental deflections varied from theoretical deflection by an average variation of 15%. This was due to reading errors and those from imperfection of loading fixtures. To improve further on accuracy of the experimental results it was recommended that inefficiency in application of loads be minimized by fabrication of stiff and rigid loading fixtures by casting. Also to achieve fully its universality it was recommended that the rig be improved further to test deflections from distributed loads. Therefore within the experimental error, the test results of the rig were found comparable to those predicted by theory hence the objectives of the project were achieved.

PRINCIPAL NOTATIONS

Symbol	Concept
σ	Direct bending stress
ε	Direct strain
θ	Angle
E	Young's modulus
R	Radius of curvature
P	Concentrated load
L	Length
Pa	Pascal (N/M^2)
T	Thickness
S	Arc length
M	Bending moment
I	Second moment of area
a	Distance from end of the beam to the point of application of load
y	deflection
c	constant of integration
EI	flexural rigidity
y_p	deflection due to point load
y_m	deflection due to moment
u	distance from neutral surface

CHAPTER 1

1.0 INTRODUCTION

1.1 Background information

Analysis of deflection in structural and machine members is of great importance in machine and structural design .Excessive deflection of structural member results in geometric distortion of the whole structure whereas in a machine excessive deflection may result in interference between moving parts increasing the rate of wear or total failure due to broken or jammed parts.

Deflection should therefore be designed not to exceed allowable space between the moving parts and the stationary ones for example casing or between the moving parts themselves .Under these conditions the part may be subjected to load whose magnitude is much less to cause failure by yielding however the geometric distortion and jamming of structural and machine parts respectively renders the structure or machine not to perform its desired function and may therefore be considered to half failed.

Knowledge on theory of deflection in beams is used in analyzing for magnitudes of deflection resulting from a given loads. In subject of solid and structural mechanics taught at second and third level of study in the degree course in mechanical engineering, students are introduced to the theory of deflection in beams. Deflections resulting from different loading situation on a given beam are analyzed for using different techniques. In all the techniques an equation governing deflection at any point in the beam span is developed and expressed as a function loads, cross-sectional and material properties of the beam.

In order to appreciate and verify this theory, students are required to perform experiments on different beams under different loading where they experimentally measure the deflections then compare with those predicted by theory. Therefore an experimental rig for measuring deflections is of importance not only in teaching solid and structural mechanics subject but also ensuring the objectives of degree course in mechanical engineering are met.

1.2 Problem statement

As mentioned above, an experimental rig for measuring beam deflection plays an important role in teaching and application of knowledge of deflection. To appreciate this knowledge, student are required to test the theory by comparing the theoretical deflections with those obtained by experimental measurement.

In the mechanical engineering laboratories the only available rig can test deflections on simply supported beam subjected to point loads only; hence the need for a universal rig that can test deflections on all types of beams under different loads.

1.3 Objectives

Due to limitations on the available rig for testing deflection we under took this project to come up with a rig that would solve these limitations. The objectives of our project included:

i. To design and fabricate an experimental rig that can be used to test deflection of all types of beams under point loads and turning moments (couple).
ii. To test the rig by comparing experimental deflections from the rig with those from theory.

CHAPTER 2

2.0 LITERATURE REVIEW

2.1 INTRODUCTION

2.1.1 Beams

Beam may be defined as member whose length is large in comparison with its thickness and is loaded with transverse loads or couples that produce significant bending effects

Beams are so common in engineering structures that their importance cannot be overemphasized. In engineering structures members that are oriented such that their lengths are horizontal are considered beams. Beams are also used in machine parts, for example, the armature shaft in a generator may be considered as a simply supported beam carrying a uniformly distributed load over a portion of its length.

Beams are generally classified according to their geometry and manner in which they are supported. Geometrical classification includes such features as the shape of cross-section, whether the beam is straight or curved and whether the beam is tapered or has a constant cross section. On the manner in which they are supported, the beams may readily be classified as cantilevers, simply supported, overhanging, continuous and fix-ended beam. Beams can be further classified according to the type of load they are carrying, for example, a cantilever beam carrying a uniformly distributed load may be classified as a uniformly loaded cantilever beam.

2.1.2 Loads

Any force that is transmitted to a body from another body by means of direct contact over an area on the surface of the first body is a load due to body contact. Loads may be classified as follows:

2.1.2.1 Concentrated load (point load):- this is a load whose area of contact is relatively small compared to the total area over the entire length of the beam.

2.1.2.2 Distributed load:-this is a load whose area of contact is large relative to the length of the beam. Distributed loads may further be classified as linearly varying or uniformly distributed loads depending on the manner in which the load vary along the length of the beam.

2.1.2.3 Couples: couple is a turning moment applied at a particular point along the beam span. This turning moment can be achieved by using mechanism for application of parallel forces whose directions of action are opposite but are separated by a distance called moment arm.

2.1.3 Reactions

As response to applied loads, the beam and the supports react by an internal force which is opposite to the applied loads in order to remain in equilibrium. The reactions at the supports give rise to an internal shear force which acts at every section of the beam. To maintain its equilibrium, the beam reacts to turning effect of external loads in form of internal bending moments which vary along the position of the beam.

2.2 BENDING STRAINS AND STRESS

2.2.1 Pure bending of beams with symmetrical sections.

2.2.1.1 Initial restrictions

Generally when a beam having an arbitrary cross-section is subjected to transverse loads, the beam will bend. In addition, twisting and buckling may be present and a problem that includes the combined effects of bending, twisting and buckling can become a complicated one.

In order to obtain bending effects alone, certain restrictions on the geometry of the beam and on the manner of loading are placed. These are:

i. It is assumed that the beam is straight, has a constant cross-section, is made of a homogeneous material and its cross-section has a longitudinal plane of symmetry
ii. It is assumed that the resultant of the applied loads lies in this plane of symmetry. These restrictions will eliminate the possibility that the beam will twist.
iii. It is also assumed that the geometry of the overall beam is such that bending and not buckling is the primary mode of failure.

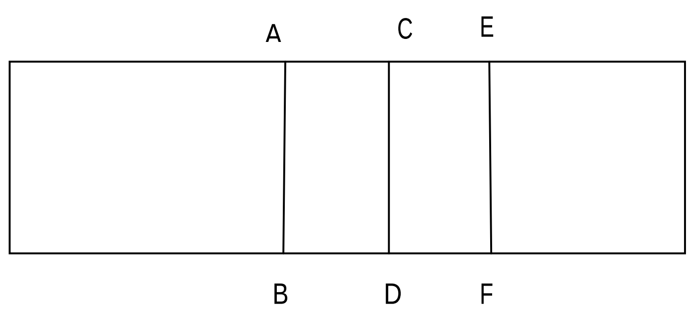
The internal reactions in any cross section of the beam may consist of a resultant normal force, a resultant shear force or a resultant couple. In order that we may examine bending effects alone, we will restrict the loading to one for which the resultant normal and shear forces are zero on any section perpendicular to the longitudinal axis of the beam.

The zero shear force implies that the bending moment is the same at every cross-section of the beam, that is $\frac{dm}{dx} = 0$ this my be visualized by considering the beam to being loaded by only pure couples at its ends remembering that these couples are assumed to be applied in the plane of symmetry. Under these conditions the beam is said to be in **pure bending** and the plane of symmetry is called **plane of bending**

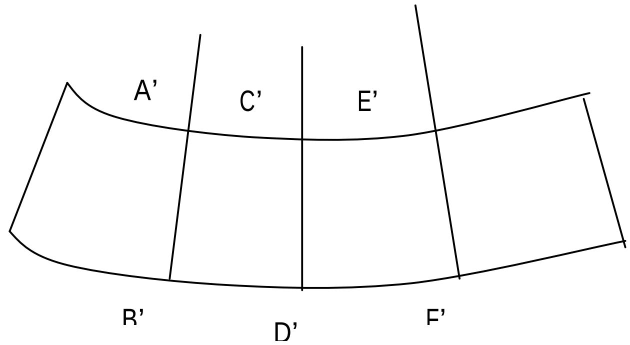
The beam now has sufficient geometry to permit reasonable arguments about its deformation.

2.2.1.2 Longitudinal deformation

Consider points on equally spaced sections AB, CD, and EF which are on plane of symmetry of a beam under pure bending



The figure below shows the plane of bending after application of couples at its ends i.e. pure bending

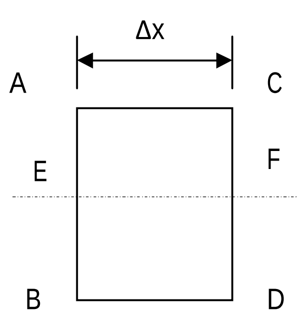


The portion of beam between plane sections AB and CD was originally geometrically identical in length and cross section with the portion between sections at CD and EF. Also each of these portions is loaded in exactly the same manner that is by couples. Therefore in order that each of these portions may be geometrically compatible in the deformed state, we may logically expect that these portions would undergo similar deformations.

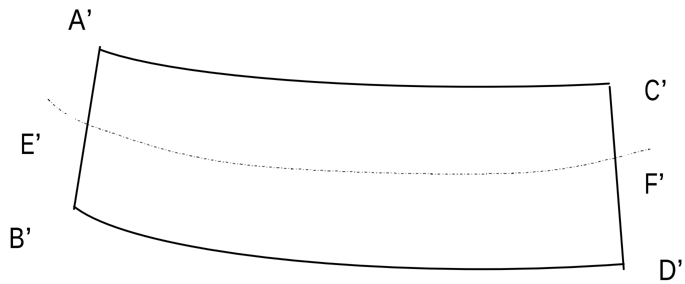
The same argument can be extended to the entire length of beam. The following conclusions can be made on characteristics of the deformation of any beam having a symmetrical cross-section and subjected to pure bending:

- Plane sections originally perpendicular to the longitudinal axis of the beam remain place and perpendicular to the longitudinal axis after bending
- In the deformed beam, the planes of these cross-sections have a common intersection; i.e. any line originally parallel to the longitudinal axis of the beam becomes an arc of a circle

Consider an element of beam having length Δx, in the plane of symmetry, in deformed state; the element is shown below:



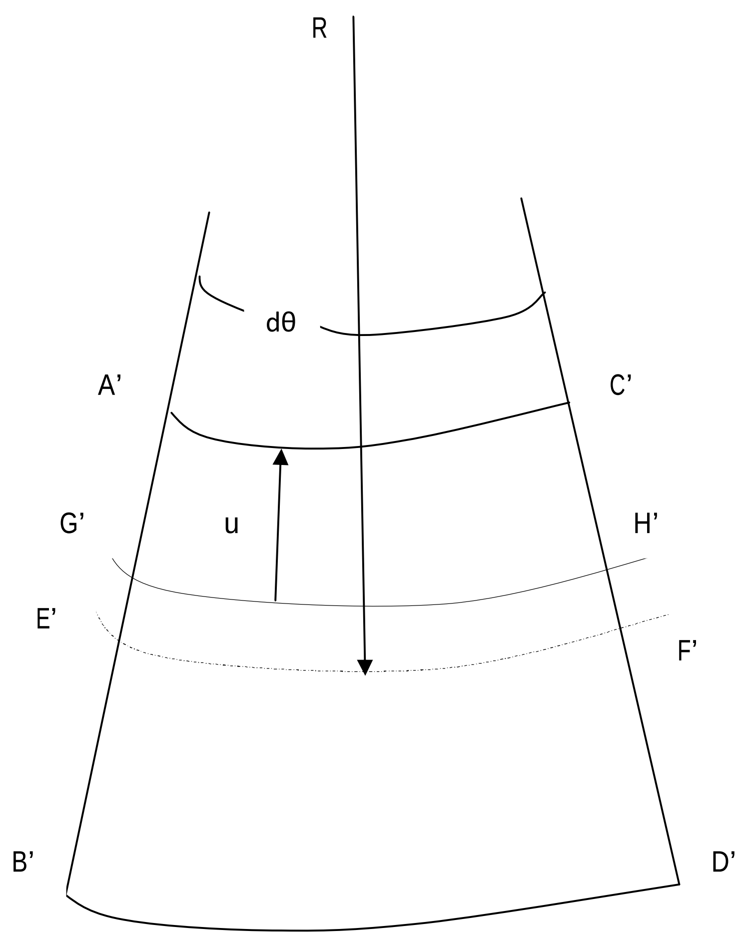
In deformed state the element is shown below where deformation is greatly exaggerated:



Under the influence of the couples M, the upper fibers of the beam are shortened while the lower fibers are elongated. However there is one surface (containing E'F') in which the fibers undergo no elongation or contraction. This longitudinal surface is called **neutral surface**. For a beam with a symmetrical cross-section and under pure bending the neutral surface is perpendicular to the longitudinal plane of bending. Also the intersection of the neutral surface with the longitudinal plane of bending of a beam is called **neutral axis.**

2.2.1.3 Strain

Consider the plane of bending of the elements of a beam shown below:



u represents the distance between the neutral axis EF and some other parallel line GH in the plane of symmetry. We assume that the distance between these lines in the deformed beam will not be significantly different from the original distance in the unloaded beam.

From the definition of the neutral surface and the geometry of above figure;

$$\Delta x = A'C' = Rd\theta$$

Also deformation (extension) on a fiber whose original position was GH becomes

$$G'H' - GH = (R-u)d\theta - \Delta x = (R-u)d\theta - Rd\theta$$

$$\therefore G'H' - GH = -ud\theta$$

By definition of axial strain;

$$\varepsilon = -\frac{ud\theta}{\Delta x} = \frac{-ud\theta}{Rd\theta}$$

$$\therefore \varepsilon = -\frac{u}{R} \dots\dots\dots\dots\dots\dots\dots (2.1)$$

Since for pure bending the radius of curvature is constant for entire length of the beam, we see that the longitudinal axial strain is directly proportional to the distance u from the neutral surface. The negative sign indicates a compressive strain for a positive value of u (fibers above neutral surface) and tensile strain for a negative value of u (fibers below the neutral surface) it is also assumed that the radius of curvature is positive (the beam is concave upwards)

2.2.1.4 Stress:

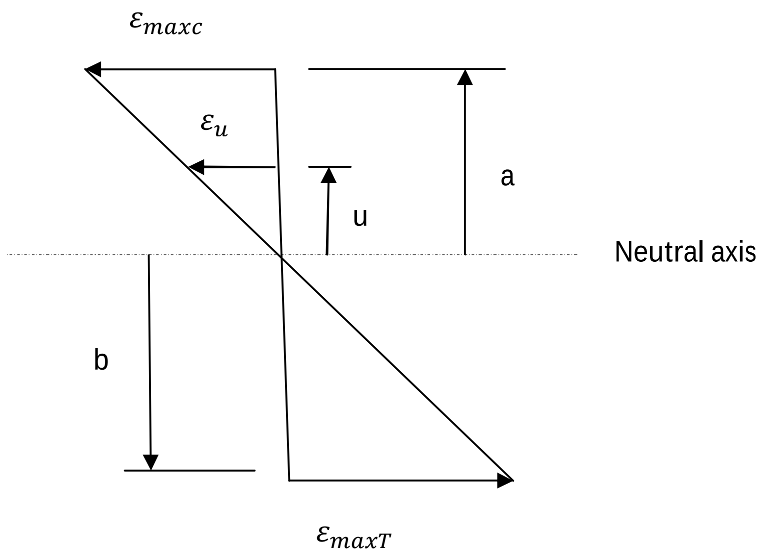
Using Hookes Law;

$\sigma = E\varepsilon$ for elastic strain

Where σ is the normal stress due to bending commonly called **flexure stress**

2.2.1.5 Strain and stress distribution

It has been shown that the longitudinal strain varies linearly from the neutral surface such a distribution across the cross-section can be illustrated graphically by means of a diagram below



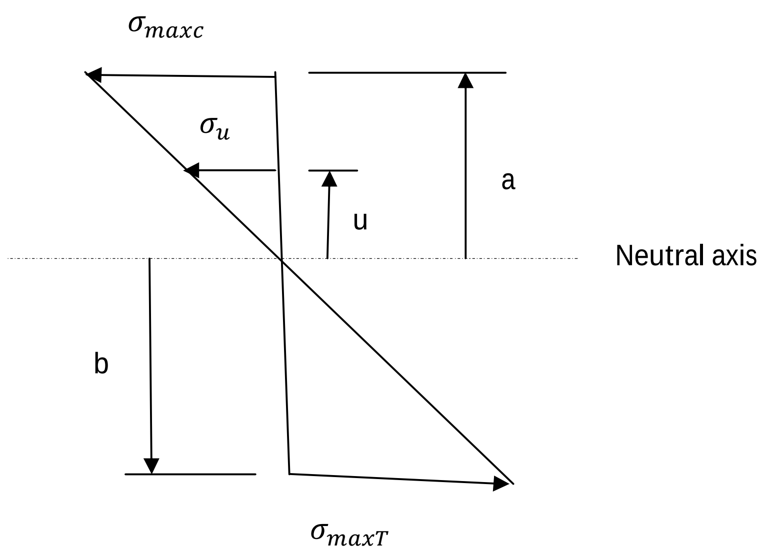
The strain at any point at a distance u from the neutral surface can be determined by the simple ratio:

$\varepsilon_u/u = \frac{\varepsilon_{maxT}}{b} = \frac{\varepsilon_{maxc}}{a}$……………(2.2)

Since the axial strain in a beam varies linearly form the neutral surface; elastic behavior will be characterized by a linear stress distribution since $\sigma = E\varepsilon$

However, it must be remembered that in a bent beam some of the fibers are in tension and others are in compression. Fortunately the modulus of elasticity for most engineering structural materials is the same in tension as in compression.

The distribution stress across the section of beam is shown below.

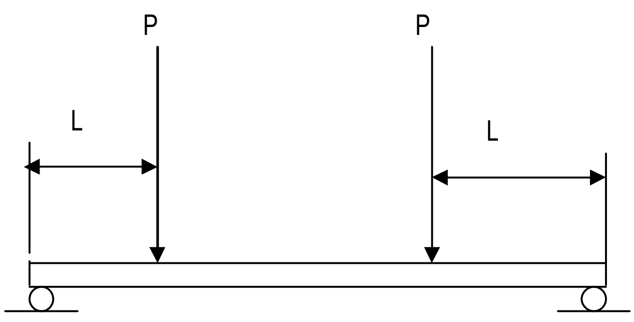


$\sigma_{maxc} = E\varepsilon_{maxc}$ and $\sigma_{maxT} = E\varepsilon_{maxT}$

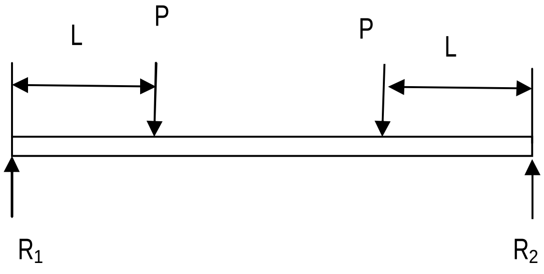
$\sigma_u/u = \frac{\sigma_{maxc}}{a} = \frac{\sigma_{maxT}}{b} = k\ (constant)$…………………(2.3)

2.2.1.6 Elastic flexure formula

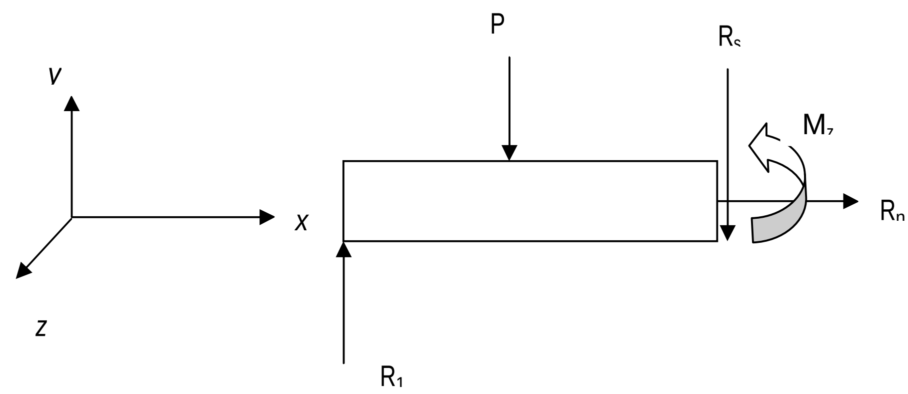
Consider a section of simply supported beam under pure bending



Free body diagram is given as:



Consider equilibrium at a section a *x* distance from the left support;



Considering static equilibrium with upward acting forces and clockwise moments being positive rise have for the whole beam:

$$R_1 = R_2 = P$$

Considering internal equilibrium of the beam with forces to the right and clockwise; moments being positive;

$$M_z = -\int (\sigma dA) u \ldots . (2.4)$$

$$R_n = \int (\sigma dA) = 0 \ldots . (2.5)$$

These equations represent the equilibrium requirements which must be satisfied by stress distribution over each cross-section of the beam.

Form the stress distribution of an elastic behavior of beam:

$$\sigma = ku$$

Substituting in (2.5)

$\int kudA = k\int udA = 0$……………(2.6)

Since k is not zero; the integral which represents the first moment of the area of cross-section about the neutral surface must be zero. This requirement implies that the neutral surface of the beam must pass through the horizontal centroidal axis of the cross section.

Thus for elastic action the strain and stress vary linearly from the transverse centroid of the cross section

Substituting for σ in (2.4) yields

$M_z = k\int u^2 dA$…………….(2.7)

Since u is measured from the centroidal axis the integral represents the second moment of the cross-sectional area about its horizontal centroid axis. This second moment is commonly called the rectangular moment of inertia. It is denoted by **I** hence equation becomes;

$M_z = -k\mathrm{I}$…………….. (2.8)

The minus sign means that a positive bending moment M_z produces compression in the upper fibers and tension in the lower fibers.

From the previous sections in general

$$k = \frac{\sigma_u}{u}$$

Substituting for k in equation of M_z we have;

$$-\frac{M_z}{I} = \frac{\sigma_u}{u}$$

Also substituting for $\sigma_u = E\varepsilon$ from strain equation where $\varepsilon = -\frac{u}{R}$ we have

$$\frac{E\,u}{R} = \sigma_u = \frac{M_z}{I}$$

Or

$\frac{E}{R} = \frac{\sigma_u}{u} = \frac{M_z}{I}$…………………….. (2.9)

The above equation is known as elastic flexure formula for beams.

2.2.2 Combined shear and bending

Although previous analysis of beams restricted to beams under- pure bending this type of loading condition in seldom encountered in an actual engineering problem. It is much more common for the resultant internal reaction to consist of a bending moment and shear force. The presence of shear force indicates a variable bending moment in the beam.

Strictly speaking the presence of the shear force and the resulting shear stresses and shear deformation would invalidate some of various statements in regard to geometry of the deformation and the resulting axial strain distribution across the beam section. Plane sections would no longer remain plane after bending and the geometry of the actual deformation would become considerably involved.

Fortunately; for abeam whose length is large in comparison with the dimensions of the cross section, the deformation effect of the shear force is relatively small and it is assumed that the longitudinal axial strains are still distributed in the same manner as for pure bending when this assumption is made the relationship of loads and stress developed previously are still considered valid. Experienced and experimental work indicates that this assumption is sufficiently accurate for most practical purposes.

When shearing effects are so large that they cannot be ignored as a design consideration other analytical method must be developed for their design.

2.3 DEFLECTION

In deformed position; the axis of the beam which was initially in a straight longitudinal line assumes some particular shape which is called deflection curve. The vertical distance between a point in neutral axis and corresponding a point in the deflection curve is called deflection at that point.

In developing the theory determining deflection of a beam, it is assumed that shear strain do not significantly influence the deformation

The deflection at any point a long the beam span is function of bending moments and property of beam material and cross section. In the theory section of this report; deflection *y(x)* equation is developed and is given in differential form as:-

$$\frac{d^2y}{dx^2} = \frac{M}{EI} = \frac{1}{R} \quad \text{.................}(2.10)$$

2.3.1 Methods of Analyzing for deflection

There are various methods of determining beam deflections common amongst these are:

i) Deflection by (Double) direct integration
ii) Deflection by moment-area method
iii) Strain- Energy or Catigliano's method
iv) Method of superposition

2.3.1.1 Deflection by direct integration

The differential deflection equation $\frac{d^2y}{dx^2} = \frac{M}{EI}$ does not contain the first term $\frac{dy}{dx}$; successive integrations with the appropriate constants of integration will give the slope θ and deflection y as function of x.The step by step procedure is as follows

1. Set up a reference coordinate system consistent with differential deflection equation.

The origin of right- hand system at the left hand end of the beam is one such coordinate system.

2. Define the equilibrium requirement expression for moment as a function of x. In cases where there are discontinuations in the loading several expressions may be necessary.
3. Determine bending modulus or flexural rigidity EI. If the cross- section varies with length the moment of inertia **I** must be expressed as function of x. If there are abrupt discontinuities in cross section or in the beam material then several expressions may be necessary.
4. Integrate the equation once for the slope and twice for the deflection y being careful to include the constants of integration. These constants are then evaluated from the conditions imposed on the deflection curve by the supports. A free- hand sketch of the deflection is often helpful in recognizing the conditions of restraint

2.3.1.2 Moment – area method for deflection

This is a semi-graphical integration technique based on geometrical interpretation of definite integrals. From differential deflection equation and definition of slope as:

$$\theta = \frac{dy}{dx} \quad \text{we have}$$

$$1/R = \frac{d\theta}{dx} = \frac{d^2y}{dx^2} \quad = \text{curvature}$$

Rewritten this equation becomes:

$$d\theta = \frac{1}{R}dl = \frac{M}{EI}dl \quad \ldots\ldots\ldots\ldots\ldots(2.11)$$

Where dl is infinitesimal change in slope of the deflection curve occurring over infinitesimal length dl. Note that $\frac{M}{EI}dl$ can be interpreted as infinitesimal area of the $\frac{M}{EI}$ (Curvature) diagram which can be easily obtained from the moment diagram by dividing moment by bending modulus EI. If the beam is homogenous and has a constant cross- section, the moment and curvature diagrams will have the same general shape

The finite change in slope between any two distinct points on the beam can be obtained by integrating equation of $d\theta$ above between the corresponding limits. Thus

$\Delta\theta_{AB} = \int_A^B d\theta = \int_A^B \frac{M}{EI} dl$ ……………….(2.12)

The deflection of the beam can be obtained indirectly by considering the tangential deviation. The tangential deviation t_{AB} is defined as the distance measured perpendicular to the undeformed neutral axis between point A on the deflection curve and a tangent line to the deflection curve drawn through the point B. Using curvature diagram; tangential deviation t_{AB} is the first moment of area of the area between A and B about point A. Thus the moment of the area on the curvature diagram is always taken about point for which tangential deviation is being determined. By combining tangential deviations between two portions of the beam; deflection at one point may be obtained.

2.3.1.3 Strain energy or Castigliano's theorem

By this method work done by external load acting on a beam over distance equal to deflection is equated to internal strains energy stored by the beam. This method is used where deflections cannot be explicitly expressed as function of loads and beam dimensions and material properties for example in curved beam.

2.3.1.4 Deflection by superposition principle

This method is used where there are combined load types acting simultaneously on the same beam. It states that the total deflection at point is a result of algebraic sum of deflection at same point due to individual loads acting alone

2.4 FACTORS TO CONSIDER IN SELECTION OF MATERIAL

In selecting material for application in as a machine part or as structural members many factors have to be considered in order to choose a material that meets the demand requirements at minimal cost. Common materials which machine or structural parts are made of are:

a) Cast iron
b) Steel

c) Copper and its alloys
d) Aluminum and its alloys
e) Plastics

The factors to be considered in selecting the material for a particular application are

i. Mechanical properties
ii. Manufacturing (fabrication) considerations
iii. Availability
iv. Cost

2.4.1 Mechanical properties

The following table shows the various mechanical properties of materials against the indicators commonly used to measure their values. The indicators are determined in laboratory tests on specimens:

Mechanical property	Measured by:-
Strength (under static loads)	Ultimate tensile strength or yield strength
Strength (repeated loads)	Endurance strength
Rigidity	Modulus of elasticity
Ductility	Percentage elongation
Hardness	Brinell/Rockwell hardness number
Toughness	Charpy or Izod impact value
Frictional properties	Coefficient of friction

Simple tension test provides measures for static strength, rigidity and ductility. An indication of toughness is also obtained from the values of strength and ductility.

2.4.2 Manufacturing factors:

The available methods for producing metal parts are

a) Cutting
b) Machining
c) Welding

d) Casting
e) Forging
f) Heat treatment
g) Rolling
h) Extrusion

The choice of material for the machine of structural part must consider the intended manufacturing method.

2.4.3 Availability

In practice the factor of availability should first be considered.

2.4.4 Cost

The cost of machine or structural part is made up of the direct cost of input (raw) material and the cost of processing it. The choice of manufacturing process must therefore consider this cost of manufacturing.

CHAPTER 3

3.0 THEORY

The word deflection generally refers to the deformed shape and position of a member subjected to bending loads. More specifically, however, deflection is used in reference to the deformed shape and position of the longitudinal axis of a beam. In deformed condition the neutral axis which is initially a straight longitudinal line assumes some particular shape which is called deflection curve.

The deviation of this curve from its initial position at any point is called deflection at that point. The deflection at any given point in the beam depends on the type of beam which is governed by the manner in which the beam is supported, the nature of loads applied to the beam any particular point within its span which can either be point or concentrated load, distributed load on the portion or the whole span. The beam may also carry couple loads any point within its span. Since deflection is a result of the internal reaction of beam, the deflection experienced by the beam depends on the ability of the beam material to resist deformation. The material property which is a measure of this ability (stiffness) is Young's Modulus, E. The stiffness of beam is also governed by the shape of the cross-section. The property of the cross section attributed to the final stiffness of the beam is the second moment of area I. the combined stiffness is called flexural rigidity EI.

Therefore an expression of deflection $y(x)$ for a given type of beam as a function the load, and flexural stiffness EI is developed in order to determine the deflection any point x along the beam. The internal reaction of beam to externally applied loads is represented by bending moment, M, hence the combined bending effect of all the externally applied loads is to cause this moment at any given section of the beam.

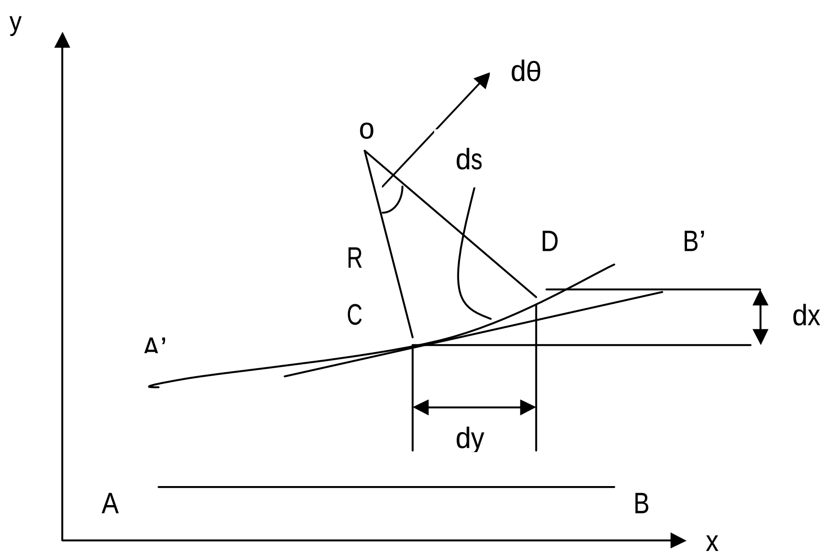
3.1 General Deflection equation

In developing the general expression of deflection in terms of the net effect of external loads inform of moment, the beam stiffness and the resulting geometry after the beam is deformed expressed as curvature $1/R$; the following assumptions are made:

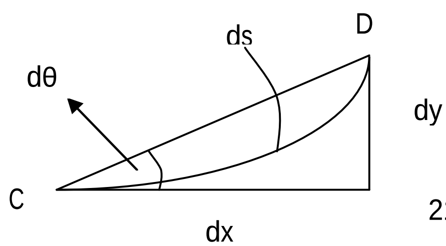
(i) The shear strain does not significantly influence the deformation. Hence the beam may be assumed to be subjected to pure bending; and thus plane section originally perpendicular to the longitudinal axis of the beam remain plane and perpendicular to the axis after bending.

(ii) Deflection at any point is very small in comparison with the length of the beam and that the horizontal projected length of the deflection curve is the same as its undeformed length. Therefore only elastic deformation is considered.

Consider undeformed beam AB which deflects to $A^{'}B^{'}$ under loading.



Using the deformed section of beam CD



$ds^2 = dx^2 + dy^2$ by Pythagoras theorem;

$$\therefore \quad \frac{d^2 s}{dx^2} = 1 + \frac{d^2 y}{dx^2}$$

$$\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

But $\quad dy/dx << 1$

Therefore; $\quad ds = dx$

Also $\quad ds = Rd\theta$

Or $\quad \frac{1}{R} = \frac{d\theta}{dx}$

Slope $\quad = \quad \tan\theta = \frac{dy}{dx}$

Since $\quad \frac{dy}{dx} << 1; \qquad \tan\theta = \theta = \frac{dy}{dx}$

Hence

$$\frac{d\theta}{dx} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d^2 y}{dx^2}$$

Therefore

$$1/R = \frac{d\theta}{dx} = \frac{d^2 y}{dx^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3.1)$$

from the general elastic flexure beam equation;

$$\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{y}$$

$$\frac{1}{R} = \frac{M}{EI}$$

Therefore

$$\frac{1}{R} = \frac{M}{EI} = \frac{d^2 y}{dx^2}$$

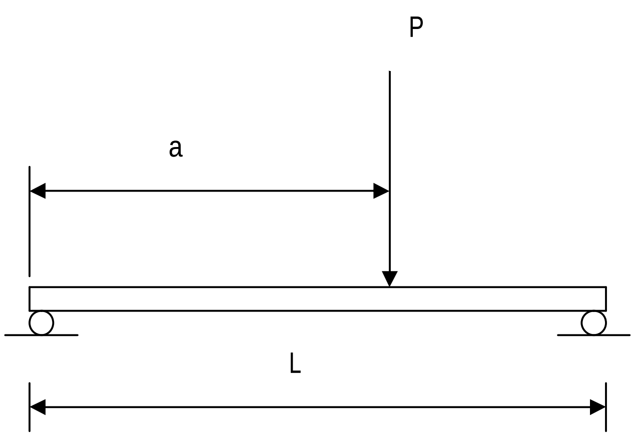
Or

$$\frac{d^2 y}{dx^2} = \frac{M}{EI} \quad \text{……………………(3.2)}$$

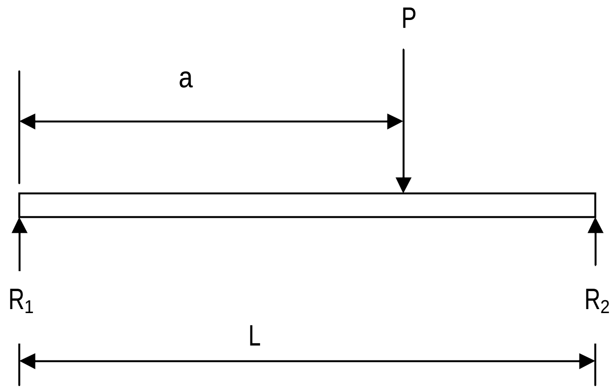
The expression for bending moment, M(x) as function of longitudinal distance x is determined for each type of beam.

3.1.1 Simply supported beam:

3.1.1.1 Simply supported beam under point load



The free body diagram of the beam is given as;



Equilibrium of forces in the vertical direction with upwards acting forces being positive gives;

$$R_1 + R_2 = P$$

Therefore $\quad R_1 = P - R_2$

Equilibrium of moments about left support with clockwise moments being positive gives

$$-R_2 L + Pa = 0$$

$$R_2 L = Pa$$

$$R_2 = \frac{a}{L} P$$

Hence

$$R_1 = P - \frac{a}{L} P$$

$$R_1 = P\left(\frac{L-a}{L}\right)$$

Equation for bending moments is obtained as;

For region $0 < x < a$

$$M(x) = R_1 x$$

$$M(x) = \left(\frac{L-a}{L}\right)Px \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.3)$$

For region $a < x < L$

$$M(x) = R_1 x - P(x-a)$$

$$= (R_1 - P)x + Pa$$

$$= \left(P\frac{(L-a)}{L} - Px\right) + pa$$

$$= Px - \frac{a}{L}xp - Px + Pa$$

$$= Pa - \frac{Pa}{L}x$$

$$M(x) = Pa(1 - \tfrac{x}{L}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.4)$$

At $x=a$

$$M(a) = M_{max} = Pa\left(1-\frac{a}{L}\right)$$

Deflection equation is given as;

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$\frac{d^2y}{dx^2} = \frac{Px}{EI}\left[\frac{(L-a)}{L}\right] \qquad \text{for } 0 < x < a$$

Obtaining deflection equation *y(x)* by direct integration

$$\frac{dy}{dx} = \frac{1}{EI}\left\{\frac{(L-a)}{2L}Px^2\right\} + C_1$$

$$y(x) = \frac{1}{EI}\left\{\frac{(L-a)}{6L}Px^3\right\} + C_1 x + C_2$$

The boundary conditions for this beam are :

$\frac{dy}{dx} = 0$ at x = a (i)

y=0 at x = 0 (ii)

y = 0 at x = L (iii)

applying (i) we have

$$C_1 = \frac{-(L-a)Pa^2}{2LEI}$$

applying (ii) we have $C_2=0$

substituting for the constants deflection equation reduces to:

$$y(x) = \frac{(L-a)Px^3}{6EIL} - \frac{(L-a)Pa^2 x}{2EIL}$$

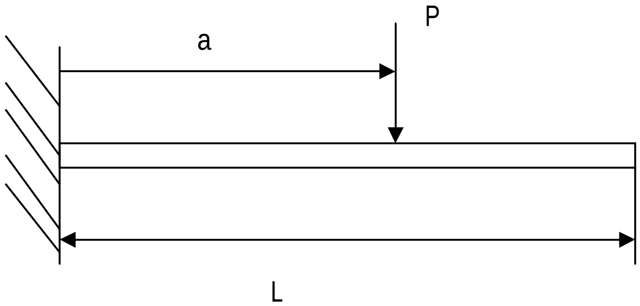
$$y(x) = \frac{(L-a)P}{EIL}\left\{\frac{x^3}{6} - \frac{a^2 x}{2}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.5)$$

For simply support with central point load i.e. a=L/2 the maximum deflection occurs at the center. Substituting a and x in the above expression we have;

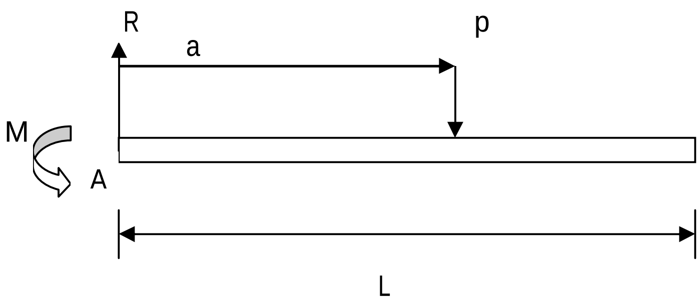
$$y_{max} = \frac{PL^3}{48EI}$$

3.1.2 Cantilever beam

3.1.2.1 Cantilever beam with point load



The free body diagram of the beam is given as;



Equilibrium of forces in the vertical direction with upwards acting forces being positive gives;

$$- P + R = 0$$

$$R = P$$

Equilibrium of moments about left support with clockwise moments being positive gives

$$\Sigma M_A = 0$$

$$- M + Pa = 0$$

$$\therefore M = Pa$$

Equation of bending moments gives

$$M(x) = Rx - M \qquad \text{for } 0 < x < a$$

M(x) = Rx – M – P (x-a) for a< x < L

R = P M = Pa

∴ M(x) = Px – Pa………… (3.6) for 0 < x < a

M (x) = 0………………(3.7) for a < x < L

Deflection equation becomes

$$\frac{d^2y}{dx^2} = \frac{M}{EI} = \frac{P(x-a)}{EI} \qquad \text{for } 0 < x < a$$

$$\frac{dy}{dx} = \frac{P}{EI}\left(\frac{x^2}{2} - ax\right) + C_1$$

$$y(x) = \frac{P}{EI}\left[\frac{x^3}{6} - \frac{ax^2}{2}\right] + C_1x + C_2$$

The boundary conditions of the beam are

$$\frac{dy}{dx} = 0 \qquad \text{at } x = 0$$

$$y = 0 \qquad \text{at } x = 0$$

Therefore C_1=0 and C_2=0 and deflection equation reduces to;

$$y(x) = \frac{P}{EI}\left[\frac{x^3}{6} - \frac{ax^2}{2}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.8)$$

Hence at x=a

$$y(a) = -\frac{Pa^3}{3EI}$$

Also for a<x<L , M (x) = 0 and deflection equation becomes,

$$\frac{d^2y}{dx^2} = 0$$

$$\frac{dy}{dx} = C_3$$

$$y\,(x) = C_3x + C_4$$

from previous section i.e. 0<x<a;

$$\left.\frac{dy}{dx}\right|_{x=a} = \frac{P}{EI}\left[\frac{a^2}{2} - a^2\right] = -\frac{Pa^2}{2EI}$$

$$y(a) = -\frac{Pa^3}{3EI}$$

Using these values of y at second region;

$$C_3 = -\frac{Pa^2}{2EI}$$

And

$$\frac{-Pa^3}{3EI} = C_3a + C_4$$

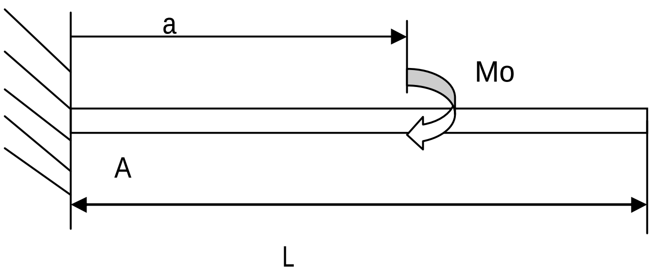
$$C_4 = \frac{-Pa^3}{6EI}$$

Therefore for a<x<L;

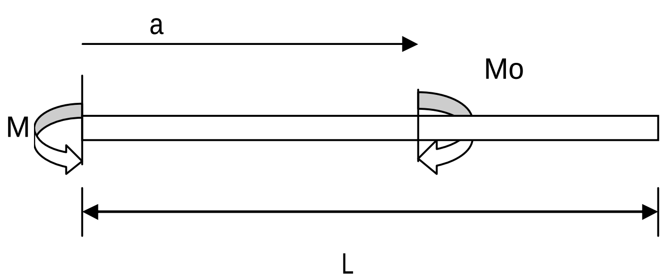
$$y(x) = \frac{-Pa^2}{2EI}x + \frac{Pa^3}{6EI}$$

$$y(x) = \frac{-Pa^2}{EI}\left[\frac{x}{2} - \frac{a}{6}\right] \qquad \text{for } a<x<L \quad \text{.....................(3.9)}$$

3.1.2.2. Cantilever beam with couple



The free body diagram of the beam is given as;



Equilibrium of moments

M = Mo

Equation of bending moments is given by;

M(x) = -M for $0< x<a$

Substituting for M

M (x) = -Mo for $0 < x < a$

And

M(x) = 0   for   a< x < L

Deflection equations for two regions are:

$$\frac{d^2y}{dx^2} = \frac{-Mo}{EI} \qquad \text{for} \quad 0 < x < a$$

$$\frac{d^2y}{dx^2} = 0 \qquad \text{for} \quad a < x < L$$

For   0< x < a

$$\frac{d^2y}{dx^2} = \frac{-Mo}{EI}$$

$$\frac{dy}{dx} = \frac{-Mo}{EI}x + C_1$$

$$y(x) = \frac{-Mo}{2E}x^2 + C_1x + C_2$$

Boundary conditions are

y= 0   at x = 0   (i)

$\frac{dy}{dx} = 0$ at x = 0   (ii)

Applying (i) and (ii) gives $C_1 = C_2 = 0$

Therefore

$$\frac{dy}{dx} = \frac{-Mo}{EI}x$$

and

$$y(x) = \frac{-Mox^2}{2EI} \quad \text{……………………(3.10)}$$

For a < x < L;

$$\frac{d^2y}{dx^2} = 0$$

$$\frac{dy}{dx} = C_3$$

$y(x) = C_3x + C_4$

Boundary conditions;

$$\frac{dy}{dx}\Big|_{x=a1} = \frac{dy}{dx}\Big|_{x=a2}$$

$$y\big|_{x=a1} = y\big|_{x=a2}$$

Where subscripts 1 and 2 refers region $0<x<a$ and $a<x<L$ respectively

Therefore

$$C_3 = -\frac{Moa}{EI}$$

$$C_3a + C_4 = \frac{Moa^2}{2EI}$$

$$\frac{-Moa^2}{EI} + C_4 = \frac{-Moa^2}{2EI}$$

$$C_4 = \frac{-Moa^2}{EI} - \frac{Moa^2}{2EI} = \frac{Moa^2}{2EI}$$

$$y(x) = \frac{-Mo}{EI}x + \frac{Mo}{2EI}a \quad \text{………………... (3.11)}$$

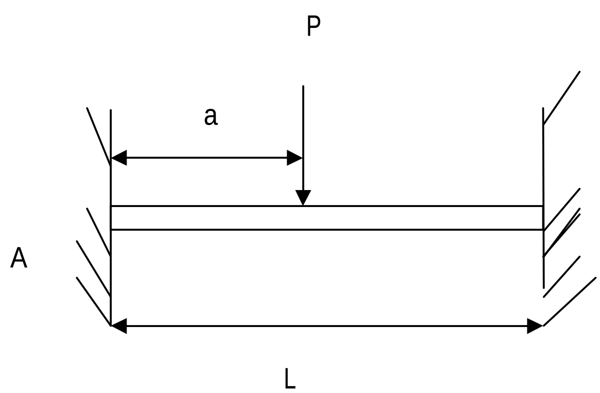
Thus for this loading;

$$y(x) = \frac{-M_o x^2}{2EI} \qquad \text{for } 0 < x < a$$

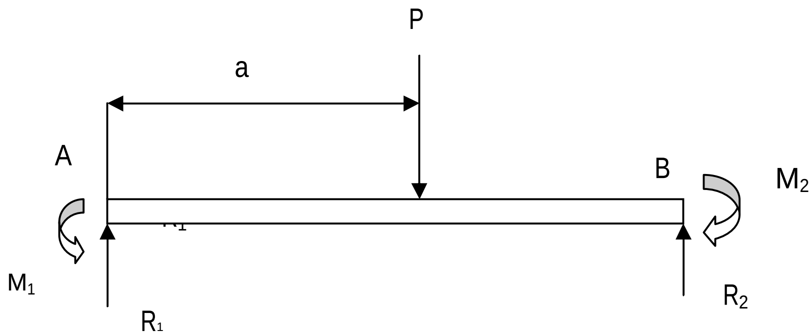
$$y(x) = \frac{-M_o x}{EI} + \frac{M_o a^2}{2EI} \qquad \text{for } a < x < L$$

3.1.3 Fixed ended beam

3.1.3.1 Fixed ended beam with point load



Free body diagram is as shown below;



Equilibrium of vertical forces with upward acting forces being positive i.e.

$\Sigma F_V = 0$

$R_1 + R_2 = P$………………(3.12)

Equilibrium of moments about the A

$\Sigma M_A = 0$

$M_1 + R_2 L = M_2 + Pa$ ………………(3.13)

Moment equations for two regions are given as:

For $0 < x < a$

$M(x) = -M_1 + R_1 x$ …………….(3.14)

For $0 < x < L$

$M(x) = -M_1 + R_1 x - P(x - a)$…………(3.15)

Equations of deflection for both regions are:

For $0 < x < a$

$$\frac{d^2y}{dx^2} = \frac{I}{EI}(-M_1 + R_1 x)$$

For $a < x < L$

$$\frac{d^2y}{dx^2} = \frac{1}{EI} - R_1 x - M_1 + Px + Pa$$

$$\frac{dy}{dx} = \frac{1}{EI}\left(-M_1 x + \frac{R_1 x^2}{2} - \frac{Px^2}{2} + Pax\right) + C_3 \ldots$$

$$y = \frac{1}{EI}\left(-\frac{M_1 x^2}{2} + \frac{R_1 x^2}{6} - \frac{Px^2}{6} + \frac{Pax^2}{2}\right) + C_3 x + C_4 \ldots$$

Boundary conditions are

$y = 0$	at $x = 0$	(i)
$dy/dx = 0$	at $x = 0$	(ii)
$\frac{dy}{dx}\Big\|_1 = \frac{dx}{dy}\Big\|_2$	at $x = a$	(iii)
$y\|_1 = y\|_2$	at $x = a$	(iv)
$y = 0$	at $x = L$	(v)

Where $|_1$ denote value for region $0 < x < a$ and $|_2$ denote value for region $0 < x < L$

respectively

Applying (i) we have $C_2 = 0$

Applying (ii) we have $C_1 = 0$

Hence for region $0<x<a$;

$$\frac{dy}{dx} = \frac{1}{EI}\left(-M_1 x + \frac{R_1 x^2}{2}\right).$$

$$y(x) = \frac{1}{EI}\left(\frac{-M_1 x^2}{2} + \frac{R_1 x^3}{6}\right). \qquad \text{……………………….(3.16)}$$

Applying (iii)

$$\left.\frac{dy}{dx}\right|_1 = \left.\frac{dy}{dx}\right|_2 \qquad \text{at } x = a$$

$$\frac{1}{EI}\left(-M_1 a + \frac{R_1 a^2}{2}\right) = \frac{1}{EI}\left(-M_1 a + \frac{R_1\, a^2}{2} - \frac{Pa^2}{2} + Pa^2\right) + C_3$$

$$0 = \frac{-Pa^2}{2EI} + \frac{Pa^2}{EI} + C_3$$

$$C_3 = -\frac{Pa^2}{2EI}$$

Applying (iv)

$$y|_1 = y|_2 \qquad \text{at } x = a$$

$$\frac{1}{EI}\left(\frac{-M_1 a^2}{2} + \frac{R_1 a^3}{6}\right) = \frac{1}{EI}\left(\frac{-M_1 a^2}{2} + \frac{R_1 a^3}{6} - \frac{Pa^3}{6} + \frac{Pa^3}{2}\right) - \frac{Pa^3}{2EI} + C_4$$

$$0 = \frac{1}{3EI} Pa^3 - \frac{Pa^3}{2EI} + C_4$$

$$C_4 = \frac{1}{6} \frac{Pa^3}{EI}$$

Replacing constants

$$d^2y \Big/ dx = \frac{1}{EI}\left(-M_1 + R_1 x\right) \qquad \text{for } 0 < x < a$$

$$dy \Big/ dx = \frac{1}{EI}\left(-M_1 x + \frac{R_1}{2} x^2\right)$$

$$y(x) = \frac{1}{EI}\left(-\frac{M_1 x^2}{2} + \frac{R_1 x^3}{6}\right)$$

$$d^2y \Big/ dx^2 = \frac{1}{EI}\left(-M_1 + R_1 x - Px + Pa\right) \qquad \text{for } a < x < a\ L$$

$$\frac{dy}{dx} = \frac{1}{EI}\left(-M_1 x + \frac{R_1 x^2}{2} - \frac{Px^2}{2} + Pax - \frac{Pa^2}{2}\right).$$

$$y(x) = \frac{1}{EI}\left(\frac{-M_1 x^2}{2} + \frac{R_1 x^3}{6} - \frac{Px^3}{6} + \frac{Pax^2}{2} - \frac{Pa^2 x}{2} + \frac{1}{6} Pa^3\right).$$

Applying (ii) for region 0 < x <a

$$-Ma + \frac{R_1 a}{2} = 0$$

$$M_1 = \frac{R_1 a}{2}$$

Applying (v) for region a<x<L

$$0 = \frac{-M_1 L^2}{2} + \frac{R_1 L^3}{6} - \frac{PL^3}{6} + \frac{PaL^2}{2} - \frac{Pa^2 L}{2} + \frac{Pa^3}{6}$$

But $M_1 = \frac{R_1 a}{2}$

$$0 = \frac{R_1 a L^2}{4} + \frac{R_1 L^3}{6} - P\left[\frac{L^3}{6} - \frac{aL^2}{2} + \frac{a^2 L}{2} - \frac{a^3}{6}\right]$$

$$= R_1\left(\frac{L^3}{6} - \frac{aL^2}{4}\right) - P\left[\frac{L^3}{6} - \frac{aL^2}{2} + \frac{a^2 L}{2} - \frac{a^3}{6}\right]$$

$$R_1 = \frac{P\left[\frac{L^3}{6} - \frac{aL^2}{2} + \frac{a^2 L}{2} - \frac{a^3}{6}\right]}{\left(\frac{L^3}{6} - \frac{aL^2}{4}\right)}$$

$$R_1 = \frac{P\left[L^3 - 3aL^2 + 3a^2 L - a^3\right]}{\left(L^3 - 1.5aL^2\right)}$$

Therefore $M_1 = \frac{R_1 a}{2} = \frac{Pa\left[L^3 - 3aL^2 + 3a^2 L - a^3\right]}{2\left(L^3 - 1.5aL^2\right)}$

From (3.12)

$R_2 = P - R_1$

From(3.13)

$M_2 = M_1 + R_2 L - Pa$

$$= \frac{Pa}{2}\alpha + PL(1-\alpha) - Pa$$

$$= \frac{Pa}{2}(\alpha - 2) + PL(1-\alpha)$$

Where

$$\alpha = \frac{L^3 - 3aL^2 + 3a^2L - a^3}{L^3 - 1.5aL^2}$$

Hence

$$M_1 = \frac{Pa\alpha}{2} \qquad \text{and} \qquad R_1 = P\alpha$$

Deflection equation reduces to:

$$y(x) = \frac{1}{EI}\left[\frac{-M_1x^2}{2} + \frac{R_1x^3}{6}\right]$$

$$y(x) = \frac{1}{EI}\left[\frac{-Pa\alpha x^2}{4} + \frac{P\alpha x^3}{6}\right]$$

$$y(x) = \frac{P}{EI}\left[\frac{-a\alpha x^2}{4} + \frac{\alpha x^3}{6}\right]$$

$$y(x) = \frac{\alpha P}{EI}\left[\frac{-ax^2}{4} + \frac{x^3}{6}\right] \quad \text{…………………(3.17)} \qquad \text{for } 0 < x < a$$

and for region a<x<L;

$$y(x) = \frac{1}{EI}\left[\frac{-M_1x^2}{2} + \frac{R_1x^3}{6} - \frac{Px^3}{6} + \frac{Pax^2}{2} - \frac{Pa^2x}{2} + \frac{Pa^3}{6}\right]$$

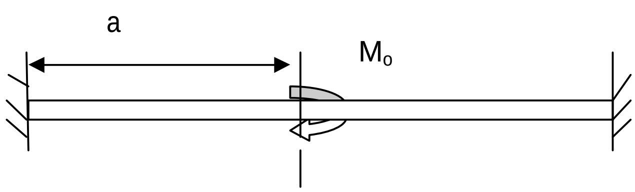
$$y(x) = \frac{1}{EI}\left[\frac{-Pa\alpha\ x^2}{4} + \frac{P\alpha x^3}{6} - \frac{Px^3}{6} + \frac{Pax^2}{2} - \frac{Pa^2x}{2} + \frac{Pa^3}{6}\right]$$

$$y(x) = \frac{P}{EI}\left[\alpha\left(-\frac{ax^2}{4} + \frac{x^3}{6}\right) - \frac{x^3}{6} + \frac{ax^2}{2} - \frac{a^2x}{2} + \frac{a^3}{6}\right]$$

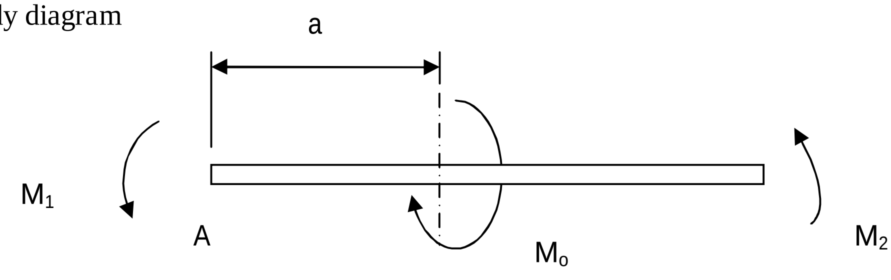
Hence;

$$y(x) = \frac{P}{EI}\left[(2-\alpha)\frac{ax^2}{4} - (1-\alpha)\frac{x^3}{6} - a^2\left(\frac{x}{2} - \frac{a^2}{6}\right)\right] \ldots\ldots\ldots\ldots\ldots(3.18) \qquad \text{for } a<x<L$$

3.1.3.2 Fixed ended beam with couple



The free body diagram



Equilibrium moments about A

$$m_1 = -\frac{m_o}{2}$$

$m_1 = -\frac{m_o}{2}$

Equation of moments

$M(x) = -M_1$ For $O < x < a$

$m_x = \frac{m_o}{2}$

Equation of deflection is given by;

$\frac{d^2y}{dx^2} = \frac{m(x)}{EI}$

Substituting for $M(x)$

$d^2y/dx^2 = \frac{m_o}{2EI}$

${dy}/{dx} = -\frac{m_o\text{x}}{2EI} + C_1$

$y(x) = -\frac{m_o}{4\text{EI}}\text{x}^2 + C_1x + C_2$

Boundary conditions are;

$at\ x = O \quad y = O \ldots\ldots\ldots.(i)$

$at\ x = O \quad \frac{dy}{dx} = O \ldots\ldots\ldots.(ii)$

Applying (i) and (ii);

$C_1 = C_2 = 0$

Equation deflection is given by

$y(x) = \frac{M_ox^2}{4EI}$ …………..(3.19) For $x < {L}/{2}$

$at\ x = {L}/{2}$

$$y_{max} = \frac{m_o L^2}{16EI}$$

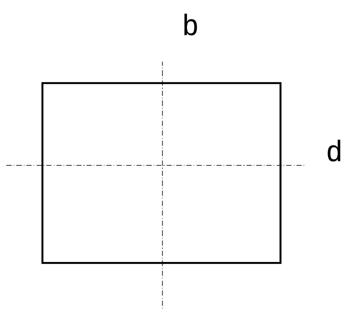
3.2 Superposition Principle:

The elastic deflection as result of combined loading is the algebraic sum of the deflections caused by individual loads. If y_p is the deflection at point due to point load only applied to a given beam and y_m is the deflection at some point due to couple only applied to the same beam, then deflection at same point due to combined point and couple load applied simultaneously is given by

$\boldsymbol{y = y_p + y_m}$………………… (3.20)

3.3 Second moment area of section I

For rectangular section;



Taking moment about an axis through centre of gravity

Second moment of area $I = \frac{bd^3}{12}$……………….. (3.21)

3.4 The three point bending experiment

Young’s modulus E, of a is determined using three point bending method. This is done by performing deflection experiment on simply supported beam made from the material. Values of deflection against loads are recorded and used to plot a graph of deflection against loads. The slope of the graph is used to get the young’s modulus E as shown below.

From equation 3.5, deflection y is given by

$$y(x) = \frac{(L-a)P}{EIL}\left\{\frac{x^3}{6} - \frac{a^2 x}{2}\right\}$$

Since deflection is taken at the centre of the beam i.e. x=L/2

Then $y_{\max} = \frac{PL^3}{48EI}$

Therefore the slope ,s of the graph is ,

$$slope = \frac{L^3}{48EI}$$

i.e. $EI = L^3/48s$……………..(3.22)

CHAPTER 4

4.0 DESIGN AND FABRICATION

The main aim of the project was to design and fabricate a rig that will accommodate all types of beams which can be loaded by concentrated loads and couple.

The rig frame was first designed as mounting for the beams. The fixtures for application of point loads and couple were then designed

4.1 DESIGN

4.1.1 Design of the rig structure

The rig structure was designed to consist of three different support frames namely: - top frame, vertical frame and base frame

Mild steel square tubes of one inch by one inch square cross-section were selected for fabrication of these frames. This material was found to be easily machinable by cutting and weldable during rig fabrication.

4.1.2 Design of beam specimen

Beam specimens were design form mild steel square tubes similar to those used in rig structures. Each beam was designed to support necessary fixtures to achieve specified loading. For couple loads, holes were drilled on the beams for attachment of coupling fixture.

The lengths of the beams were chosen such that they could fit into the rig structure with reasonable length remaining as beam span.

4.1.3 Design of loading fixture

The loading fixtures designed included:

i. Concentrated/point loads fixture
ii. Coupling fixture

4.1.3.1 Point load fixture:

Point load fixtures were made for easy application of point loads. This is done by hanging at specific position on the beams.

The fixtures were designed to consist of two flat plates of dimensions 64mm by 18mm connected at the top and bottom by cylindrical rods as shown in drawing no: 4. These parts were made from mild steel.

4.1.3.2 Coupling fixtures

Coupling fixtures was designed to convert two forces acting in different directions separated by a distance into a turning moment. This was achieved by combining two rigid plates to make an L-shaped fixture. At the edge of the longer side loads were hanged while the shorter side was attached another flat plate. The complete fixture appear as in drawing no. 4

4.1.4 Design of Rig Mounting

The mounting of the rig was designed to provide a strong base for the whole structure. Mobility of the rig was achieved by attaching industrial castor wheels

Base frames were made from 40mm by 40mm heavy gauge square tubes. These frames provided a heavy base for stability of the rig.

4.1.5 Design of coupling rail

The purpose of coupling rail was to carry the pulleys. The pulleys were meant to carry the wire that will be used to apply couple on the beam

4.2 FABRICATION

The fabrication process of the fully complete rig consisted of a number of fabrication processes for different parts of the rig. These parts were made form different mild steel bars of different cross-section. The complete parts were later assembled to make the rig.

The following methods of fabrication were used

i. Cutting
ii. Machining
iii. Drilling
iv. Grinding
v. Welding

4.2.1 Fabrication of rig structure

The rig structure consisted of three support frames each which of was made from 1 inch square tubes of mild steel. The tubes were marked to the required dimensions and cut using hand hack –

saw. The tubes were then joined together by welding to form rectangular frame. Care was taken to ensure squareness of the corners. This was done by using try square during welding.

The rectangular frames were joined together to form the rig structure. The frames were first spot welded while squareness was tested. The frames were then permanently welded to form a rigid support structure for the experimental rig

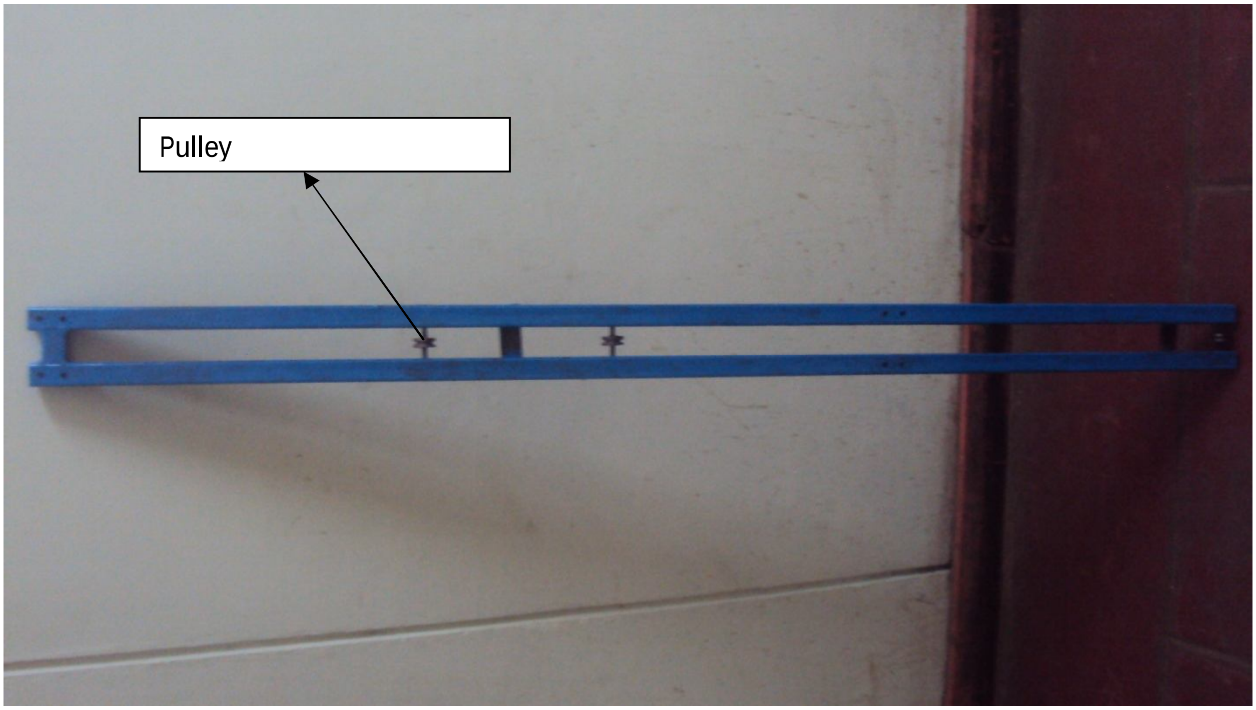
4.2.2 Fabrication of coupling rail

Coupling rail was made from mild steel square tubes similar to those used in making the rig structure. It consisted of two tubes of 1450mm length with pulleys attached at given distance to correspond to the position of application of turning moment (couple) at the beam.

The required lengths were marked and cut from the mild steel tubes. The locations of pulleys were marked on each tube for drilling holes for pulley axle. The pulley axles we made from cylindrical rods of diameter 6 mm and 85mm length

Pulley wheels were machined from 25mm diameter mild steel rod. A V-groove on each pulley was cut using lathe machine. A total of three pulleys were made.

The pulleys were assembled on the axles by welding them at their centers. These were then assembled into the two square tubes to form a coupling rail. The two were welded together at the edges by a square tube. Two mild steel strips were welded at their ends and another at the mid-point



Photograph of coupling rail

4.2.3 Fabrication of point load and coupling fixtures

The point load and coupling fixtures were designed as shown in the photograph in the next page. Point load fixture was made from mild steel plates and cylindrical rods of given dimensions by welding. The required dimensions were marked and cut using hand hack saw. The holes were drilled using drilling machine.

The coupling fixture was made from T-section mild steel and two flat plates, of the given dimensions. The dimensions were marked and cut using hand hacksaw. The holes were drilled at the lower flat plate for attachment to the beam at location of application of couple. A moment arm of 15 cm was marked and the holes for hanging masses were drilled at the T-section beam.

Photographs of Coupling fixture

Point load fixture

4.2.4 Fabrication of knife edged supports

The knifed edged supports for simply supported beams were made by grinding knife edge at one of the sides of mild steel angle iron. A central axis of the support was clearly marked for symmetrical positioning of beam during experiment

4.2.5 Fabrication of specimen beams

Specimens were made from mild steel tubes similar to those used for rig structure. For each type of beam, beam span with allowance for attachment to the rig structure were marked off. Also the positions for application of couple were marked.

Holes were then drilled to correspond to those of coupling fixture.

4.2.6 Base mounting

Base mounting was made from heavy gauge square tubes of 40mm by 40mm cross section. it consisted of 100cm by 100cm square frame supported by four 30cm long square tubes at each corner to form the vertical supports.

For mobility of the rig, industrial castor wheels were attached to the vertical supports by 6mm thick square plate of length 100mm. 3/8 inch diameter holes drilled on these plates enabled fastening of the castor wheels to the vertical supports by use of bolt and nut as shown below

Base mounting

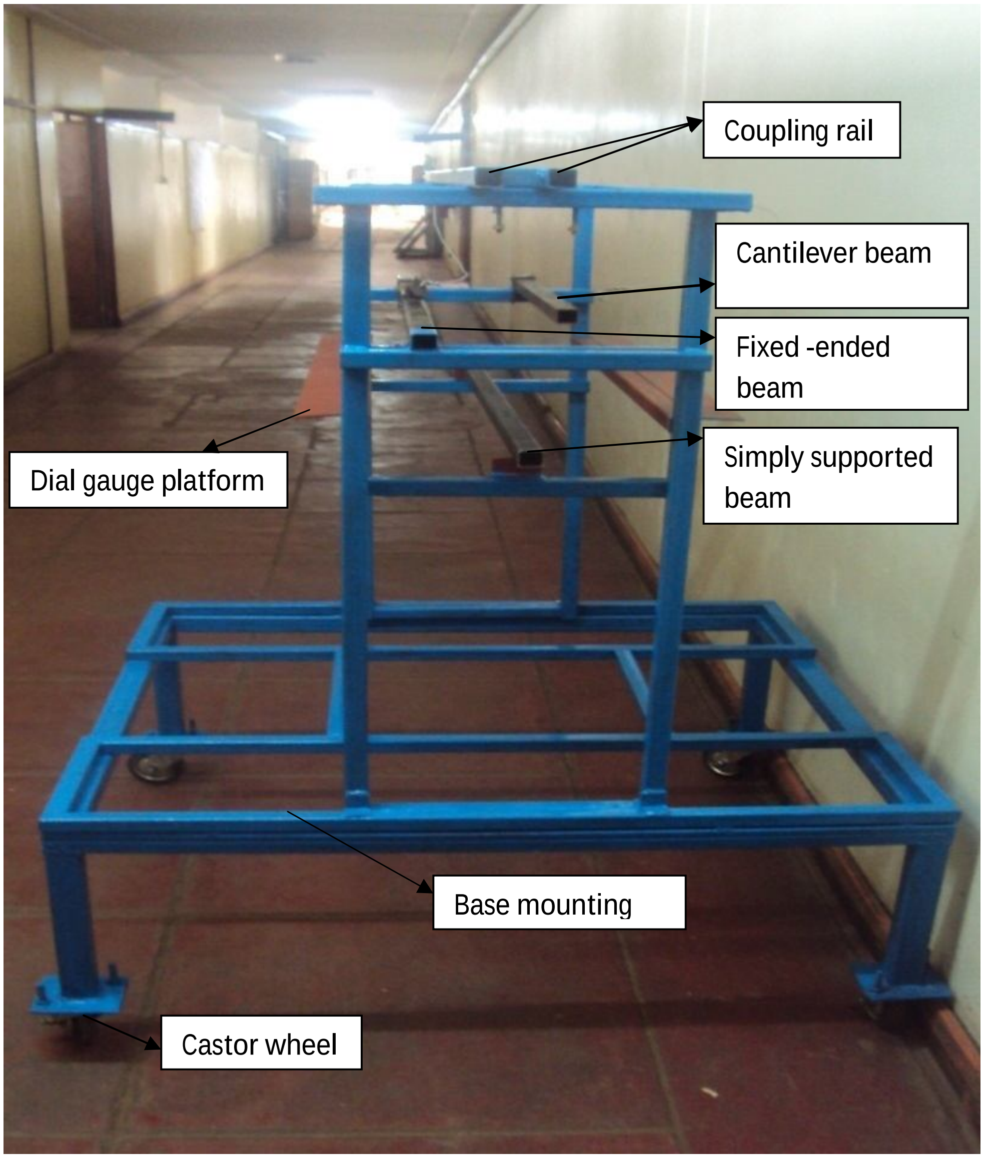
4.3 ASSEMBLY

Different parts made were assembled onto the rig structure to correspond to their positions as per designed drawing.

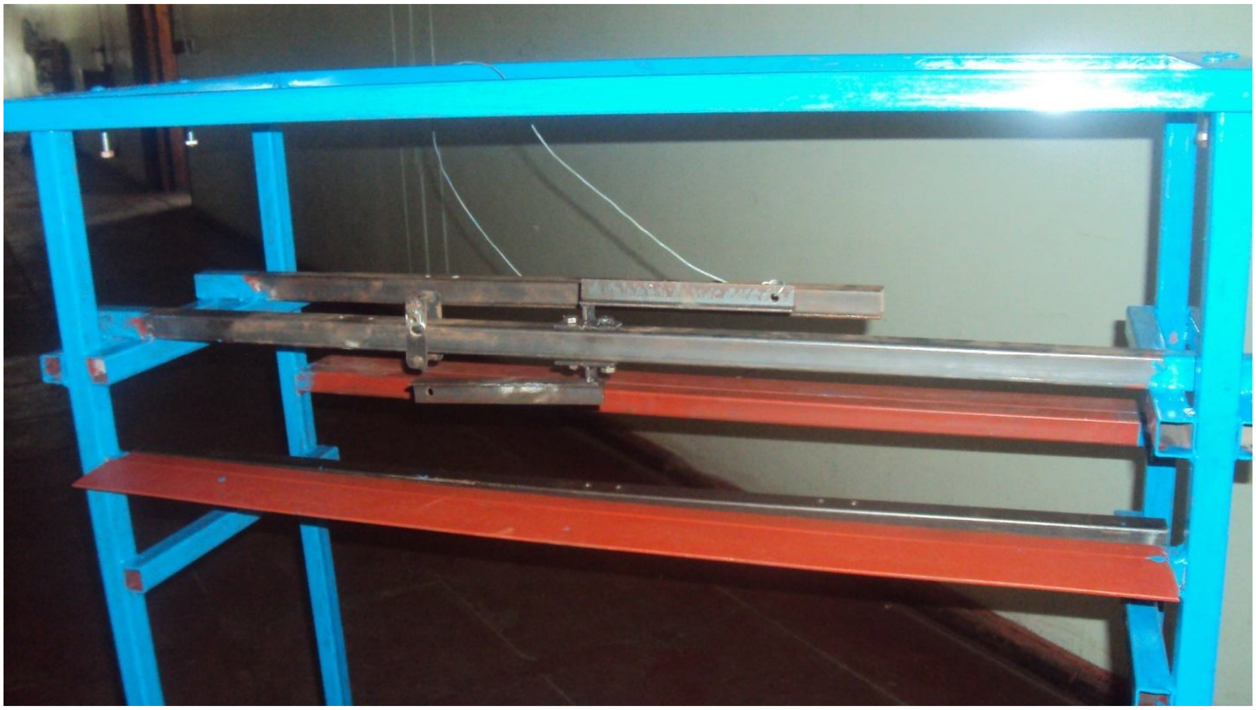
Coupling rail was assembled into the rig structure by fastening with bolts. The position of the rail on the top frame of the rig was marked to correspond to central axis of the beam of interests. The rail was then clamped and through holes drilled from the rail to the top frame.

The rail was then fastened into the rig structure by bolts and nuts

The specimen beams and knife edge supports were welded onto their position as per the design drawing.



The final rig structure



Beams with loading fixtures

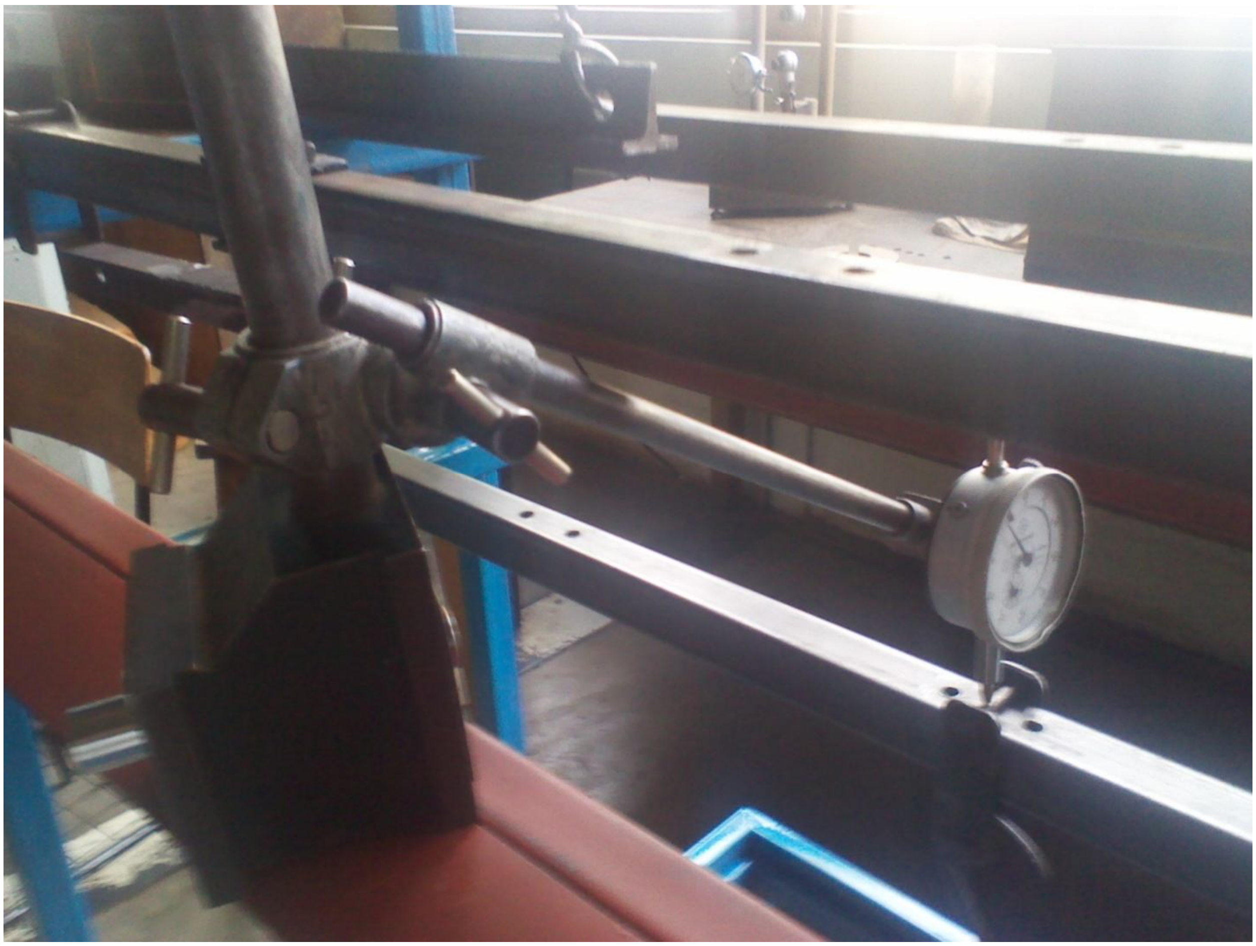
CHAPTER 5

5.0 EXPERIMENTAL PROCEDURE

5.1 Point loads

5.1.1 Simply supported beam

1. Effective length of 84.5cm was marked on a beam of one metre length
2. The beam was placed on the two knife edge plates mounted on the rig such that the effective length was 84.5cm.
3. Dial gauge was fixed at the centre of the plate in the diagram as shown below.
4. Masses from 1kg to 15kg were hanged at the centre of the beam at interval of 1kg.
5. The deflection corresponding to each weight hanged was taken and tabulated.



Experimental deflection for simply supported beam with central point load.

5.1.2 Cantilever beam

1. A beam of effective length 63.6cm was permanently welded to the rig on one end.
2. Dial gauge was fixed from the top at distance of $\frac{2}{3}L$ (42.4cm) from the fixed end.
3. Masses of 100g to 1000g were hanged at the centre of the beam at interval of 100g.
4. The deflections corresponding to each mass was recorded.
5. Table of deflections against the loads hanged in Newton was drawn.

5.1.3 Fixed beam.

1. A beam of 1m length was permanently fixed on both ends such that the effective length was 84.7cm.
2. Dial gauge was fixed at the centre of the plate.
3. Masses of 1kg to 15kg were hanged at the centre of the beam at interval of 1kg.
4. The deflections corresponding to each mass hanged was recorded.

5.2 Couple

5.2.1Cantilever beam

1. The coupling fixture was tightly fixed on the beam at $\frac{2}{3}L$ (42.4cm) from the fixed end.
2. Equal masses were hanged on both sides of the coupling fixture to produce moment about the point at which the fixture was fixed to the beam. The moment arm was 30cm.
3. Dial gauge was fixed on the beam at the free end.
4. Masses of 1kg to 10kg were hang at interval of 1kg.The deflections corresponding to each mass were read on dial gauge and recorded against their corresponding couple.

5.2.2 Fixed-ended beam

1. Coupling fixture was tightly fixed at the centre of the already fixed plate.
2. The couple was applied by hanging equal mass on both ends of the coupling fixture such that the moment arm is 30cm.
3. Dial gauge was fixed on the arm at the point of application of couple i.e. 42.35cm.
4. Masses of 1kg to 10kg were hang at interval of 1kg.Deflections were recorded for each couple applied.

5.3 Combination of point load and a couple

5.3.1Cantilever beam

1. The coupling fixture is tightly fixed on cantilever beam at 42.4cm from the fixed end.
2. Dial gauge was then fixed at 21.2cm from the fixed end.

3. Mass of 15kg was hanged on both sides of the coupling fixture to produce a constant couple throughout the experiment.
4. Masses of 0.1kg, 0.5kg, 1kg, 1.5kg, 2kg, 2.5kg, 3kg and 3.5kg were hang on the cantilever beam at 21.2cm mark to give point loads.
5. The deflections corresponding to each mass hang were recorded.

5.3.2 Fixed beam

1. The coupling fixture was tightly fixed on fixed ended beam at distance of 42.35cm.
2. Dial gauge was fixed at distance of 21.2cm from one of the fixed ends.
3. Mass of 15kg was hanged on both sides of the coupling fixture to produce a turning moment of 14.715Nm.Throughout the experiment.
4. Masses of 1kg, 4kg, 6kg, 8kg, 12kg, 16kg, and 20kg were hanged at 21.2cm mark to provide varying point load.
5. The deflections corresponding to each mass were recorded against the corresponding load.

CHAPTER 6

6.0 RESULTS AND ANALYSIS

6.1 Experimental results

6.1.1 Simply supported beam (SSB)

Load (N)	Deflection $\boldsymbol{x10^{-5}}$m at $\boldsymbol{x = L/2}$
9.81	5
19.62	10
29.43	15
49.05	26
58.86	31
68.67	37
78.48	42
98.1	54
107.91	60
117.72	65
127.53	70
147.15	81

6.1.2 Cantilever beam

6.1.2.1 Cantilever with point load at $^2/_3\,L$

Load (N)	Deflection $x10^{-5}$m at $x = {}^2/_3\,L$
0.981	1
1.962	2
2.943	3
3.924	4
4.905	5
5.886	6
6.867	7
7.848	8
8.829	9
9.81	10
11.772	11
12.753	12
13.734	13
14.715	14

6.1.2.2 Cantilever with couple at $^2/_3\,L$

Deflection at $x = L$

Load (N)	Moment arm (r)	Moment M_o (Nm)	Deflection $x10^{-5}m\ at\ x = L$
9.81	0.3	2.943	4
19.62	0.3	5.886	9
29.43	0.3	8.830	13

39.24	0.3	11.797	18
49.05	0.3	14.75	23
58.86	0.3	17.658	28
68.67	0.3	20.601	32
78.48	0.3	23.544	37
88.29	0.3	26.487	41
98.1	0.3	29.43	45

6.1.2.3: Cantilever beam with varying point load at $^1/_3\, L$ and constant couple at $^2/_3\, L$

Couple, $m_0 = 14.715\ Nm$

Load (N)	Deflections $x10^{-5}$m at $x = {}^1/_3\, L$
0.981	10
4.905	10.5
9.81	11
14.715	11.5
19.62	12
24.525	12.5
29.43	12.5
34.335	13.0

6.1.3. Fixed ended beam

6.1.3.1 Fixed ended beam with central point load

Load	Deflection $x10^{-5}$m at $x = {}^L/_2$
9.81	1

19.62	2
29.43	3
39.24	4
49.05	5
58.86	6
68.67	7
78.48	8
88.29	9
98.1	10
107.91	11
117.72	12
127.53	13
147.15	14

6.1.3.2 Fixed ended beam with a couple at $\frac{L}{2}$ deflection at $x = {}^{L}/_{2}$

Load (N)	Moment arm (r)	M (Nm)	Deflection$x10^{-5}m\ at\ x = L$
9.81	0.3	2.943	4
19.62	0.3	5.886	8
29.43	0.3	8.830	12
39.24	0.3	11.772	16
49.05	0.3	14.715	24
68.67	0.3	20.601	29
78.48	0.3	23.544	**32**
88.29	0.3	26.487	37

6.1.3.3 Fixed –ended beam with varying point load at $L/4$ and a constant couple M_0 at $L/2$

Load (N)	M_O (Nm)	Deflection $x10^{-5}m, at\ x = L/4$
9.81	14.715	5
39.24	14.715	5.5
58.86	14.715	6.0
78.48	14.715	6.5
117.72	14.715	7.0
156.96	14.715	7.5
196.2	14.715	8.0

6.1.4 Experimental results for determination of Young's modulus E

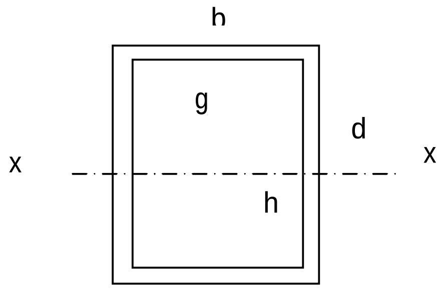
6.1.4.1 RESULTS OF 3 POINT BENDING TEST OF THE SIMPLY SUPPORTED BEAM

Load (N)	Deflection $x10^{-5}m$
9.81	6
19.62	11
29.43	16
39.24	21
49.05	26
58.86	32
68.67	37
78.48	43

6.2 ANALYSIS

6.2.1 Determination of second moment of area, I, of the beams

All the beams used have the same square cross-sections of the following dimensions



Taking moments about the neutral axis $x - x$ from theory

I=$\frac{1}{12}[bd^3 - gh^3]$

For our beams

b = 25.93mm

d=25.03mm

t=(1.38 + 1.3 + 1.7 + 1.2) × $\frac{1}{4}$ mm

t=1.395mm

$g = b - 2t = 25.93 - 2.79 = 23.14mm$

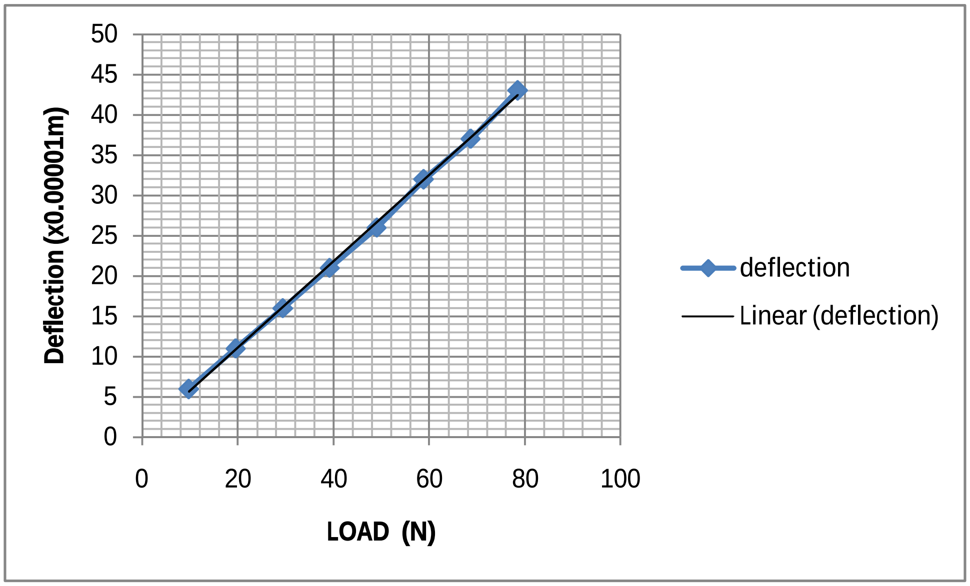
$h = d - 2t = 25.03 - 2.79 = 22.24mm$

I=$\frac{1}{12}[bd^3 - gh^3]$

I=$\frac{1}{12}[25.93 \times 10^{-3} \times (25.03 \times 10^{-3})^3 - 23.14 \times 10^{-3}(22.24 \times 10^{-3})^3]$

$\boldsymbol{I = 1.2672448032 \times 10^{-8} m^4}$

6.2.2 Determination of Young's modulus E of beam materials:



From the graph using the points (9.81, 6) and (49.05, 26)

$$Slope = \frac{(26 - 6) \times 10^{-5}}{49.05 - 9.81}$$

$$= 0.50964 \times 10^{-5} mN^{-1}$$

But from the deflection equation of simply supported beam with central point loads central deflection is given by:

$$y = \frac{PL^3}{48EI}$$

Therefore

$$slope = \frac{L^3}{48EI}$$

Thus $slope = 0.50964 \times 10^{-5} = \frac{0.845^3}{48EI}$

From which;

EI = 2466.1977 Nm^2 and substituting for I from above we have E = 195 GN/m^2

6.2.3 Determination of flexural rigidity of the beams

From the graphical analysis above

Flexural rigidity = $\boldsymbol{EI = 195 \times 10^9 \times 1.267248032 \times 10^{-8} m^4}$

$$\boldsymbol{EI = 2466.1977 Nm^2}$$

6.2.4 Calculation of theoretical deflections:

6.2.4.1 Simply supported beam:

Deflections equation for simply supported beam loaded at a distance a, form the left support is given by

$$y(x) = \frac{(L-a)P}{EIL}\left\{\frac{x^3}{6} - \frac{a^2 x}{2}\right\} \quad \text{for } 0 \le x \le \alpha$$

For central loading, $a = L/2$

$$y(x) = \frac{P}{2EI}\left[\frac{L^3}{48} - \frac{L^2 x}{8}\right]$$

Deflections at the centre, i. e

$$y = -\frac{PL^3}{48EI}$$

Substituting for values of EI and L

EI = 2446.1977Nm^2

L = 0.845m

$$y = \frac{-P(0.845)^3}{48 \times (2466.1977)}$$

$$y = -5.09684\text{P} \times 10^{-6}(m)$$

Sample calculation

$$y = -5.09684\text{P} \times 10^{-6}$$

For load of 58.86 N, deflection at the centre,

$$y = -5.09684 \times 58.86 \times 10^{-6}$$

$$=3.025 \times 10^{-4} m$$

Similarly, deflections for other loads were calculate and tabulated as follows

Load, P(N)	Central deflection, $y \times 10^{-5}(m)$
9.81	5.041
19.62	10.082
29.43	15.123
39.24	20.164
49.05	25.204
58.86	30.245
68.67	35.29
78.48	40.327
88.29	45.368
98.1	50.409
107.91	55.45
117.72	60.49
127.53	65.53
147.15	75.613

6.2.4.2 Cantilever beam:

6.2.4.2.1 Cantilever beam with point load:

The general expression for deflection of cantilever beam with point load acting at distance a from the fixed end is given by

$y(x) = \frac{P}{EI}\left[\frac{x^3}{6} - \frac{ax^2}{2}\right]$ For $o \leq x \leq \alpha$

$for \quad a = {}^{2}/_{3L} \; and \quad x = {}^{2}/_{3}L$

$$y = \frac{-8PL^3}{81EI}$$

Substituting for the values of EI and L

$$y = \frac{8(0.636)^3}{81 \times 2466.1977}\mathrm{P} = -1.03869\mathrm{P} \times 10^{-5}\mathrm{m}$$

Sample Calculation

For load of 9.81N, deflection, y is

$$y = -1.0386 \times 10^{-5} \times 9.81$$

$$y = 10.19 \times 10^{-5}\mathrm{m}$$

Similarly, deflection for other loads were calculated and tabulated as below

Load (N)	Deflections $\times 10^{-5}(m)$
0.981	1.019
1.962	2.038
2.943	3.057
3.924	4.076
4.905	5.095
5.886	6.114

6.867	7.133
7.848	8.152
8.829	9.171
9.81	10.190
10.791	11.209
11.772	12.228
12.753	13.246
13.734	14.265
14.715	15.284

6.2.4.2.2 Cantilever beam with a couple

The general expression for cantilever beam with a couple at distance a from the fixed is given by

$$y(x) = \frac{-M_o ax}{2EI} + \frac{M_o a^2}{2EI}$$

$$= \frac{M_O a}{2EI}[a - x] \qquad a \le x \le L$$

For $x = L \ and \ a = {}^{2}/_{3} L$

y= $-0.1111 \frac{M_0 L^2}{EI}$

Substituting L=0.636m and EI=2466.1977Nm^2

y= $-1.82238 \times 10^{-5} Mo$ m

Sample calculations

For load of 9.81N

$M_0 = r \times P$

Where r= Moment arm of the fixture P= Load,

r= 0.3 m P = 9.81N

M_0 = 0.3 × 9.81

= 2.943Nm

Therefore deflection at free end is

$y = -1.82238 \times 2.943 \times 10^{-5}\, m$

$= 5.363 \times 10^{-5} m$

Similarly, deflections by other loads were determined using the same procedure and were tabulated below

Load, P	M_o (Nm)	Deflections, $(\times 10^{-5})(m)$
9.81	2.943	5.363
19.62	5.886	10.727
29.43	8.830	16.090
39.24	11.772	21.453
49.05	14.715	26.816
58.86	17.658	32.18
68.67	20.601	37.543
78.48	23.544	42.906
88.29	26.487	48.269
98.1	29.43	53.632

6.2.4.2.3 Cantilever with point load and a couple

By superposition principle

$y_T = y_p + y_m$

Where

y_T = $Total\ deflection$

y_p = $deflection\ due\ to\ point\ load\ only$

y_m = $deflection\ due\ to\ couple\ alone$

From theory

$$y_p = \frac{-Pa^2}{EI}\left[\frac{x}{2} - {}^{a}/_{6}\right]$$

$$y_m = \frac{M_o x^2}{2EI}$$

$$y_T = \frac{-Pa^2}{EI}\left[\frac{x}{2} - \frac{a}{6}\right] - \frac{M_o x^2}{2EI}$$

$$at \quad x = {}^{l}/_{3} \qquad a = {}^{L}/_{3}$$

$$y_T = -\frac{P}{EI}\left(\frac{L}{3}\right)^2\left[\frac{L}{26} - \frac{L}{18}\right] - \frac{M_o L^2}{18EI}$$

$$y_T = -\frac{PL^3}{81EI} - \frac{M_o L^2}{18EI}$$

Sample calculation

For,

$M_o = 14.715\text{Nm} \qquad P = 0.981N \qquad \text{EI} = 2466.1977\text{Nm}^2 \qquad L = 0.636m$

$$y_T = \frac{0.981 \times (0.636)^3}{81 \times 2466.1977} - \frac{14.715(0.636)^2}{18 \times 2466.1977}$$

$$= 13.5 \times 10^{-5} m$$

Similarly, the deflection for other loads were calculated tabulated below

P (N)	M_o (Nm)	Deflection $y_T \times 10^{-5} m$
0.981	14.715	13.5
4.905	14.715	14.0
9.810	14.715	14.7
14.715	14.715	15.3
19.62	14.715	15.9
24.525	14.715	16.6
29.43	14.715	17.2
34.335	14.715	17.8

6.2.4.3 Fixed –ended beam

6.2.4.3.1 Fixed–ended beam with a point load:

From theory, the general expression is given by

$$y(x) = \frac{\propto p}{EI}\left[\frac{-ax^2}{4} + \frac{x^3}{6}\right]$$

Where $\propto = \frac{L^3 - 3aL^2 + 3a^2L - a^3}{L^3 - 1.5aL^2}$

For $a = L/2$

$\propto = 0.5$

$$therefore\ y(x) = \frac{p}{2EI}\left[\frac{-x^2L}{8} + \frac{x^3}{6}\right]$$

$at \quad x = L/2$

$$y = -\frac{PL^3}{192EI}$$

Sample calculation

For load

$P = 9.81N \quad EI = 2466.1977Nm^2 \quad L = 0.847m$

$$y = \frac{9.81 \times (0.847)^3}{192 \times 2466.1977} = 1.2589 \times 10^{-5}m$$

Similarly deflections for other loads were calculated and tabulated below

Load, P (N)	Deflection $\times 10^{-5}m$
9.81	1.259
19.62	2.518
29.43	3.777
39.24	5.036
49.05	6.294
58.86	7.553
68.67	8.812
78.48	10.071
88.29	11.330
98.1	12.589
107.91	14.105
117.72	15.107
127.53	16.366
147.15	18.884

6.2.4.3.3 Fixed ended beam with couple

From theory

$$y(x) = \frac{-M_o x^2}{4EI}$$

For $x = {}^{L}\!/_{2}$

$$y = \frac{-M_o ({}^{L}\!/_{2})}{4EI}$$

$$y = \frac{-M_o L^2}{16EI}$$

Sample calculation

For a load =2.943 Nm

$EI = 2466.1977 Nm^2$

$L = 0.847m$

$$y = \frac{-2.943 \times (0.847)^2}{16 \times (2466.1977)}$$

$y = 5.351 \times 10^{-5} m$

Similarly deflection from other loads were calculated and tabulated as shown below

M Nm	Deflection $y \times 10^{-5} m$
2.943	5.351
5.886	10.701
8.830	16.054
11.772	21.403
14.715	26.753
17.658	32.104
20.601	37.455
23.544	42.805
26.483	48.149
29.43	

6.2.4.3.2 Fixed ended beam with combined loads

For fixed ended beam with point load and couple by superposition principle;

$y_T(x) = y_p(x) + y_m(x)$

Where y_p = $deflection\ form\ point\ load\ only$

y_m = $deflection\ form\ couple\ alone$

From theory;

$$y_p = \frac{\propto P}{EI}\left(-\frac{ax^2}{4} + \frac{x^3}{6}\right)$$

Where $\propto = \frac{L^3 - 3aL^2 + 3a^2L - a^3}{L^3 - 1.5aL^2}$

For $a = {}^{L}/_{4}$

$\propto = 0.675$

$$y_p = \frac{0.675P}{EI}\left(-\frac{x^2L}{16} + \frac{x^3}{6}\right)$$

Taking deflection at $x = {}^{L}/_{4}$

$$Y_p = \left(-\frac{0.878906}{EI}PL^3\right) \times 10^{-3}$$

$$y_{m=}\frac{-M_o x^2}{4EI}$$

Taking deflection at $x = {}^{L}/_{4}$

$$y_m = \frac{-ML^2}{64EI}$$

Therefore

$$Y_T = \left(\frac{-0.8789063PL^3}{EI}\right) \times 10^{-3} - \frac{M_0L^2}{64EI}$$

$M_0 = 14.715\ Nm$ $\qquad EI = 2466.1977NM^2$ $\qquad L = 0.847m$

M_0 = 14.715 Nm

y_T = $-(2.1656P \times 10^{-8} + 6.6884 \times 10^{-5})$m

Sample calculation

For point load of 9.81 with

$y_T = -(2.1656P \times 10^{-8} + 6.6884 \times 10^{-5})m$

$y_T = -(2.1656 \times 9.81 \times 10^{-8} + 6.6884 \times 10^{-5})$

$y_T = -6.691 \times 10^{-5}m$

Similarly deflection due to other values of point load with same couple were calculated and tabulated below:

P (N)	M_o Nm	Deflection $y_T \times 10^{-5}m$
9.81	14.715	6.691
39.24	14.715	7.538
58.86	14.715	7.963
78.48	14.715	8.388
117.72	14.715	9.238
156.96	14.715	10.088
196.2	14.715	10.937

6.3 COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS

6.3.1 Simply supported beam with central point load

Load P (N)	Experimental deflection $\times 10^{-5} m$	Theoretical deflection $\times 10^{-5} m$	% variation
9.81	5	5.041	0.8
19.62	10	10.082	0.8
29.43	15	15.123	0.8
49.05	26	25.20	0.79
58.86	31	30.245	0.8
68.67	37	35.290	0.82
78.48	42	40.327	0.8
98.1	54	50.409	0.8
107.91	60	55.45	0.8
117.72	65	60.49	0.8
127.53	70	65.53	0.8
147.15	81	75.613	0.8

6.3.2 Cantilever beam

6.3.2.1 Cantilever with point load.

Load P (N)	Experimental deflection $\times 10^{-5} m$	Theoretical deflection $\times 10^{-5} m$	% variation
0.981	1	1.019	1.9
1.962	2	2.038	1.9
2.943	3	3.057	1.9
3.924	4	4.076	1.9
4.905	5	5.095	1.9
5.886	6	6.114	1.9

6.867	7	7.133	1.9
7.848	8	8.152	1.9
8.829	9	9.171	1.9
9.81	10	10.19	1.9
11.772	11	12.228	10
12.753	12	13.246	9.41
13.734	13	14.265	8.87
14.715	14	15.284	8.4

6.3.2.2 Cantilever beam with couple.

Couple applied $^2/_3 L$

Deflection measured at free end x=L

Couple(Nm)	Experimental deflection $\times 10^{-5}m$	Theoretical deflection $\times 10^{-5}m$	% variation
2.943	4	5.363	25.4
5.886	9	10.727	16.10
8.830	13	16.090	18.28
11.797	18	21.453	16.10
14.75	23	26.816	14.2
17.658	28	32.18	13.0
20.601	32	37.543	14.80
23.544	37	42.906	13.76
26.487	41	48.269	15.06
29.43	45	53.632	16.1

6.3.2.3Cantilever beam with varying point load and constant couple

Couple applied at $^2/_3 L$, $M_o = 14.715Nm$

Point load applied at $^1/_3 L$

Deflection measured at $^1/_3 L$

Load P (N)	Experimental deflection $\times 10^{-5}m$	Theoretical deflection $\times 10^{-5}m$	% variation
0.981	10	13.5	25.9
4.905	10.5	14.0	27.0
9.810	11.0	14.7	25.20
14.715	11.5	15.3	24.84
19.62	12	15.9	24.5
4.525	12.5	16.6	24.68
29.43	12.5	17.2	27.30
34.335	13	17.8	26.97

5.3. 3 Fixed ended beam

6.3.3.1 Fixed ended beam with central point load

Deflections measured at $x = {}^L/_2$

Load P (N)	Experimental deflection $\times 10^{-5}m$	Theoretical deflection $\times 10^{-5}m$	% variation
9.81	1	1.259	20.57
19.62	2	2.518	20.57
29.43	3	3.777	20.57
39.24	4	5.036	20.57
49.05	5	6.294	20.57
58.86	6	7.553	20.57
68.67	7	8.812	20.57

78.48	8	10.071	20.57
88.29	9	11.330	20.57
98.1	10	12.589	20.57
107.91	11	14.105	20.57
117.72	12	15.107	20.57
127.53	13	16.366	20.57
147.15	14	18.884	25.86

6.3.3.2 Fixed ended beam with central couple

Deflection measured at $x = {}^{L}/_{2}$.

Couple Mo(Nm)	Experimental deflection $\times 10^{-5}m$	Theoretical deflection $\times 10^{-5}m$	% variation
2.943	4	5.351	25.25
5.886	8	10.701	25.24
8.830	12	16.054	25.25
11.772	16	21.403	25.24
14.715	20	26.753	25.24
17.658	24	32.104	25.24
20.601	29	37.455	22.57
23.544	33	42.805	22.91
26.487	37	48.149	23.14

6.3.3.3 Fixed ended beam with varying point load and constant couple

Couple applied at $\frac{L}{2,}$ $M_o = 14.715Nm$

Point load applied at $\frac{L}{4}$

Deflection measure at $\frac{L}{4}$

Load P (N)	Experimental deflection $\times 10^{-5}$m	Theoretical deflection $\times 10^{-5}$m	% variation
9.81	5	6.691	25.26
39.24	5.5	7.538	27.04
58.86	6.0	7.963	24.65
78.48	6.5	8.388	22.51
117.72	7.0	9.238	24.22
196.2	8	10.937	26.85
156.96	7.5	10.088	25.65

CHAPTER SEVEN

DISCUSSION, CONCLUSION AND RECOMMENDATIONS

7.0 DISCUSSION

Our discussion will be based on two major areas

- Design and fabrication of the rig
- Testing and results

7.0.1: Design and fabrication

For the rig to be functional, several factors had to be considered in the designing and fabrication process. These factors included the following:-

i. Choice of material
ii. Determination of suitable rig dimensions
iii. Mounting the specimen on the rig
iv. Making the rig portable
v. Assembly of the frame structure

7.0.1.1 Choice of material

For the rig to be used for this purpose, it had to posses the property of being rigid, light in weight and high tensile strength. For this reason the material to be used for fabrication of the rig had to have several properties like machineability, weldability, high tensile strength, rigidity, light in weight.

Comparing with other materials mild steel was found to posses all the above mentioned properties except for the weight. To overcome this problem of weight square tubes of mild steel were used. For the rig structure we opted to use one inch (25.4mm) square tubes of light gauge were used. For the mounting, heavy gauge square tubes of cross section 40mm by 40mm were used.

Mild steel is easy to machine by various machine operations available in the workshop. This made easy cutting and drilling operations during fabrication.

Mild steel also posses high tensile strength and therefore can withstand high loadings. Steel possesses high elastic modulus of about 200-210 GPA. And therefore can withstand high loads without excessive deflections. Mild steel is also the most easily available material for various engineering applications. Therefore it could readily be obtained.

7.0.1.2 Determination of suitable rig dimensions

Some of the factors considered in deciding the size of the rig included:-

- Since our objective was to investigate elastic deflections which are usually small we decided to use specimen of span length 0.8. This guided us to get a rig length of 1m.
- For stability of the rig, a width of 1m was found suitable; therefore the base dimensions are 1m by 1m
- The rig structure was also to be made small enough so as not to be bulky. Height of 1m together with the above dimensions was found to fit adequately into the available workshop space.

7.0.1.3 Mounting the specimens on the rig

Cantilever and fixed –ended beams were mounted onto the rig structure by welding. For the experiment to be successful the specimens were to be tested in horizontal position. For this reason the specimens had to be mounted accurately in position such that they were symmetrical about the horizontal axis. Similarly knife edge supports were mounted for testing of simple supported beams

7.0.1.4 Assembly of the structure

The final structure had to posses the property of rigidity and stability for this reason, the method of assembly was of great importance in achieving this properties.

Welding provides a strong coalescence of two parts as compared to other joining processes; this was why welding was opted for joining the parts together.

It is also quick to use welding as a joining process hence more efficient.

7.0.1.5 Making the rig portable

For the purpose of making the rig mobile, four castor wheels were bolted onto the base mounting. This ensures that the rig can be moved easily form one point to another

7.0.2 Testing and results

Here we shall consider the following:

- Dial gauge setting on the specimen beams

- Testing of results and the variation between the experimental and the expected theoretical results.
- Causes of errors that contributed to the deviations in the results.

7.0.2.1 Setting of the dial gauge on the specimens:

Two flat mild steel plates were mounted on either sides of the rig parallel the specimen beams. The plates acted as dial gauge supports while taking the readings (deflections)on the beams. Proper mounting of dial gauge was achieved by:

(i) Ensuring that the flat plate were perfectly flat and wide enough to accommodate the whole dial gauge base

(ii) Magnetic base of the dial gauge attract the steel plates, therefore it cannot topple over once on the flat plate.

7.0.2.2 Test results and deviations

In this section comparison between results obtained experimentally with the expected theoretical calculations was made and from our analysis it was found that the average variations for different beams under different loads were as follows:

- Simply supported beam with central point loads was 0.79% to0.82%
- Cantilever beam with point load was 1.9% to 10%
- Cantilever beam with couple was 13% to 25.4%
- Cantilever beam with varying point loads and constant couple was 25.20% to 27.30%.
- Fixed ended beam with central point loads was 20.57% to 25.86%.
- Fixed ended beam with a central couple was 22.57% to25.25%.
- Fixed beam with varying point loads and a constant couple was 22.51% to 27.55%

7.0.3 Sources of errors

From the above mentioned deviations it is evident that our results were influenced by different sources of errors. Also from the results higher magnitudes of deviations were found application of couple or in combined loads situations. Comparing with the errors when point loads alone are acting it can be concluded that most of the errors arose from the coupling fixture. This is because the coupling fixture affects the stiffness of the beam and also deflection of moment arms may reduce the effective couple applied.

Secondly for cantilever beam with point loads, large deviations were caused by application of heavy loads. This implied the deflection at heavy weights were inaccurate.

Other source of errors includes:

- The way the specimen beams were mounted may have been perfectly symmetrical about a horizontal axis due limitation of caused by geometry of square tubes hence fixing the specimen properly in reference to the horizontal datum was difficult.
- Reading errors: these were contributed to by errors in reading the dimensions of the specimen beams and errors in reading the dial gauge deflections.

7.1 CONCLUSION

The objectives of this project were successfully achieved. This is due the proven fact that it is impossible to design, fabricate and test a universal rig used to measure beam deflections. A comparison of the experimental values with the expected theoretical values showed an average variation of 0.8%, 1.9% and 20.57% for simply supported, cantilever and fixed ended beam respectively with point loads. For couple application the average was found to be between 13.0% to 25.4% for both cantilever and fixed ended beams. The error for combination of point loads and couple was found to vary between 24.22% to 27.0%. These variation were due to experimental errors highlighted in detail in the discussion section are reasonable variations hence the functionality of the rig has been ascertained.

Thus the design and fabrication of this rig enable the practical appreciation of theory of deflections in beams.

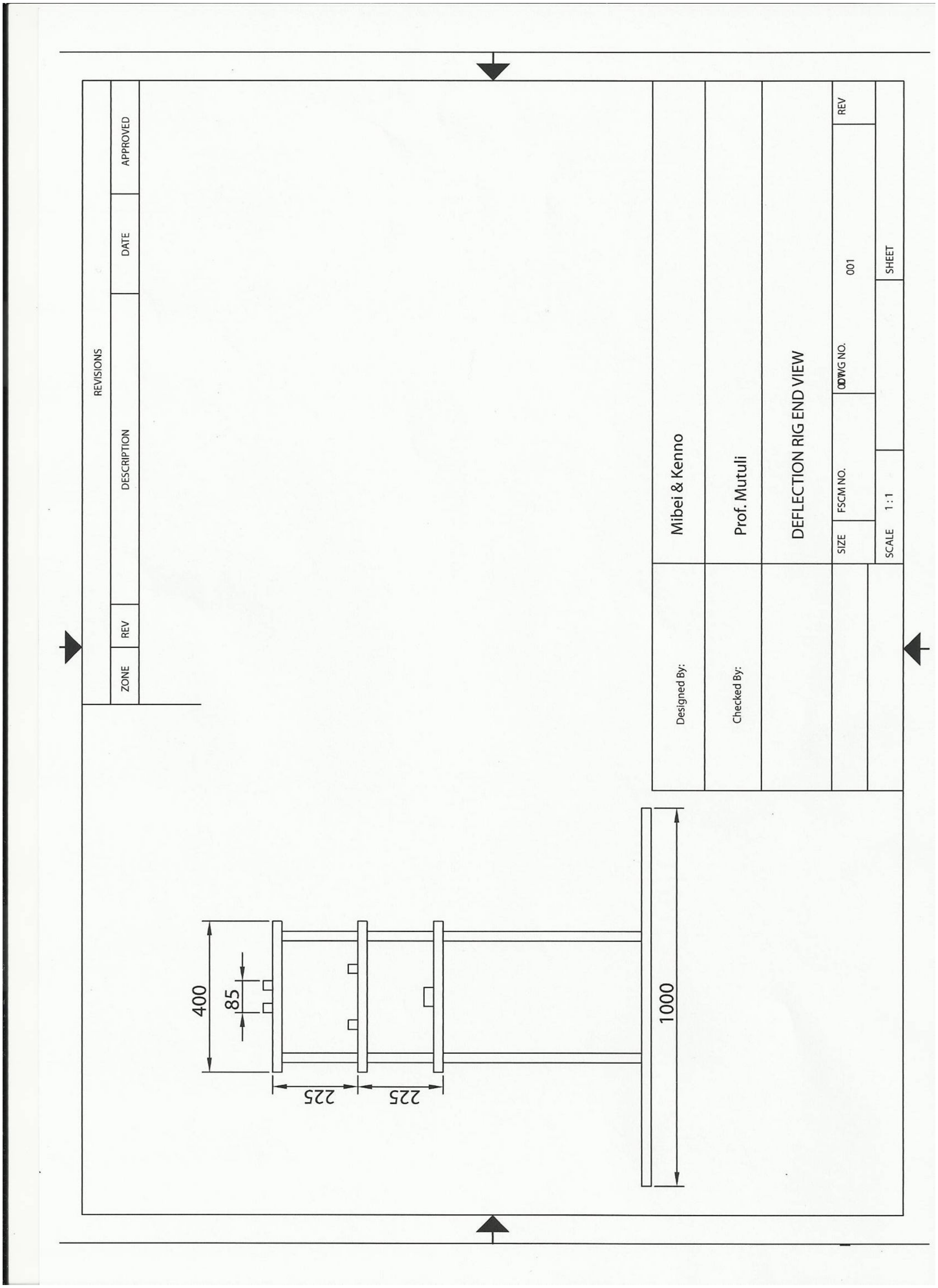
7.2 RECOMMENDATIONS

- The coupling fixture should be improved further on its stiffness and reduce its effects on effective stiffness of the beam. These will increase the accuracy of the measured deflections. This can be done by casting it to eliminate joints which reduce its stiffness and rigidity.
- The frictional effects on the load hanging mechanism on the coupling rail should be further reduced by use of roller bearings to further reduce errors during couple application.
- The rig should be improved further to measure of deflections from uniformly and linearly varying distributed loads in order to complete its universality.

8 REFFERENCES

1. Byars EF, (1963) ***Engineering Mechanics of Deformable Bodies*** ,International Textbook Company, Scranton Pennsylvannia (US)
2. FME 202 and FME 301 ***Solid and Structural Mechanics*** class notes.

APPENDIX: DESIGN DRAWINGS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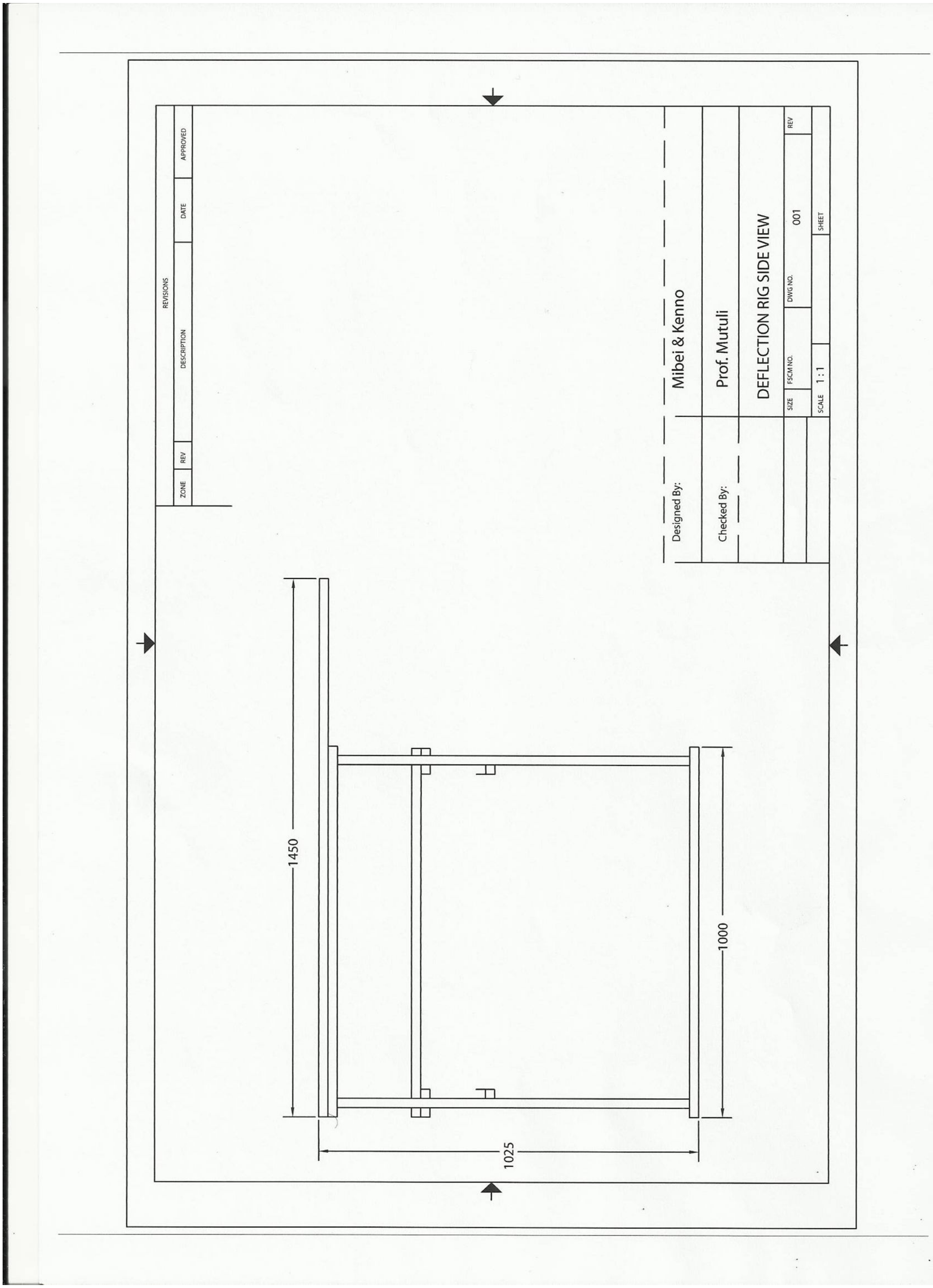
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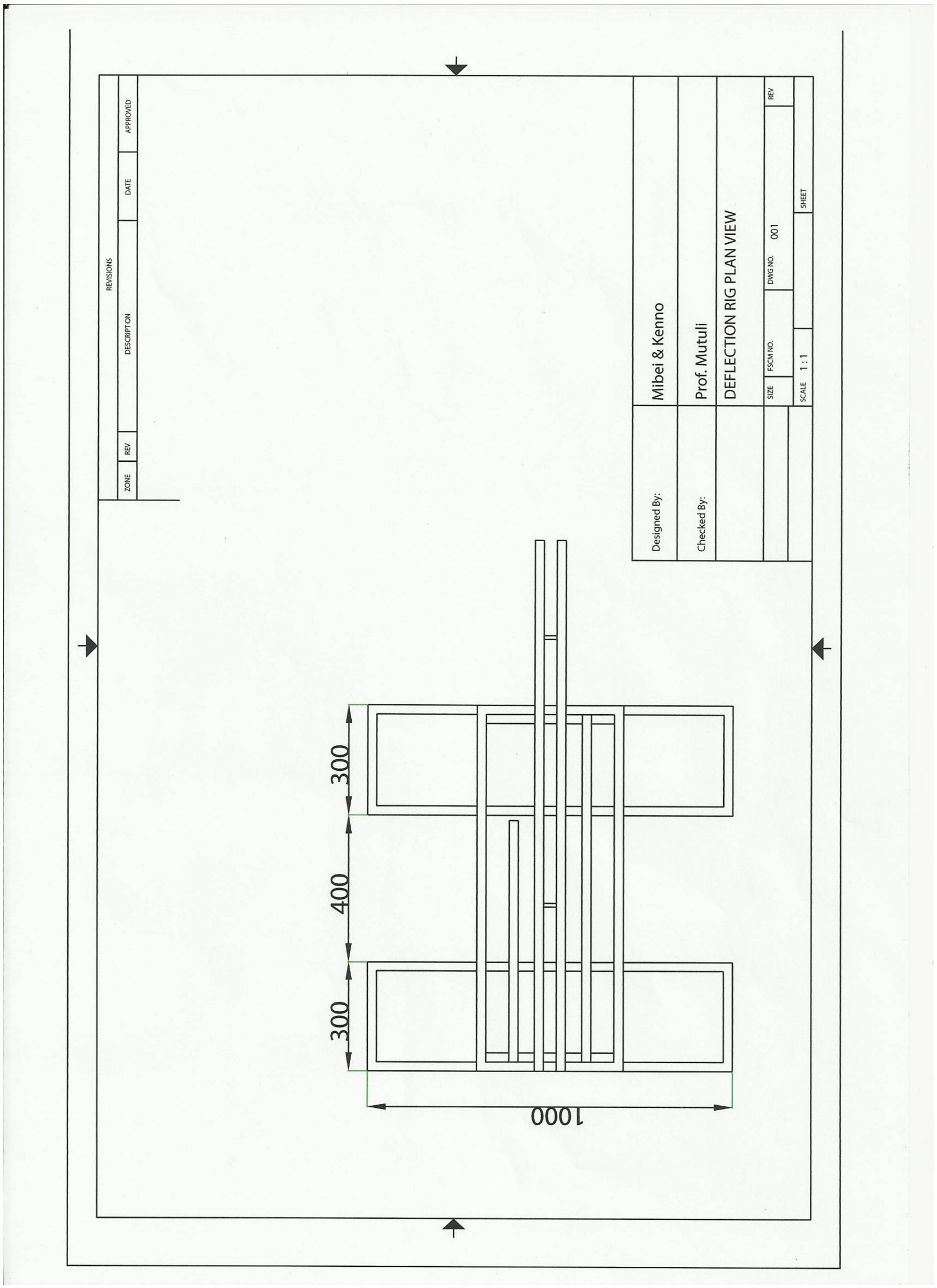
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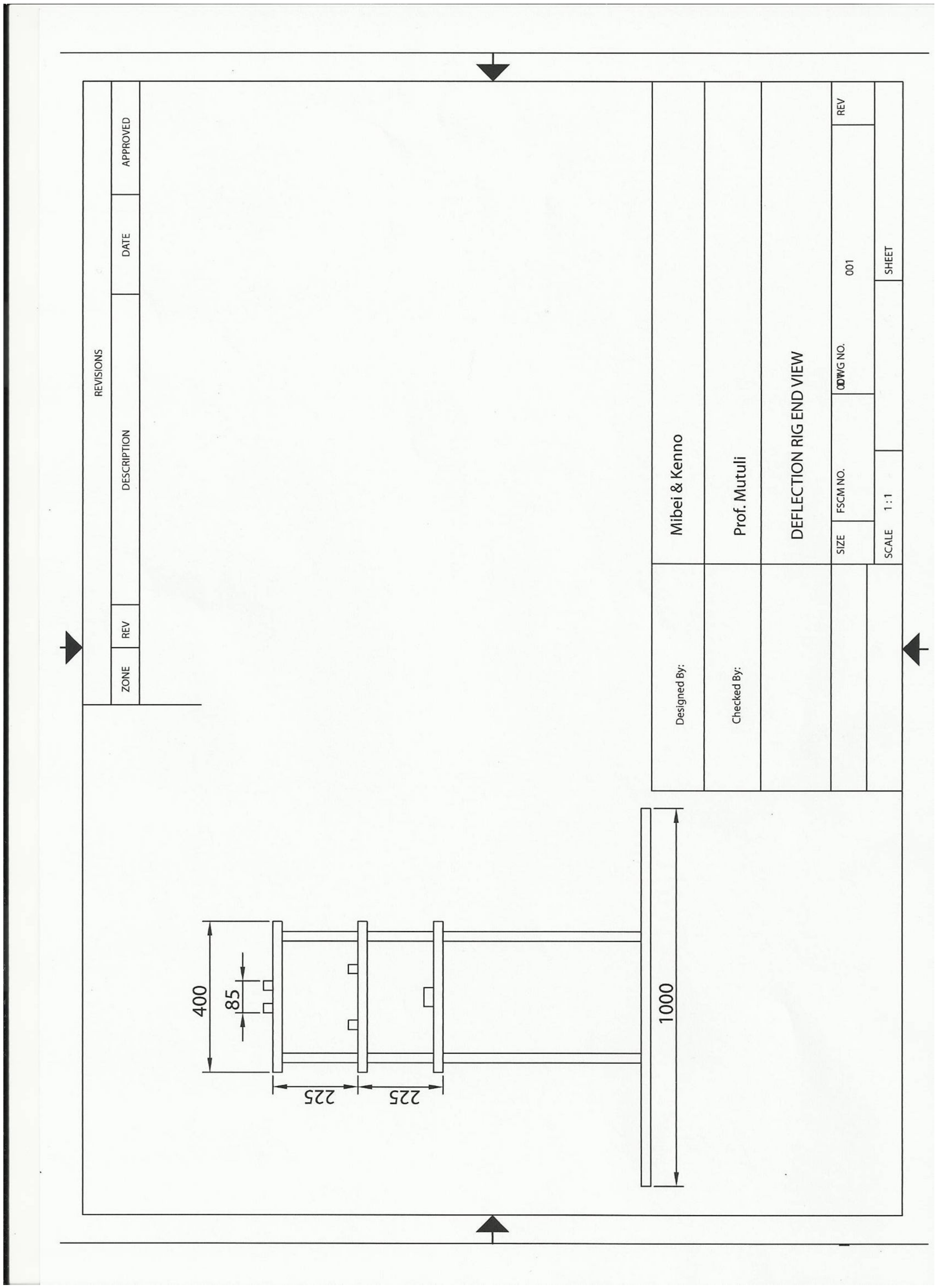
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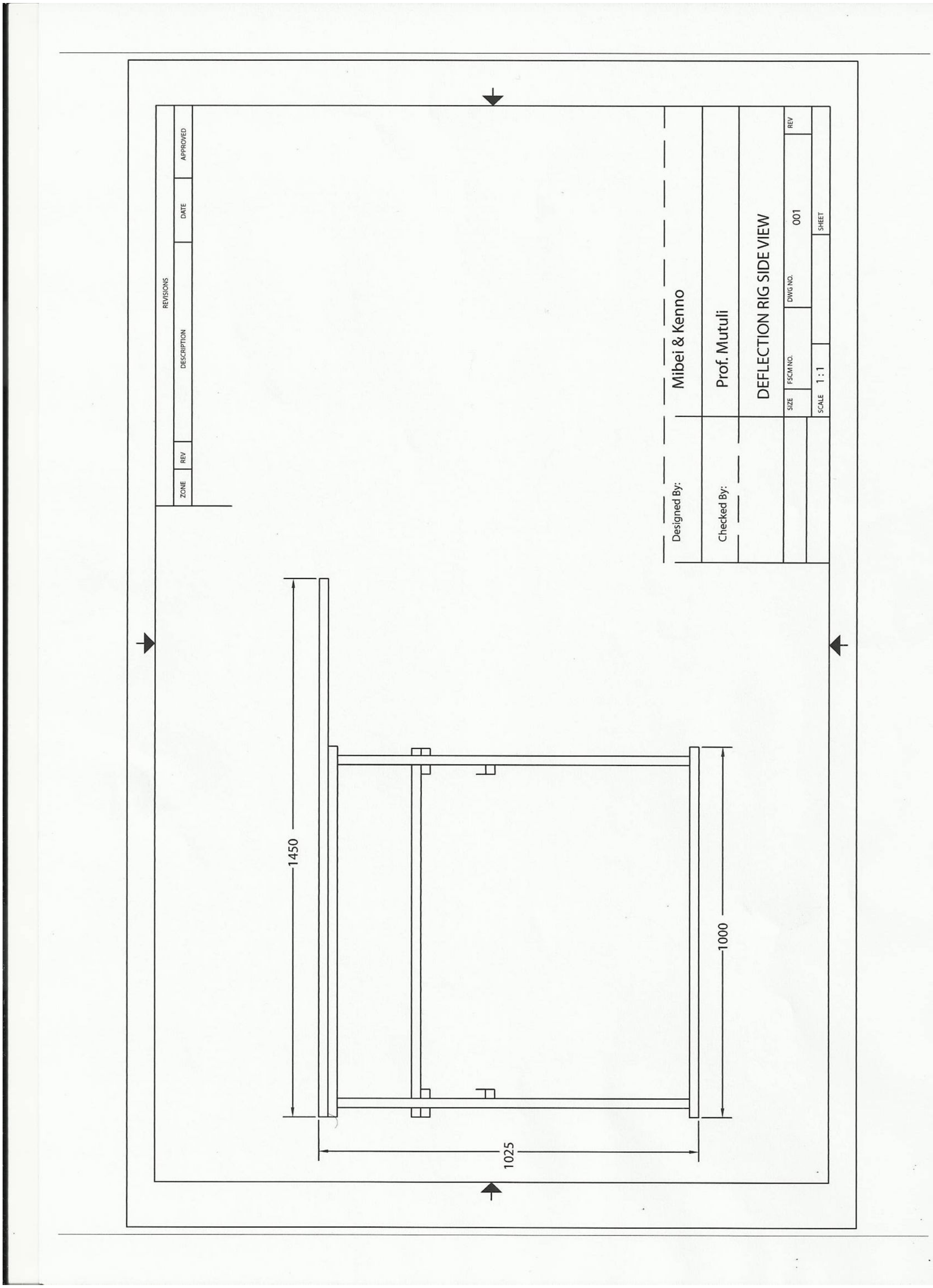

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