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KINEMATIC AND STATIC FORCE TRANSMISSION ANALYSIS OF A SINGLE TOGGLE JAW CRUSHER

A final year project report submitted in partial fulfillment of the award of the Bachelor’s degree in Mechanical Engineering of the University of Nairobi.

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DECLARATION

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ABSTRACT
Stone crushing is important in many sectors of economy such as in construction and in mining. Two types of equipment that are used in the crushing of stone are the horizontal Pittman stone crusher and the Single toggle stone crusher. Both have different characteristics and mechanisms that influence their application in crushing rocks. The following report aims at studying the kinematic and static force characteristics of each mechanism in order to determine how the two compare. The comparison began with the study of previously analyzed kinematics and torque transmission characteristics of the horizontal Pittman stone crusher. The study was extended by examining the torque transmission characteristics of the single toggle stone crusher. This was done by theoretical analysis followed by an experiment on a modified single toggle stone crusher, in order to collect results that were useful in establishing a relationship with a large scale stone crusher. There are many different characteristics that each mechanism can be compared against. Hence the choice of the mechanism to use is dependent on the application of the machine.
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ABBREVIATIONS AND SYMBOLS

$\theta_i$ - Angle of each member relative to the vertical

$r_i$ - Length of each member.

Y - Vertical axis direction.

Z - Horizontal axis direction.

P - Random location of a point on the swing jaw.

$Zp$ - Location of the point $P$ relative to the horizontal axis direction

$Yp$ - Location of the point $P$ relative to the vertical axis direction.

$\alpha$ - Angular acceleration.

$\alpha_{pv}$ - Angular acceleration at a point $P$ in the vertical axis direction.

$\alpha_{ph}$ - Angular acceleration at a point $P$ in the vertical axis direction.

$\phi$ - Angular position of point $P$ on the swing jaw, relative to the vertical.

$F_i$ - Force experienced by each member.

Fy - The force resolved in the vertical direction.

Fz - The force resolved in the horizontal direction.

$V_{pv}$ - Velocity at a point in the vertical axis.

$V_{ph}$ - Velocity at a point in the horizontal axis.

$T_i$ - Torque on the members.

Where $i = 1, 2, 3, 4...$
CHAPTER 1

1.1 Introduction

Over the recent past there has been an increased attention paid to the Kenyan mining sector. This has been credited mainly with the discovery of oil. This has raised the profile of Kenya as a mining country. This is with the exploration of other minerals as well. One of the mining machines that have found use in these operations is the stone crushers. The application of the stone crusher in mining is to help in the breaking down of the ore to allow for processing. There are various sizes of these crushers, the size being dependent on the application and the work load required of the crusher. The stone crusher’s other application are in recycling and in quarrying. In recycling the crusher is used to break down concrete to allow for easier disposal.

One area of application of the stone crusher that has found minimal use in Kenya and one that would be of greatest potential is in that of quarrying. Currently most of the quarrying operations in the country are undertaken by technology that is past its time. They usually involve the breaking down of rocks by simple hand tools in small scale. These techniques are largely labor intensive, produce inconsistencies in the aggregates that are produced and are limited in terms of the production capacities. One of the proposed solutions to these issues may be in the adoption of stone crushing technologies.

There are a number of factors that will play a role in the determination of what technologies will be applied in the quarrying process. The physical and chemical characteristics of the rocks to be crushed are one of these factors. They may be collectively thought of as the geological characteristics of the rock. These are factors such as the hardness of the rocks, the structural components of the rock and the abrasive nature of the rock amongst other factors. The consideration also lies in the output that one seeks. Different technologies provide different end products which may be suitable for different forms of applications. One factor that will be of consideration in this paper is as regards the type of technology that will be applied. This is from the realization that technologies may have limitations in their applications.

Stone crushing involves the use of machines that as earlier pointed out will vary in terms of the work load that is applied. Stone crushers are mainly classed depending on the pivoting mechanism of the swing jaw. In this major category one has the Blake crusher that is pivoted at the top which provides it with fixed amount of feed. This is then categorized into the double
toggle and the single toggle stone crushers. The other two varieties are dodge crusher which is pivoted at the bottom to provide fixed delivery and who use is limited to laboratory use. There is also the universal crusher that is pivoted at the mid position of the swinging jaw (Willis, 2006).

The different types of jaw crushers have different advantages and disadvantages that dictate the areas in which they are used. The construction of the double toggle jaw crusher enables its application as a crusher for strong abrasive rocks, buts its use is limited to its size and cost. On the other end the single toggle jaw crusher is favored for use due to its ability to provide greater throughput with less abrasion. It is also able to provide better crushing capacity as compared to the double toggle due to the angle of the jaw. The single toggle has a greater angle as compared to the double toggle which allows it to handle a greater amount of aggregate.

In the making of aggregate as proposed in this research paper, stone crushers are ideal for three main reasons. The first one is that they are efficient in the process of crushing stone. There is more that can be done in a little amount of time and this can be done without a lot of wastage of resources. The second is that it has cost saving benefits. The initial installation costs may be high, but the benefits that will accrue from the savings in terms of labor will more than offset this initial cost. Finally the stone crusher is able to provide consistency in the output which can be controlled with certain types of crushers as desired. This is especially important in the construction industry where the aggregate size determines the strength properties of the concrete used.

Despite these advantages there is the need to study the application of the stone crusher to determine its effectiveness in the application to which it has been placed under. There are various factors that would need to be considered in this case. Some of these factors include the cost that may be divided into initial cost of purchase of the crusher and the operational cost of the crusher. The other cost lies in the processing ability of the crusher that one is using. This may be dependent on the input that is fed to the crusher or the expected output from the crusher. These two may be compared to achieve a given scale that may be used in the decision making process. The most important of these considerations though may lie in the relative efficiencies of the different types of crushers.
1.1.1 Theory of Breaking down and Crumbling of Rocks

Essential requirements of breaking rocks

Rock material breaks if tensile stresses inside the particle exceed critical value causing a permanent deformation as crack propagates. Tensile stresses are obtained by loading the particle between two rigid surfaces or through impaction. A relative compression of about 0.3-0.4 per cent is needed in order to break a rock. Rock material breakage involves a series of steps in which the amount of stress application determines the degree of eventual size reduction. This fundamental concept known as a brittle rock fracture is presented in Figure 1.

Figure 1: Brittle fracture process of a rock and related theories [Heikkilä, 1991]

The stages of brittle rock fracture are as follows:

1. Inner cracks are closed when small amount of stress is applied to corresponding rock
2. After crack closing, material is subjected to linear elastic deformation
3. Fracture initiates when tensile stresses inside the particle exceeds the critical value resulting a stable fracture propagation
4. If particle stresses exceed the critical energy release the fracture propagation will turn unstable, resulting in the total rupture of a rock.

Rock material breakage patterns for single particle breakage

As presented in previous figure, an amount of stress application determines the degree of size reduction in rock material breakage. Three different breakage patterns are identified for a single particle breakage. Those patterns are shatter, cleavage and attrition.

Shatter

First breakage pattern is called shatter. Shatter takes place due to rapid application of compressive stress. This is a fracture mechanism that produces a broad spectrum of product sizes as material undergoes a total rupture. The formation of product size distribution in shattering of parent particle is presented in Figure 2.

![Figure 2: Size distribution originating from single particle breakage in shattering [King, 2001]](image)

Shattering process consists of series of consecutive steps in which originating particle is fractured as breakage of the parent particle is followed by sequential fracturing of successive generations of daughter fragments until all energy available for fracture is dissipated. The population of progeny particles is made of number of sub-populations; those from primary fracture process and those from successive re-breakage. Shattering is the most common mode of
fracture for crushers using dynamic breakage, like vertical and horizontal shaft imp actors. Shattering is also the breakage mode requiring the largest amount of crushing energy.

**Cleavage**

If there is not enough force for multiple fractures of daughter fragments, particle will be broken into two particles close to equal in size. This breakage pattern is known as cleavage. Cleavage is most likely to happen in uni-axial loading conditions where particle is loaded between two rigid surfaces. Size distribution of product particles will be relatively narrow, often bimodal or sometimes multi-modal as particles fall into two main categories. Formation of daughter fragments is presented in Figure 3.

![Figure 3: Formation of progeny particles in cleavage.](image)

Most of the fine particles produced originate from contact surfaces along the crack lines due to shear stresses. This breakage pattern is the most common in crushers using compressive action like cone and gyratory crushers.
Attrition (abrasion) and Chipping

When stresses are not large enough to cause a fracture, attrition occurs at the weakest parts of a particle. Parent particle size hardly changes but particle shape is affected so that corners are chipped away. Generally this means that the particles become more cubic in shape. Formation of progeny particles via attrition is presented in Figure 4. In order to achieve conditions suitable for attrition, particle must be loaded from multiple points. Typically, such conditions can be achieved in an inter particle crushing, where generation of particles are compressed against each other. Such breakage pattern is dominant in fine crushers used in tertiary and quaternary stage, where reduction ratio is low.

![Figure 4: Formation of progeny particles in attrition [King, 2001]](image)

**1.1.2 Summary of Breakage Modes**

Breakage pattern is controlled by the intensity of applied crushing energy; the higher the intensity, the more complete rupture of originating particle is. This dependency between applied energy and degree of size reduction can be seen clearly from Figure 2, presenting the uniaxial compression tests of single particle.

These tests also reveal that attrition can be achieved with the lower crushing energy than cleavage, whereas the shatter requires clearly most crushing energy of all. For every crusher type
there is a primary breakage mode that is the dominant over the other two. Thus every crusher type has its own characteristic output curve.

Figure 5: Effect of applied crushing energy on degree of size reduction during uni-axial compression. Applied compression ratios were: (from right to left) 0.05, 0.17, 0.25 and crushing energies: 0.007, 0.010, 0.488 kWh/t.

1.2 Problem Statement

In the efficiency one is looking at the input and comparing this to the output that the crusher is giving in terms of the work that is input into the crusher. In the research paper the examination will look into the work that is supplied to the driving mechanisms from the motor and how this is transmitted through the different mechanisms to yield the desired output. The efficiencies of the different crushers may then be used to make an argument as to which crusher is mechanically better than the others. This examination will go towards developing the major considerations that will go into the final decision making in terms of the best crusher. This is despite the fact that this may not be the only factor that is taken into consideration when considering the type of crusher to use.
1.3 Aim of the Project

The aim of the project was to carry out a force analysis of a crank rocker single toggle mechanism and compare it with the Pittman type stone crusher. Furthermore, make recommendations on the conclusion drawn from the two mechanisms and make determinations as to the best mechanism for use in stone crushing.

1.4 Methodology

- The project will begin by looking at past literature that is available on the existing crusher mechanisms.
- Visiting various large scale stone crushing facilities in the country to determine their needs.
- Studying the laboratory stone crusher to understand its operational mechanisms and output.
- Analyze data from theoretical single toggle force analysis and comparing this with laboratory experiment data.
CHAPTER 2

2.1 Literature Review

In considering the literature that is available on the jaw crusher there are two basic approaches that one may take. The first would be to consider the theoretical background that has been developed on the different types of jaw crushers. This considers texts on mechanics that look at crushers and different academic journals that have researched the development of the available information on the jaw crusher. The second approach would be to look at the information that is presented for industrial crushers.

Hartman in his text indicate that the first jaw crusher was patented in 1830 and that there have been many different types since then. This includes the single toggle, dodge type and the Blake type also known as the double toggle. The double toggle finds use where the more heavy duty crushing is being applied (Hartman et al. 1992). They introduce the first major difference between the different type of crushers as well as some of the terminologies used.

Hendricks has built on this by examining the working mechanisms of the different types of jaw crushers. This examination provides an introductory and basic look at the working mechanism of the jaw crushers without examining the technical aspects (Hendricks, 2000). Additional information on this is provided by Swain, who also goes ahead to consider some preliminary scientific deduction on the workings of the jaw crusher. This is by providing mathematical calculations by which the capacity of the jaw crusher may be determined (Swain, Patra & Roy, 2011).

Others such as Cao et al. presented data that has been used in the kinematic study of single toggle jaw crushers (Cao et al. 2006). This includes by authors such as Deepak who presented mathematical developed kinematic equations using this data. This data was also used by Garnaik in the development of graphs using a mathematical program called MATLAB (Garnaik, 2010).
CHAPTER 3

3.1 Functional Analysis

3.1.1 Jaw crusher Working Principle

The jaw crusher is composed of a main frame, eccentric shaft, flywheel, toggle plate, jaw plate, tension rod, fixed jaw, and movable jaw and so on. The motor or internal combustion engine transmits power through a belt drive, drives the moving jaw, through the eccentric shaft, to execute periodic motion towards, and away from the fixed jaw. The angle between the toggle plate and the moving jaw plate varies as the moving jaw moves through its cycle. The feed material will be crushed in this process. The final crushed material will be discharged from the outlet, at the bottom of the crusher. The moving jaw of the crusher enables periodic crushing and discharging to realize batch production.

Figure 6: Cutaway view of a jaw crusher
3.1.2 Jaw Crusher Components

1. **Frame:** Is a heavy duty design steel plates welded construction. It is welded by $CO_2$ shield arc process. Stress is relieved after fabrication.

2. **Crashing Chamber:** Deep crushing chamber with sharp nip angle ensures high crushing capacity.

3. **Swing Jaw and Main Bearing Housing:** Swing jaw is a robust steel casting with fully machined face to support swing jaw. Main bearing housing allows removal from frame as an integral sub assembly and permits of change over unit in the event bearings damage.

4. **Flywheel:** Balances the inertia forces and promotes smooth operation of the crusher.

5. **Eccentric Shaft:** Is a forged from hardened and tempered chrome molybdenum steel. They have large diameters to suit heavy duty application

6. **Bearing:** Fag or SKF heavy duty self-aligning double row bearings on swing jaw and main frame Bath oil lubrication ensures positive lubrication. Bearing life under typical quarry application is in excess of 8 years.

7. **Hydraulic Cylinder:** The cylinder allows easy adjustment of discharge setting by moving toggle block to the desired setting.

8. **Shim:** Allows easy adjustment of closed side setting.

9. **Toggle plate:** Designed to shear protecting crusher components if none crushable objects is induced to crushers

10. **Check plates:** Plates are made of manganese steel castings that allow easy replacement.

11. **Jaw plate:** High manganese steel castings can be reversed allowing extended life. Wedge or lug system allows easy replacement.

The single toggle jaw crusher is versatile and it can be used to crush rocks, whose hardness may range from medium-hard to extremely hard, as well as different kinds of ore, building rubble and glass, among other hard materials. It is widely used in a variety of demolition, extraction, reclamation and recycling industries, but especially, in the mining and construction sectors (AUBEMA Jaw Crushers, 2013; SBM Mining and Construction Machinery, 2013).
The heart of the crushing mechanism of a jaw crusher consists of two metallic jaw plates that are slightly and oppositely inclined away from the vertical to form a V Shaped crushing zone with a wide upper opening and a narrow lower opening. One of the jaw plates are fixed while the other is movable and referred to as the swing jaw. When in operation, the charge of material to be crushed is fed into the crushing zone through the upper opening. The swing jaw is driven to execute a cyclic reversing motion and to apply cyclic intermittent compressive forces that crush the charge of material against the fixed jaw. As the larger lumps of material are crushed into smaller lumps, they fall, under gravity, into the narrower lower sections of the crushing zone, where they are crushed again into even smaller lumps. This process is repeated until the charge of material is all crushed into aggregates that are small enough to fall out of the crusher, through the opening at the lower end of the crushing zone (Gupta and Yan, 2006). The crushing mechanism is enclosed in a box-like steel frame. The jaw crusher can be crawler track-mounted or trailer-mounted to realize a mobile unit that can be repositioned, when the need arises, even as the work advances.

Moreover, in many cases, the jaw crusher can be easily disassembled for relocation or access to confined places (Carter, 1999). This enables the jaw crusher to be used in both surface and underground mining. Other advantages of the jaw crusher include its simplicity in structure and mechanism, reliability, ease of maintenance, and high capacity, as compared to other types of crusher such as the Cone crusher, the gyratory crusher and the various designs of impact crushers (Zhong and Chen, 2010).

Today, the most commonly used types of jaw crusher are the single toggle and the double toggle designs. The original double toggle jaw crusher was designed by Eli Whitney Blake in the USA in 1857 (Mular et. al., 2002). The motion of the swing jaw in a double toggle crusher is such that it applies an almost purely compressive force upon the material being crushed. This minimizes wear on the crushing surfaces of the jaws and makes the double toggle jaw crusher suitable for crushing highly abrasive and very hard materials. Even today, the Blake design, with some comparatively minor improvements, can still be found in mines and quarries around the world.

The single toggle design, which was developed between the 1920s and the 1950s, is a simpler, lighter crusher (Mular et. al., 2002). Its swing jaw has an elliptical rolling motion such that it applies a compressive as well as a rubbing force on the material being crushed. This has a force-
feeding effect that improves the throughput of the device but it also tends to cause rapid wear of the crushing surfaces of the jaws.

However, the single toggle jaw crusher has a lower installed cost, as compared to the double toggle design. Improvements in materials and design have made the single toggle jaw crusher more common today as the primary crusher in quarrying operations (The Institute of Quarrying Australia, 2013). According to Carter (1999), sales of the single toggle jaw crusher exceed those of the double toggle jaw crusher by a factor of at least nine to one. The crushing action of a jaw crusher is brought about by the motion of its swing jaw and the forces that it exerts on the material being crushed (Gupta and Yan, 2006; Pennsylvania Crusher Corporation, 2006). Therefore, in the study and design of the jaw crusher, it is important to understand the kinematics of the swing jaw.

This paper sets out to obtain a complete kinematical description of the single toggle jaw crusher, from first principles. The Pittman jaw crusher on the other hand, are Jaw crushers most easily recognized any quarrying operation. They are also probably the oldest design of mechanical crushers, except for spalling hammers and stamping batteries.

They generally consist of a heavy-duty steel box-like frame, fitted with a vertical, or nearly vertical, fixed crushing jaw, at one end, and a moving swing jaw, at the opposite end. The swing jaw is provided with a mechanism that drives it and causes it to move with a cyclic oscillating motion. Thus, the swing jaw continuously moves towards and away from the fixed jaw, and in so doing it subjects the charge of material to be crushed to continuous waves of compression. This movement can be provided a motor or internal combustion engine fixed as part of the crusher. The motor or internal combustion engine transmits power through a belt drive, driving the movable jaw, through the eccentric shaft, to execute the periodic motion towards and away from the fixed jaw.
CHAPTER 4

4.1 Kinematic Analysis of the Jaw Crusher

The mechanism that was proposed consists of a four bar eccentric shaft and rocker mechanism with the rocker being the swing jaw. A simple line diagram of this mechanism is shown below.

In analysis of the kinematics of the above crusher, an understanding of the motion of the rocker, relative to the fixed jaw as the crank rotates through a complete cycle is mandatory. All angular displacements are taken counter clockwise, relative to the Y direction.

Figure 7: kinematic Model of a Single Toggle Jaw Crusher
4.1.1 Position and Displacement Analysis

The analysis of the position and displacement can be accomplished through use of the well-known vector loop closure method, which is illustrated in figure 8 below.

![Figure 8: Vector-Loop Closure](image)

In Fig. 8, the vector loop equation can be written as follows:

\[ \mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4 = 0 \]  

Equation 1 above can be re-written in complex notation as follows:
\[ r_1 e^{j\theta_1} + r_2 e^{j\theta_2} + r_3 e^{j\theta_3} + r_4 e^{j\theta_4} \]  \hspace{1cm} (2)

Noting the Euler Identities:

\[
\begin{align*}
e^{j\theta} &= \cos \theta + j \sin \theta \\
e^{-j\theta} &= \cos \theta - j \sin \theta
\end{align*}
\]  \hspace{1cm} (3)

For conciseness the following notation can also be introduced:

\[
\begin{align*}
\cos \theta_i &= c_i \\
\sin \theta_i &= s_i
\end{align*}
\]  \hspace{1cm} (4)

Using equations (3) and (4), equation (2) can be re-written as:

\[ r_1(c_1 + js_1) + r_2(c_2 + js_2) + r_3(c_3 + js_3) + r_4(c_4 + js_4) = 0 \]  \hspace{1cm} (5)

The real and imaginary parts of the equation (5) could be separated to obtain:

\[
\begin{align*}
r_1c_1 + r_2c_2 &= -(r_3c_3 + r_4c_4) \\
r_1s_1 + r_2s_2 &= -(r_3s_3 + r_4s_4)
\end{align*}
\]  \hspace{1cm} (6)

Furthermore, both equations above could be squared to yield the following equations:

\[
\begin{align*}
r_1^2c_1^2 + 2r_1r_2c_1c_2 + r_2^2c_2^2 &= r_3^2c_3^2 + 2r_3r_4c_3c_4 + r_4^2c_4^2 \\
r_1^2s_1^2 + 2r_1r_2s_1s_2 + r_2^2s_2^2 &= r_3^2s_3^2 + 2r_3r_4s_3s_4 + r_4^2s_4^2
\end{align*}
\]  \hspace{1cm} (7)

By adding the corresponding terms in equation (7) above and noting that \(c_i^2 + s_i^2 = 1\), we obtain the following:

\[ r_1^2 + 2r_1r_2(c_1c_2 + s_1s_2) + r_2^2 = r_3^2 + 2r_3r_4(c_3c_3 + s_3s_3) + r_4^2 \]  \hspace{1cm} (8)

Rearranging equation (6) we have:

\[
\begin{align*}
r_1c_3 &= -(r_1c_1 + r_2c_2 + r_4c_4) \\
r_1s_3 &= -(r_1s_1 + r_2s_2 + r_4s_4)
\end{align*}
\]  \hspace{1cm} (9)

From trigonometry:

\[ \cos \theta_i \cos \theta_k + \sin \theta_i \sin \theta_k = \cos(\theta_i - \theta_k) \]  \hspace{1cm} (10)

By substituting equation (9) into (8) and using the identity in equation (10), we obtain:
\[ 2r_1r_2 \cos(\theta_2 - \theta_1) + 2r_2r_4 \cos(\theta_4 - \theta_1) + r_1^2 + r_2^2 - r_3^2 + r_4^2 = -2r_2r_4 \cos(\theta_4 - \theta_2) \]  

(11)

From figure (11) above, \( \theta_1 \) is a fixed quantity and for given values of \( r_1, r_2, r_3 \) and \( r_4 \) the value of \( \theta_1 \) will be known.

Also, motion of the crank \( O_2O_3 \) is the input motion. It may be considered to be a rotation at uniform angular velocity \( \omega_2 \). At an instant time \( t \), after commencement of motion, the value \( \theta_2 \) in radians will be determined as follows:

\[ \theta_2(t) = \omega_2 t \]  

(12)

For given lengths of the four links in the mechanism, equation (11) can be used to determine the values of \( \theta_4 \) that correspond to any given value of \( \theta_2 \).

Equation (11) describes all possible spatial configurations of the mechanism, for given lengths of the four links. For the case where \( \theta_1 = 0 \) , equation (11) becomes:

\[ 2r_1r_2 \cos \theta_2 + 2r_2r_4 \cos \theta_4 + r_1^2 + r_2^2 - r_3^2 + r_4^2 = -2r_2r_4 \cos(\theta_4 - \theta_2) \]  

(13)

Dividing each term by \( 2r_2r_4 \), we obtain the following equation:

\[ \frac{r_1}{r_4} \cos \theta_2 + \frac{r_1}{r_2} \cos \theta_4 + \frac{r_1^2 + r_2^2 - r_3^2 + r_4^2}{2r_2r_4} = \cos(\theta_4 - \theta_2) \]  

(14)

The above equation can also be written as follows:

\[ K_1 \cos \theta_2 + K_2 \cos \theta_4 + K_3 = \cos(\theta_4 - \theta_2) \]

\[ \begin{align*}
K_1 &= \frac{r_1}{r_4} \\
K_2 &= \frac{r_1}{r_2} \\
K_3 &= \frac{r_1^2 + r_2^2 - r_3^2 + r_4^2}{2r_2r_4}
\end{align*} \]  

(15)
Equation (15) is called the Freudenstein equation. For given values of the lengths of the four links, the equation can be used to determine values of \( \theta_4 \) that correspond to any given values of \( \theta_1 \).

The data in Table 1 shall be used to demonstrate how the kinematic equations are applied.

<table>
<thead>
<tr>
<th>( r_1 \sin \theta_1 ) (mm)</th>
<th>( r_1 \cos \theta_1 ) (mm)</th>
<th>( r_2 ) (mm)</th>
<th>( r_3 ) (mm)</th>
<th>( r_4 ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>45.3</td>
<td>815.7</td>
<td>12</td>
<td>1085</td>
<td>455</td>
</tr>
</tbody>
</table>

From the data in Table 1 above, it follows to good approximation that, \( r_1 = 817 \text{ mm} \) and \( \theta_1 = 3.18^\circ \). This data will be used for the analysis of the crusher.

4.1.2 Angular displacement of the Swing Jaw

Equation (6) can be re-arranged to obtain the following equation:

\[
\begin{align*}
\left\{ 
\begin{array}{c}
r_1c_1 + r_2c_2 + r_3c_3 = -r_4c_4 \\
r_1s_1 + r_2s_2 + r_3s_3 = -r_4s_4
\end{array}
\right. \\
(16)
\end{align*}
\]

Equation (16) can be substituted into equation (8) and using equation (10), the following is obtained:

\[
\begin{align*}
2r_1r_2 \cos(\theta_2 - \theta_1) + 2r_2r_3 \cos(\theta_3 - \theta_2) + r_1^2 + r_2^2 + r_3^2 - r_4^2 &= -2r_3r_1 \cos(\theta_3 - \theta_1) \\
(17)
\end{align*}
\]

For given lengths of the four links in the mechanism, along with the value of \( \theta_1 \), equation (17) can be used to determine corresponding values of \( \theta_3 \) for the given values of \( \theta_2 \). When compared to equation (11), equation (17) is of greater utility in describing the motion of the swing jaw, relative to that of the crank.

If \( \theta_1 = 0 \) equation (17) reduces to the following:

\[
\begin{align*}
2r_1r_2 \cos \theta_2 + 2r_2r_1 \cos \theta_3 + r_4^2 - r_3^2 - r_2^2 - r_1^2 &= -2r_2r_3 \cos(\theta_3 - \theta_2) \\
(18)
\end{align*}
\]
Equation (18) can be divided through by $2r_2 r_3$ to obtain the following:

$$\frac{r_1}{r_3}\cos\theta_2 + \frac{r_1}{r_2}\cos\theta_1 + \frac{r_1^2 - r_3^2 - r_2^2 + r_1^2}{2r_2 r_3} = -\cos(\theta_3 - \theta_2) \quad (19)$$

The equation (19) above could be rewritten as follows:

$$\begin{align*}
K_1 \cos\theta_2 + K_2 \cos\theta_3 + K_3 &= -\cos(\theta_3 - \theta_2) \\
K_1 &= \frac{r_1}{r_3} \\
K_2 &= \frac{r_1}{r_2} \\
K_3 &= \frac{r_4^2 - r_3^2 - r_2^2 - r_1^2}{2r_2 r_3} 
\end{align*} \quad (20)$$

Equation (20) may be regarded as another version of Freudenstein’s equation. For the given values of the lengths of the four links, the equation can be used to determine the values of $\theta_3$ that correspond to given values of $\theta_2$.

By using the data in table 1, equation (17) could be reduced as follows:

$$\begin{align*}
K_1 \cos\theta_3 + K_2 \sin\theta_3 + K_3 &= 0 \\
K_1 &= \cos\theta_2 + 68 \\
K_2 &= \sin\theta_2 + 3.8 \\
K_3 &= 62.9 + 0.75\cos\theta_2 + 0.042\sin\theta_2 
\end{align*} \quad (21)$$

In equation (21), for any given value of $\theta_2$, $K_1$, $K_2$ and $K_3$ can be determined.

Moreover, the first of equations (21) may be re-written as follows:

$$K_1 \cos\theta_3 = -(K_2 \sin\theta_3 + K_3) \quad (22)$$

By squaring both sides of equation (37), using the well-known trigonometric identity $\cos^2 \theta = 1 - \sin^2 \theta$ and then re-arranging the result, the following can be obtained:
For any given value of $\theta_2$, the equation (23) above is quadratic in $\sin \theta_3$ and can therefore be solved to yield 2 values of $\theta_3$. Thus there are two possible configurations of the four bar mechanism in figure 8 for any possible value of $\theta_2$.

However, it is important to note that only one configuration is proper, that is, the one giving value of $\theta_3$ lying between $90^\circ$ and $180^\circ$ as shown in figure 9 below.

The angle $\theta_3$ being greater than $180^\circ$ would be absurd as per the crusher design and is therefore unsuitable.
Table 2: Analytically determined values of $\theta_3$ for a given values of $\theta_2$

<table>
<thead>
<tr>
<th>$\theta_2$ (Degrees)</th>
<th>$\theta_3$ (Degrees)</th>
<th>$\theta_2$ (Degrees)</th>
<th>$\theta_3$ (Degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>160.2</td>
<td>195</td>
<td>160.8</td>
</tr>
<tr>
<td>15</td>
<td>160.5</td>
<td>210</td>
<td>160.6</td>
</tr>
<tr>
<td>30</td>
<td>160.7</td>
<td>225</td>
<td>160.4</td>
</tr>
<tr>
<td>45</td>
<td>160.9</td>
<td>240</td>
<td>160.1</td>
</tr>
<tr>
<td>60</td>
<td>161.1</td>
<td>255</td>
<td>160.1</td>
</tr>
<tr>
<td>75</td>
<td>161.3</td>
<td>270</td>
<td>160</td>
</tr>
<tr>
<td>90</td>
<td>161.5</td>
<td>285</td>
<td>159.7</td>
</tr>
<tr>
<td>105</td>
<td>161.5</td>
<td>300</td>
<td>159.7</td>
</tr>
<tr>
<td>120</td>
<td>161.6</td>
<td>315</td>
<td>159.8</td>
</tr>
<tr>
<td>135</td>
<td>161.5</td>
<td>330</td>
<td>159.9</td>
</tr>
<tr>
<td>150</td>
<td>161.4</td>
<td>345</td>
<td>160</td>
</tr>
<tr>
<td>165</td>
<td>161.3</td>
<td>360</td>
<td>160.2</td>
</tr>
<tr>
<td>180</td>
<td>161.1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For one full cycle of the crank, the minimum value of $\theta_3$ is 159.7°, whereas the maximum value of $\theta_3$ is 161.6°. Thus, the range of variation of the inclination of the rocker, swing jaw, to the vertical is about 1.9°. With the length of the rocker being 1085 mm, this range of angular oscillation of the swing jaw translates to a throw of about 24 mm at the lower end of the swing jaw. A graph of the variation of $\theta_3$ with $\theta_2$ is shown in figure 10 below:
By using equation (23) and the values of constants $K_1$, $K_2$ and $K_3$, the following is obtained:

\[
\begin{align*}
4635.45 \sin^2 \theta_3 - 8564.46 \sin \theta_3 + 3955.84 &= 0 \\
A &= K_1^2 + K_2^2 = 4635.45 \\
B &= 2K_2K_3 = -8564.46 \\
C &= K_3^2 - K_1^2 = 3955.84
\end{align*}
\]

Equation (24) can be solved using the quadratic formula to yield two values of $\theta_3$ which are $166.64^\circ$ and $166.34^\circ$.

During the cycle of motion of the mechanism, two particular phases are of interest. These special phases, which are known as toggle positions occur when the crank $O_2O_3$ and the coupler $O_3O_4$ fall on the same straight line. For this to happen, either $\theta_3$ must be equal to $(\theta_2 + 180)^\circ$ or $\theta_3$ must be equal to $\theta_2$. When these conditions are used in equation (17), along with the data in Table 1, it is found that the toggle positions will occur when $\theta_2 = 161.35^\circ$ and when $\theta_2 = 340^\circ$. In determining
the toggle positions, due regard must be given to the fact that, for each value of \( \theta_2 \), there will be two possible configurations of the mechanism, only one of which will be suitable for the proper operation of the crusher.

4.1.3 **Position and Displacement of a Point in the Swing Jaw**

It should be informative to be able to determine the motion of a point in the swing jaw, particularly on the crushing surface of the swing jaw, as it varies with the motion of the crank. The position of such a point would be fixed relative to that of the swing jaw \( O_1O_4 \) (Fig. 7). For the purpose of locating such a point, the coordinate system illustrated in Fig. 11 will be used.
In Fig. 11, the $Y'Z'$ coordinate reference frame has its origin at $O_3$ and it is fixed in the swing jaw. The point $P$ too is fixed in the swing jaw and its position is located by the vector $r_5$, of magnitude $r_5$, whose origin is at $O_3$ and whose direction is indicated by the angle $\phi_5$. Thus:
\[ y' = r_5 \cos \phi_5 \]
\[ z' = r_5 \sin \phi_5 \]  \hspace{1cm} (25)

The direction of the vector \( \mathbf{r}_5 \) taken relative to the \( Y \) direction is indicated by the angle \( \theta_5 \), such that:

\[ \theta_5 = (\theta_3 + \phi_3 - 90)^\circ \]  \hspace{1cm} (26)

Thus the following equation is obtained:

\[ \sin \theta_5 = -\cos(\theta_3 + \phi_3) = \sin \theta_3 \sin \phi_5 - \cos \theta_3 \cos \phi_5 \]
\[ \cos \theta_5 = \sin(\theta_3 + \phi_3) = \sin \theta_3 \cos \phi_5 + \cos \theta_3 \sin \phi_5 \]  \hspace{1cm} (27)

In the special case where \( \phi_5 = 90^\circ \), \( \theta_5 \) becomes equal to \( \theta_3 \) and the point \( P \) then lies on the line \( O_3 O_4 \), at a distance of \( r_3 \) from \( O_3 \).

In Fig. 15, the location of point \( P \) relative to the \( YZ \) coordinate reference frame may now be expressed as follows:

\[ y_p = r_1 \cos \theta_1 + r_2 \cos \theta_2 + r_5 \cos \theta_5 \]
\[ z_p = r_1 \sin \theta_1 + r_2 \sin \theta_2 + r_5 \sin \theta_5 \]  \hspace{1cm} (28)

By using equations (27), equations (28) can be re-written as follows:

\[ y_p = r_1 \cos \theta_1 + r_2 \cos \theta_2 + r_5 (\sin \theta_3 \cos \phi_5 + \cos \theta_3 \sin \phi_5) \]
\[ z_p = r_1 \sin \theta_1 + r_2 \sin \theta_2 + r_5 (\sin \theta_3 \sin \phi_5 - \cos \theta_3 \cos \phi_5) \]  \hspace{1cm} (29)

In the special case where \( \phi_5 = 90^\circ \), equations (29) reduce to the following:

\[ y_p = r_1 \cos \theta_1 + r_2 \cos \theta_2 + r_5 \cos \theta_3 \]
\[ z_p = r_1 \sin \theta_1 + r_2 \sin \theta_2 + r_5 \sin \theta_3 \]  \hspace{1cm} (30)

Given the lengths \( r_1, r_2, r_3 \) and \( r_4 \) of the links, along with the angle \( \theta_1 \), equations (17) and (30) can be used to determine the locus of any point on the line \( O_3 O_4 \), for a complete cycle of rotation of the crank \( O_2 O_3 \), provided that the distance \( r_5 \) of that point from \( O_3 \) is known.

Five points were selected along the length of the line \( O_3 O_4 \), whose distances from \( O_3 \) are given in Table 3 below. The point \( P1 \) is coincident with \( O_3 \) and the point \( P5 \) is coincident with \( O_4 \). The rest of the points are uniformly spaced along the length of the line \( O_3 O_4 \).
Table 3: Locations of Selected Points along the Coupler

<table>
<thead>
<tr>
<th>Point</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_5 ) (mm)</td>
<td>0</td>
<td>271.25</td>
<td>542.5</td>
<td>813.75</td>
<td>1085</td>
</tr>
</tbody>
</table>

Using equations (30), along with the data given in Tables 1 and 3, the positions of the points \( P1 \) through to \( P5 \) were determined for one complete cycle of motion of the mechanism. The data in Tables 4 and 5 are representative of the results.

Table 4: Ranges of Displacements in the \( Y \) Direction

<table>
<thead>
<tr>
<th>Point</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_{\text{max}} ) (mm)</td>
<td>803.75</td>
<td>547.09</td>
<td>290.32</td>
<td>33.46</td>
<td>-223.47</td>
</tr>
<tr>
<td>( y_{\text{min}} ) (mm)</td>
<td>827.75</td>
<td>572.54</td>
<td>317.45</td>
<td>62.46</td>
<td>-192.44</td>
</tr>
<tr>
<td>Range of ( y )</td>
<td>24</td>
<td>25.45</td>
<td>27.13</td>
<td>29</td>
<td>31.03</td>
</tr>
</tbody>
</table>

As can be seen in Table 4, the range of displacement in the \( Y \) direction increases monotonously, and at a slightly increasing rate, as we move from point \( P1 \) through to \( P5 \). Motion of the swing jaw in the \( Y \) direction causes a rubbing action between the material being crushed and the swing jaw, thereby causing the crushing surface of the swing jaw to wear. One could therefore expect an increasing wear rate as we move from \( P1 \) through to \( P5 \). On the other hand, during the crushing stroke, any motion of the swing jaw in the downward vertical direction forcefully feeds the material being crushed into the crushing chamber, which is desirable because it increases the throughput of the crusher.
Table 5: Ranges of Displacements in the Z Direction

<table>
<thead>
<tr>
<th>Point</th>
<th>$P1$</th>
<th>$P2$</th>
<th>$P3$</th>
<th>$P4$</th>
<th>$P5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_{\text{min}}$ (mm)</td>
<td>33.18</td>
<td>126.57</td>
<td>219.02</td>
<td>308.98</td>
<td>396.52</td>
</tr>
<tr>
<td>$z_{\text{max}}$ (mm)</td>
<td>57.18</td>
<td>143.67</td>
<td>231.02</td>
<td>320.44</td>
<td>412.44</td>
</tr>
<tr>
<td>Range of $z$</td>
<td>24</td>
<td>17.10</td>
<td>12.00</td>
<td>11.46</td>
<td>15.92</td>
</tr>
</tbody>
</table>

In Table 5, the range of displacement in the Z direction decreases at a decreasing rate, as we move from point $P1$ through to $P4$ but then increases as we move from point $P4$ to $P5$. Displacement of the swing jaw in the Z direction should be the greater contributor to the crushing action.

The loci of points $P1$ through to $P5$, for one complete cycle of motion, are shown in Figs. 12 to 16. These loci have been referred to as coupler curves.
In Figs. 12 to 16, the scales on the $Y$ and the $Z$ axes should be equal in order for the forms of the loci to be correct. In Fig. 12, the locus of point $P1$ is the circle that is described by the crankpin $O_3$ and centered at $O_2$. With the data that were used to determine these loci, this circle has a radius of $12\,\text{mm}$. As can be seen in Figs. 13, 14 and 15, the loci of points $P2$, $P3$ and $P4$ appear to be ellipses of varying proportions. As we move from point $P2$ to $P3$ and on to $P4$, the major axis of the ellipses grows longer while the minor axis grows shorter. Moreover, the major and minor axes of these ellipses are increasingly angled relative to the $YZ$ coordinate reference frame.
Figure 13: The Locus of Point P2 for One Complete Cycle of Motion
Figure 14: The Locus of Point P3 for One Complete Cycle of Motion
Figure 15: The Locus of Point P4 for One Complete Cycle of Motion
4.1.4 Angular Velocity of the Swing Jaw

An expression for the angular velocity of the coupler (swing jaw) can be obtained by differentiating equation (17) with respect to time. In doing so, it should be noted that \( r_1, \theta_1, r_2, r_3 \) and \( r_4 \) are all constants with respect to time. The result of the differentiation is then as follows:

\[
\left. \begin{align*}
& r_1 r_2 \sin (\theta_2 - \theta_1) \frac{d\theta_2}{dt} + r_3 r_1 \sin (\theta_3 - \theta_1) \frac{d\theta_3}{dt} + r_2 r_3 \sin (\theta_3 - \theta_2) \frac{d\theta_3}{dt} \\
= & + r_2 r_3 \sin (\theta_3 - \theta_2) \frac{d\theta_3}{dt}
\end{align*} \right\} 
\]

\[ (31) \]

Equation (31) can be divided through by \( r_1 r_2 \) to obtain the following:
\[
\frac{r_1 \sin (\theta_3 - \theta_1) \frac{d\theta_1}{dt}}{r_2} + \frac{r_2 \sin (\theta_2 - \theta_1) \frac{d\theta_2}{dt}}{r_3} + \sin (\theta_3 - \theta_2) \frac{d\theta_3}{dt} = \sin (\theta_3 - \theta_2) \frac{d\theta_3}{dt}
\]

Moreover, the following additional notation can be introduced:

\[
\begin{align*}
\frac{d\theta_2}{dt} &= \omega_2 \\
\frac{d\theta_3}{dt} &= \omega_3
\end{align*}
\]

While \( \omega_3 \) is expected to vary with time, the crank (eccentric shaft) is assumed to rotate at constant rotational velocity and therefore \( \omega_2 \) should be constant. According to manufacturer’s specifications, for a PE 400× 600 single toggle jaw crusher, \( \omega_2 \) may be taken to be 275 rpm or 28.8 rad/s\(^{-1}\). Thus, with the data given in Table 4, equation (32) may be re-written as follows:

\[
\omega_3 = \left( \frac{K_2 - K_1}{K_2 + K_3} \right) \omega_2 = 28.8 \left( \frac{K_2 - K_1}{K_2 + K_3} \right)
\]

\[
\begin{align*}
K_1 &= \frac{r_1}{r_3} \sin (\theta_2 - \theta_1) = 0.753 \sin (\theta_2 - \theta_1) \\
K_2 &= \sin (\theta_3 - \theta_2) \\
K_3 &= \frac{r_1}{r_2} \sin (\theta_3 - \theta_1) = 68.08 \sin (\theta_3 - \theta_1)
\end{align*}
\]

In equation (34), for any given value of \( \theta_1 \), with the corresponding value of \( \theta_4 \) having been determined, \( K_1, K_2 \) and \( K_3 \) will be constants. Equations (34) were used to determine the values of \( \omega_3 \), as \( \theta_2 \) varied from 0\(^\circ\) to 360\(^\circ\). Some of these calculated values are given in Table 6 as shown below.
Table 6: Analytically Determined Values of $\omega_3$ for Given Values of $\theta_2$

<table>
<thead>
<tr>
<th>$\theta_2$ (degrees)</th>
<th>$\omega_3$ (rad/s)</th>
<th>$\theta_2$ (degrees)</th>
<th>$\omega_3$ (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.407</td>
<td>195</td>
<td>-0.463</td>
</tr>
<tr>
<td>15</td>
<td>0.443</td>
<td>210</td>
<td>-0.476</td>
</tr>
<tr>
<td>30</td>
<td>0.450</td>
<td>225</td>
<td>-0.454</td>
</tr>
<tr>
<td>45</td>
<td>0.429</td>
<td>240</td>
<td>-0.397</td>
</tr>
<tr>
<td>60</td>
<td>0.381</td>
<td>255</td>
<td>-0.373</td>
</tr>
<tr>
<td>75</td>
<td>0.309</td>
<td>270</td>
<td>-0.206</td>
</tr>
<tr>
<td>90</td>
<td>0.216</td>
<td>285</td>
<td>-0.087</td>
</tr>
<tr>
<td>105</td>
<td>0.108</td>
<td>300</td>
<td>-0.036</td>
</tr>
<tr>
<td>120</td>
<td>-0.009</td>
<td>315</td>
<td>-0.154</td>
</tr>
<tr>
<td>135</td>
<td>-0.129</td>
<td>330</td>
<td>-0.259</td>
</tr>
<tr>
<td>150</td>
<td>-0.242</td>
<td>345</td>
<td>-0.345</td>
</tr>
<tr>
<td>165</td>
<td>-0.341</td>
<td>360</td>
<td>-0.407</td>
</tr>
<tr>
<td>180</td>
<td>-0.417</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For one full cycle of rotation of the crank, the minimum value of $\omega_3$ was found to be $-0.476\text{rads}^{-1}$ or $-4.55\text{rpm}$ while the maximum value of $\omega_3$ was found to be $0.451\text{rads}^{-1}$ or $4.3\text{rpm}$. Thus, the angular velocity of the coupler (swing jaw) is generally small.

A graph of the variation of $\omega_3$ with $\theta_2$ is shown in Fig. 21 below:
Figure 17: Variation of Swing Jaw Velocity $\omega_3$ with Crank Angle $\theta_2$

A graph of the variation of $\omega_3$ with $\theta_2$ is shown in Fig. 16. Note in Fig. 16 that the angular velocity of the swing jaw becomes zero at the instances where the angular displacement of the swing jaw attained maximum and minimum values, as can be seen in Fig. 16. This is to be expected since it is at these instances that the rate of change of angular displacement of the swing jaw instantaneously becomes zero.

4.1.5 Velocity of a Point in the Swing Jaw

The position of a point in the swing jaw is determined by equations (29). In the particular case where the point falls on line $O_3O_4$, its position is then described by equations (30). Thus the vertical and horizontal components of the velocity of a point on line $O_3O_4$ can be determined by differentiating equations (30) with respect to time, to obtain the following:

$$
\begin{align*}
    v_{pv} &= \dot{y}_p = \omega_2 r_2 \sin \theta_2 - \omega_3 r_3 \sin \theta_3 \\
    v_{ph} &= \dot{z}_p = \omega_2 r_2 \cos \theta_2 + \omega_3 r_3 \cos \theta_3
\end{align*}
$$

(35)
With the data given in Table 1 and with the value of $\omega_2$ taken to be 28.8 radians per second, equations (30) can be re-written as follows:

$$
\begin{align*}
\nu_{pv} &= \dot{y}_p = -0.3456 \sin \theta_2 - \omega_5 r_5 \sin \theta_5 \\
\nu_{ph} &= \dot{z}_p = 0.3456 \cos \theta_2 + \omega_5 r_5 \cos \theta_5
\end{align*}
$$

(36)

In equations (36), if $r_5$ is given in meters, then, for any given value of $\theta_2$, the corresponding values of $\theta_3$ and $\omega_3$ can be determined, as was earlier done, and therefore the velocity components $\nu_{pv}$ and $\nu_{ph}$ can be determined in meters per second.

For the values of $r_5$ given in Table 6, the values of the velocity components $\nu_{pv}$ and $\nu_{ph}$ were determined for one complete cycle of motion of the mechanism. The data in Tables 7 and 8 are representative of the results.

Table 7: Velocities in the Y Direction

<table>
<thead>
<tr>
<th>Point</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{y}_{\min}$ (m/s)</td>
<td>-0.346</td>
<td>-0.366</td>
<td>-0.389</td>
<td>-0.414</td>
<td>-0.442</td>
</tr>
<tr>
<td>$\dot{y}_{\max}$ (m/s)</td>
<td>0.346</td>
<td>0.367</td>
<td>0.393</td>
<td>0.421</td>
<td>0.452</td>
</tr>
</tbody>
</table>

In Table 7, negative values of velocity indicate a vertically downward motion while positive values indicate a vertically upward motion. As can be seen in Table 7, the maximum value of the component of velocity in the Y direction, whether it is directed upward or downward, increases monotonously, and at a slightly increasing rate, as we move from point $P1$ through to $P5$. Once again, this could suggest an increasing rate of wear as we move from $P1$ through to $P5$. Moreover, as we move from point $P2$ through to $P5$, slightly greater velocities are achieved in the vertically upward direction, as compared to the vertically downward direction, though the difference is small.

The vertical components of velocity for points $P1$ through to $P5$ are compared graphically in Fig. 18, for one complete rotation of the crank. It can be seen in Fig. 17 that the angular
oscillation of the swing jaw instantaneously stops when $\theta_s \approx 118.81^\circ$ and when $\theta_s \approx 295.625^\circ$. With no angular oscillation of the swing jaw, its motion becomes a pure translation and all the points in it have the same vertical components of velocity, as can be seen in Fig. 18.

Figure 18: Vertical Components of Velocity of Points in the Swing Jaw
Table 8: Velocities in the Z Direction

<table>
<thead>
<tr>
<th>Point</th>
<th>( \dot{z}_{\text{min}} ) (m/s)</th>
<th>( \dot{z}_{\text{max}} ) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 )</td>
<td>( -0.346 )</td>
<td>( 0.346 )</td>
</tr>
<tr>
<td>( P_2 )</td>
<td>( -0.383 )</td>
<td>( 0.383 )</td>
</tr>
<tr>
<td>( P_3 )</td>
<td>( -0.421 )</td>
<td>( 0.421 )</td>
</tr>
<tr>
<td>( P_4 )</td>
<td>( -0.46 )</td>
<td>( 0.46 )</td>
</tr>
<tr>
<td>( P_5 )</td>
<td>( -0.50 )</td>
<td>( 0.50 )</td>
</tr>
</tbody>
</table>

In Table 8, negative values of velocity indicate that the swing jaw is moving away from the fixed jaw while positive values indicate that the swing jaw is moving towards the fixed jaw. It can be seen in Table 8, the maximum value of the component of velocity in the Z direction increases at an almost constant rate, as we move from point \( P_1 \) through to \( P_5 \). Moreover, as we move from point \( P_1 \) through to \( P_5 \), the maximum value of the component of velocity in the Z direction appears to remain unchanged, whether the swing jaw is moving towards the fixed jaw or away from the fixed jaw.

The horizontal components of velocity for points \( P_1 \) through \( P_5 \) are compared graphically in Fig. 19, for one complete rotation of the crank. Again, the instances when the angular oscillation of the swing jaw instantaneously stops are evidenced in Fig. 19 by the crank positions at which all the points in the swing jaw have equal horizontal (as well as vertical) components of velocity.
In Figs. 18 and 19, it can be seen that, for approximately the first quarter of rotation of the crank, the swing jaw moves vertically downward and horizontally towards the fixed jaw, thus forcefully feeding the charge of material into the crushing chamber and simultaneously crushing it. For the second quarter of rotation of the crank, the swing jaw still moves vertically downward but horizontally away from the fixed jaw, thus letting the crushed material fall through the crushing chamber. For the third quarter of rotation of the crank, the swing jaw moves vertically upwards and horizontally away from the fixed jaw, still letting the crushed material fall through the crushing chamber.

Finally, in the last quarter of rotation of the crank, the swing jaw continues to move vertically upwards but horizontally towards the fixed jaw, thus beginning another crushing cycle.

Figure 19: Horizontal Components of Velocity of Points in the Swing Jaw
4.1.6 Angular acceleration of the Swing Jaw

An expression for the angular acceleration of the coupler can be obtained by differentiating equation (31) with respect to time. In doing so, it should be borne in mind that \( r_1 \), \( \theta_1 \), \( r_2 \), \( r_3 \), and \( \omega_2 \) are all constants with respect to time. The result of the differentiation is then as follows:

\[
\begin{align*}
& r_1 r_2 \omega_2 \cos(\theta_2 - \theta_1) + r_2 r_3 (\omega_3 - \omega_2) \cos(\theta_3 - \theta_2) \\
& + r_2 r_3 \sin(\theta_3 - \theta_2) \frac{d\omega_3}{dt} + r_3 r_1 \omega_1^2 \cos(\theta_1 - \theta_2) \\
& + r_3 r_1 \sin(\theta_3 - \theta_1) \frac{d\omega_1}{dt} \\
& = r_2 r_3 \omega_2 (\omega_3 - \omega_2) \cos(\theta_3 - \theta_2)
\end{align*}
\]

(37)

Here, the following additional notation can be introduced:

\[
\frac{d\omega_3}{dt} = \alpha_3
\]

(38)

With the use of equation (38), equation (37) can now be re-arranged into the following:

\[
\begin{align*}
& r_1 r_2 \omega_2^2 \cos(\theta_2 - \theta_1) + r_2 r_3 (\omega_3 - \omega_2)^2 \cos(\theta_3 - \theta_2) \\
& + r_2 r_3 \omega_3^2 \cos(\theta_3 - \theta_1) \\
& = [r_2 r_3 \sin(\theta_3 - \theta_2) + r_3 r_1 \sin(\theta_3 - \theta_1)] \alpha_3
\end{align*}
\]

(39)

Equation (39) can be divided through by \( r_2 r_3 \) to obtain the following:

\[
\begin{align*}
& \frac{r_1}{r_3} \omega_2^2 \cos(\theta_2 - \theta_1) + (\omega_3 - \omega_2)^2 \cos(\theta_3 - \theta_2) + \frac{r_3}{r_2} \omega_3^2 \cos(\theta_3 - \theta_1) \\
& = \left[ \sin(\theta_3 - \theta_2) + \frac{r_1}{r_2} \sin(\theta_3 - \theta_1) \right] \alpha_3
\end{align*}
\]

(40)

By letting \( \omega_2 = 28.8 \text{rads}^{-1} \), as was done before, and using the data in Table 1, equation (40) may be re-written as follows:

\[
\begin{align*}
\alpha_3 &= (K_1 + K_2 + K_3) / K_4 \\
K_1 &= 624.56 \cos(\theta_2 - \theta_1) \\
K_2 &= (\omega_3 - 28.8)^2 \cos(\theta_3 - \theta_2) \\
K_3 &= 68.08 \omega_3^2 \cos(\theta_3 - \theta_1) \\
K_4 &= -[\sin(\theta_3 - \theta_2) + 68.08 \sin(\theta_3 - \theta_1)]
\end{align*}
\]

(41)
With the values of $\theta_3$ and $\omega_3$ that correspond to given values of $\theta_2$ having been determined; equations (41) were used to determine the values of $\alpha_3$ as $\theta_2$ was varied from $0^\circ$ to $360^\circ$. Some of the calculated values are given in Table 9.

For one complete cycle of motion of the swing jaw, the minimum value of its angular acceleration occurred at $\theta_2 = 123.9^\circ$ and was found to be $-13.208$ radians per square second while the maximum value of its angular acceleration occurred at $\theta_2 = 291.2^\circ$ and was found to be $13.573$ radians per square second. Thus, the angular acceleration of the swing jaw can attain substantial magnitudes.
Table 9: Analytically Determined Values of $\alpha_3$ for Given Values of $\theta_2$

<table>
<thead>
<tr>
<th>$\theta_2$ (degrees)</th>
<th>$\alpha_3$ (rad/s$^2$)</th>
<th>$\theta_2$ (degrees)</th>
<th>$\alpha_3$ (rad/s$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5.415</td>
<td>195</td>
<td>-3.315</td>
</tr>
<tr>
<td>15</td>
<td>2.362</td>
<td>210</td>
<td>0.543</td>
</tr>
<tr>
<td>30</td>
<td>-0.767</td>
<td>225</td>
<td>4.384</td>
</tr>
<tr>
<td>45</td>
<td>-3.820</td>
<td>240</td>
<td>7.858</td>
</tr>
<tr>
<td>60</td>
<td>-6.657</td>
<td>255</td>
<td>10.659</td>
</tr>
<tr>
<td>75</td>
<td>-9.175</td>
<td>270</td>
<td>12.573</td>
</tr>
<tr>
<td>90</td>
<td>-11.150</td>
<td>285</td>
<td>13.490</td>
</tr>
<tr>
<td>105</td>
<td>-12.538</td>
<td>300</td>
<td>13.406</td>
</tr>
<tr>
<td>120</td>
<td>-13.179</td>
<td>315</td>
<td>12.407</td>
</tr>
<tr>
<td>135</td>
<td>-12.960</td>
<td>330</td>
<td>10.617</td>
</tr>
<tr>
<td>150</td>
<td>-11.813</td>
<td>345</td>
<td>8.226</td>
</tr>
<tr>
<td>165</td>
<td>-9.741</td>
<td>360</td>
<td>5.415</td>
</tr>
<tr>
<td>180</td>
<td>-6.841</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A graph of the variation of $\alpha_3$ with $\theta_2$ is shown in Fig. 20 below:
4.1.7 Acceleration of a Point in the Swing Jaw

The vertical and horizontal components of the acceleration of a point on line $O_3O_4$ can be determined by differentiating equations (35) with respect to time, to obtain the following:

\[
\begin{align*}
a_{py} &= \ddot{y}_p = -\omega_2^2 r_2 \cos \theta_2 - \omega_3^2 r_3 \cos \theta_3 - \alpha_3 r_5 \sin \theta_3 \\
a_{pz} &= \ddot{z}_p = -\omega_2^2 r_2 \sin \theta_2 - \omega_3^2 r_3 \sin \theta_3 + \alpha_3 r_5 \sin \theta_3
\end{align*}
\]

(42)

In equations (42), if $r_5$ is given, then, for any given value of $\theta_2$, the corresponding values of $\theta_3$, $\omega_3$, and $\alpha_3$ can be determined, and therefore the acceleration components $a_{py}$ and $a_{pz}$ can also be determined.

For the values of $r_5$ given in Table 6, the values of the acceleration components $a_{py}$ and $a_{pz}$ were determined, using equations (42), for one complete cycle of motion of the mechanism. The data in Tables 10 and 11 are representative of the results.
### Table 10: Accelerations in the Y Direction

<table>
<thead>
<tr>
<th>Point</th>
<th>( P1 )</th>
<th>( P2 )</th>
<th>( P3 )</th>
<th>( P4 )</th>
<th>( P5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \dddot{y}_{\text{min}} ) (m/s(^2))</td>
<td>-9.953</td>
<td>-10.467</td>
<td>-11.092</td>
<td>-11.817</td>
<td>-12.629</td>
</tr>
<tr>
<td>( \dddot{y}_{\text{max}} ) (m/s(^2))</td>
<td>9.953</td>
<td>10.467</td>
<td>11.420</td>
<td>12.252</td>
<td>13.132</td>
</tr>
</tbody>
</table>

In Table 10, negative values indicate an acceleration that is directed vertically downward and would either slow down the vertical component of velocity of the swing jaw, if it were moving in the upward direction, or speed up the vertical component of velocity of the swing jaw, if it were moving in the downward direction. Positive values indicate an acceleration that is directed vertically upward and would either slow down the vertical component of velocity of the swing jaw, if it were moving in the downward direction, or speed up the vertical component of velocity of the swing jaw, if it were moving in the upward direction. It can be seen in Table 10, the maximum value of the component of acceleration in the \( Y \) direction increases monotonously. This is at a slightly increasing rate, as we move from point \( P1 \) through to \( P5 \).

### Table 11: Accelerations in the Z Direction

<table>
<thead>
<tr>
<th>Point</th>
<th>( P1 )</th>
<th>( P2 )</th>
<th>( P3 )</th>
<th>( P4 )</th>
<th>( P5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \dddot{z}_{\text{min}} ) (m/s(^2))</td>
<td>-9.953</td>
<td>-8.993</td>
<td>-8.064</td>
<td>-7.176</td>
<td>-6.339</td>
</tr>
<tr>
<td>( \dddot{z}_{\text{max}} ) (m/s(^2))</td>
<td>9.953</td>
<td>8.807</td>
<td>7.716</td>
<td>6.720</td>
<td>5.887</td>
</tr>
</tbody>
</table>

In Table 11, negative values indicate an acceleration that is directed horizontally away from the fixed jaw and would either slow down the horizontal component of velocity of the swing jaw, if it were moving towards the fixed jaw, or speed up the horizontal component of velocity of the swing jaw, if it were moving away from the fixed jaw. Positive values indicate an acceleration that is directed horizontally towards the fixed jaw and would either slow down the horizontal
component of velocity of the swing jaw, if it were moving away from the fixed jaw, or speed up the horizontal component of velocity of the swing jaw, if it were moving towards the fixed jaw.

As can be seen in Table 11, the maximum value of the component of acceleration in the Z direction decreases monotonously and at a slightly decreasing rate, as we move from point \( P1 \) through to \( P5 \).

The vertical components of acceleration for points \( P1 \) through to \( P5 \) are compared graphically in Fig. 25, for one complete rotation of the crank as shown below.

![Figure 21: Vertical Components of Acceleration of Points in the Swing Jaw](image)

It can be seen in Fig. 21 that the angular velocity of the swing jaw instantaneously becomes zero when \( \theta_2 = 26.32^\circ \) and when \( \theta_2 = 207.92^\circ \). At these instances, the acceleration of the swing jaw becomes purely translational and all the points in it have the same vertical component of acceleration, as can be seen in Fig. 21. The horizontal components of acceleration for points \( P1 \) through to \( P5 \) are compared graphically in Fig. 22, for one complete rotation of the crank. Again it can be seen in Fig. 21 that the angular velocity of the swing jaw instantaneously becomes zero.
when $\theta_2 = 26.32^\circ$ and when $\theta_2 = 207.92^\circ$. At these instances, the acceleration of the swing jaw becomes purely translational and all the points in it have the same horizontal (and vertical) component of acceleration, as can be seen in Fig. 22.

Figure 22: Horizontal Components of Acceleration of Points in the Swing Jaw
4.2 Static Force Analysis of the Crusher Mechanism

The forces acting in the links of the proposed mechanism are illustrated in Fig. 23 below:

In performing the static force analysis it shall be assumed that the masses of the links, as well as friction forces are negligible. \( T_2 \) is the driving torque, applied at the crank axis \( O_2 \) to drive the crank and the entire crusher mechanism. \( T_3 \) is the torque, acting about the swing jaw axis \( O_3 \), due \( F_2 \) and \( F_3 \) are the forces in links and 3 , respectively and they are all assumed to be compressive. Considering the motion of the crank, the free body diagram in Fig. 24 below is obtained:
The equilibrium of moments on the crank, about the joint $O_2$, leads to the following equation:

$$0 = -F_{Y2} r_2 \sin \theta_2 + F_{Z2} r_2 \cos \theta_2 + T_2 \bigg\}$$

$$T_2 = \left[ F_{Y2} \sin \theta_2 - F_{Z2} \cos \theta_2 \right] r_2$$  \hspace{1cm} (43)

Considering the motion of the rocker link, the following free body diagram could be drawn:
The equilibrium of moments on the rocker, about the joint $O_4$, yields the following equation:

$$0 = -F_{Y43} r_3 \sin \theta_3 - F_{Z43} r_3 \cos \theta_3 + T_3$$

$$T_3 = [F_{Y43} r_3 \sin \theta_3 - F_{Z43} r_3 \cos \theta_3] r_3$$  \hspace{1cm} (44)$$

If the equilibrium of forces at joint $O_4$ is considered in the Z direction, the following equation is obtained:
\[ F_{Z43} + F_3 \cos(\theta_3 + 90^\circ) = 0 \]
\[ F_{Z43} + F_3 \sin \theta_3 = 0 \]
\[ F_{Z43} = -F_3 \sin \theta_3 = F_{Z32} \]  
\hspace{1cm} \text{(45)}

From equation (45) it follows that:
\[ F_{Z23} = F_{Z34} = F_3 \sin \theta_3 \]  
\hspace{1cm} \text{(46)}

Similarly, considering the force acting on the Y direction at joint \(O_4\), the following equation is obtained:
\[ F_{Y43} + F_3 \sin(\theta_3 - 90^\circ) = 0 \]
\[ F_{Y43} - F_3 \cos \theta_3 = 0 \]
\[ F_{Y43} = F_3 \cos \theta_3 = F_{Y32} \]  
\hspace{1cm} \text{(47)}

Equations (46) and (47) can be substituted into equation (43) to obtain the following equation:
\[ T_2 = \left[ F_3 \cos \theta_3 \sin \theta_2 + F_3 \sin \theta_3 \cos \theta_2 \right] f_2 \]
\[ T_2 = r_2 F_3 \left[ \cos \theta_3 \sin \theta_2 + \sin \theta_3 \cos \theta_2 \right] \]
\[ T_2 = r_2 F_3 \left[ \theta_2 + \theta_3 \right] \]  
\hspace{1cm} \text{(48)}

Similarly, equations (45), (46) and (47) are substituted into equation 44 to yield the following result:
\[ T_3 = \left[ F_3 \cos \theta_3 \sin \theta_3 + F_3 \sin \theta_3 \cos \theta_3 \right] f_3 \]
\[ T_3 = r_3 F_3 \left[ \cos \theta_3 \sin \theta_3 + F_3 \sin \theta_3 \cos \theta_3 \right] \]
\[ T_3 = r_3 F_3 \left[ \sin 2\theta_3 \right] \]  
\hspace{1cm} \text{(49)}

A relationship between \(T_2\) and \(T_3\) can be obtained by dividing equation (49) by equation (48), to obtain the following equation:
\[ \frac{T_3 r_2}{T_2 r_3} = \frac{\sin(2\theta_3)}{\sin(\theta_2 + \theta_3)} \]  
\hspace{1cm} \text{(50)}
4.1.1 Determination of $T_3$ Experimentally

The torque applied on the swing jaw $T_3$ was obtained experimentally by using a modified single toggle small scale stone crusher whose mechanism is shown by the picture below:

![Figure 26: Mechanism of a Modified small scale Single Toggle Stone Crusher](image)

Assumptions made during the Experimental Analysis

From Figure 26 above, it is seen that the rocker is not supported directly by the crank like in the single toggle stone crusher. It has another link that connects the crank to the swing jaw. This difference led to some assumptions being made for proper analysis.

The assumption that was made when conducting the experiment was that equation (50) can be used to obtain the torque ratios in both mechanisms. Furthermore, the data used in Table 1 are dimensions of a large scale stone crusher therefore; another assumption made was that the dimensions of a large scale stone crusher could be used in conjunction with the power consumed and angular velocity of a small scale stone crusher to obtain the torque $T_3$. 
4.2.2 Determination of Power Consumed

The values of voltage and current used to operate the stone crusher were read off from a clamp meter as 235rms and 8.07rms respectively.

\[ P = VI \quad (51) \]

Where:

\[ P = \text{Power consumed in kilowatts} \]
\[ V = \text{Voltage measured in volts} \]
\[ I = \text{Current measured in amperes} \]

Substituting the values of voltage and current in equation (51) the power consumed is was determined as follows:

\[ P = \frac{(235 \times \sqrt{2})(8.07 \times \sqrt{2})}{1000} = 3.7929\text{Kw} \]

4.4.3 Determination of Torque applied on the Swing Jaw

The torque applied on the rocker can be determined by using the following equation:

\[ T = \frac{P}{\omega} \quad (52) \]

Where:

\[ \omega = \text{Angular speed of the motor in rad/s} \]
\[ T = \text{Applied torque in Nm} \]

But: \( \omega = 337\text{rpm} \) (obtained experimentally by use of a tachometer)

Using the information given above and equation (52) the applied torque \( T_3 \) is obtained as follows:

\[ T_3 = \frac{3.7929 \times 1000}{337 \times \frac{2\pi}{60}} = 107.4763\text{Nm} \]
Equation (50) was then used in conjunction with the data in Table 1 to obtain the torque $T_2$ applied at different crank angles. Moreover, the normalized torque ratios $T_3/T_2$ at different crank angles $\theta_2$ were obtained. The following table is representative of the results obtained.

Table 12: Values of $T_2$ and $T_3/T_2$ at different Crank Angles

<table>
<thead>
<tr>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
<th>$T_3/T_2$</th>
<th>$T_2$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
<th>$T_3/T_2$</th>
<th>$T_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>160.235</td>
<td>0</td>
<td></td>
<td>195</td>
<td>160.8539</td>
<td>-51.3573</td>
<td>-23.8854</td>
</tr>
<tr>
<td>15</td>
<td>160.4574</td>
<td>196.5741</td>
<td>6.240344</td>
<td>210</td>
<td>160.6075</td>
<td>745.6186</td>
<td>1.645198</td>
</tr>
<tr>
<td>30</td>
<td>160.6913</td>
<td>-33.982</td>
<td>-36.0982</td>
<td>225</td>
<td>160.3634</td>
<td>-71.122</td>
<td>-17.2477</td>
</tr>
<tr>
<td>45</td>
<td>160.9217</td>
<td>-81.7182</td>
<td>-15.0112</td>
<td>240</td>
<td>160.1398</td>
<td>-60.8192</td>
<td>-20.1695</td>
</tr>
<tr>
<td>60</td>
<td>161.1343</td>
<td>55.84326</td>
<td>21.96666</td>
<td>255</td>
<td>159.9529</td>
<td>304.1136</td>
<td>4.033658</td>
</tr>
<tr>
<td>75</td>
<td>161.3157</td>
<td>100.7792</td>
<td>12.17206</td>
<td>270</td>
<td>159.816</td>
<td>-56.9966</td>
<td>-21.5222</td>
</tr>
<tr>
<td>90</td>
<td>161.4541</td>
<td>-573.255</td>
<td>-2.13987</td>
<td>285</td>
<td>159.7383</td>
<td>90.48816</td>
<td>13.55636</td>
</tr>
<tr>
<td>105</td>
<td>161.54</td>
<td>88.96087</td>
<td>13.7891</td>
<td>300</td>
<td>159.7241</td>
<td>4.601326</td>
<td>266.5949</td>
</tr>
<tr>
<td>135</td>
<td>161.5314</td>
<td>-16.939</td>
<td>-72.4179</td>
<td>330</td>
<td>159.8805</td>
<td>-114.954</td>
<td>-10.6711</td>
</tr>
<tr>
<td>150</td>
<td>161.435</td>
<td>223.0355</td>
<td>5.499976</td>
<td>345</td>
<td>160.0383</td>
<td>-120.048</td>
<td>-10.2183</td>
</tr>
<tr>
<td>165</td>
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A graph was drawn for a complete crank rotation. It was observed that the first spike in Fig. 26 indicates the great amplification of the crushing force that occurs at the toggle position, which corresponds to a crank angle of about $90^\circ$. Theoretically, the crushing force amplification should be infinite at this toggle position. The second spike in Fig. 26 occurs at a crank angle of about $210^\circ$. This spike corresponds to the second toggle position of the mechanism. However, as the crank rotates from $90^\circ$ to $210^\circ$, the crusher would be on the idle stroke with the swing jaw being retracted and no work being done in crushing the feed material. Therefore, the second spike has no physical meaning in so far as crushing of the feed material is concerned.
CHAPTER 5

5.1 Discussion

The main objective of the experiment was to compare the characteristics of a horizontal Pittman toggle stone crusher with the single toggle stone crusher. This comparison was used in determining which of these methods was better. The analysis major consideration was the force transmission characteristics of the two crushers.

In this project, we set out to analyze the force transmission characteristics of a single toggle stone crusher. This began with the theoretical analysis of the force transmission characteristics of the single toggle stone crusher. This was achieved by deriving the equations of the forces along each member of the free body diagram in figure 23.

The swing jaw positional displacement was determined using the following equation,

\[
y_p = r_1 \cos \theta_1 + r_2 \cos \theta_2 + r_3 \cos \theta_3 \\
z_p = r_1 \sin \theta_1 + r_2 \sin \theta_2 + r_3 \sin \theta_3
\]

Graphs were drawn for different values of \( \theta_3 \) and selected points, \( P1 \) to \( P5 \). In table 5, the range of displacement in the \( Z \) direction increases, as we move from point \( P1 \) through to \( P5 \). Displacement of the swing jaw in the \( Z \) direction is the greater contributor to the crushing action. The loci of points \( P2 \) through to \( P5 \), for one complete cycle of motion, are shown in Figs. 13 to 16. As can be seen in Figs. 13, 14 and 15, the loci of points \( P2 \), \( P3 \) and \( P4 \) are straight lines of varying lengths. As we move from point \( P1 \) to \( P5 \), the lines increase in length in both axes.

The following equation (32) was used to determine the velocity, \( \omega_3 \), as \( \theta_2 \) varied from 0° to 360°.

\[
r_1 r_2 \sin (\theta_2 - \theta_1) \frac{d \theta_2}{dt} + r_3 r_1 \sin (\theta_3 - \theta_1) \frac{d \theta_3}{dt} + r_2 r_3 \sin (\theta_3 - \theta_2) \frac{d \theta_3}{dt} = +r_2 r_3 \sin (\theta_3 - \theta_2) \frac{d \theta_3}{dt}
\]
For one full cycle of rotation of the crank, the minimum value of $\omega_3$ was found to be $-0.42rad/s$, while the maximum value of $\omega_3$ was found to be $0.415rad/s$. Thus, the angular velocity of the coupler (swing jaw) is generally small.

The velocity of a point on the swing jaw was calculated using the following equation (36):

$$

\begin{align*}
\nu_{PV} &= \dot{y}_P = -0.3456 \sin \theta_2 - \omega_3 r_3 \sin \theta_3 \\
\nu_{PH} &= \dot{z}_P = 0.3456 \cos \theta_2 + \omega_3 r_3 \cos \theta_3
\end{align*}
$$

Data was tabulated and a graph of the velocity variation on the different points $P_1$ to $P_5$ was shown in Figs. 18 and 19.

It was seen that, for approximately the first quarter of rotation of the crank, the swing jaw moves vertically downward and horizontally towards the fixed jaw, thus forcefully feeding the charge of material into the crushing chamber and simultaneously crushing it. For the second quarter of rotation of the crank, the swing jaw still moves vertically downward but horizontally away from the fixed jaw, thus letting the crushed material fall through the crushing chamber. For the third quarter of rotation of the crank, the swing jaw moves vertically upwards and horizontally away from the fixed jaw, still letting the crushed material fall through the crushing chamber. Finally, in the last quarter of rotation of the crank, the swing jaw continues to move vertically upwards but horizontally towards the fixed jaw, thus beginning another crushing cycle.

The angular acceleration was analyzed using the following equation, (40);

$$
\frac{r_1}{r_3} \omega_2^2 \cos(\theta_2 - \theta_1) + (\omega_3 - \omega_2)^2 \cos(\theta_3 - \theta_2) + \frac{r_1}{r_2} \omega_3^2 \cos(\theta_3 - \theta_1)
\]

$$

For one complete cycle of motion of the swing jaw, the minimum value of its angular acceleration occurred at $\theta_2 = 125^\circ$ and was found to be $13rad^2s^{-2}$ radians per second while the maximum value of its angular acceleration occurred at $\theta_2 = 290^\circ$ and was found to be $13.5rad^2s^{-2}$. Thus, the angular acceleration of the swing jaw can attain substantial magnitudes.

The normalized torque was analyzed by the following equation, (50);
\[
\frac{T_1 r_2}{T_2 r_3} = \frac{\sin(2\theta_1)}{\sin(\theta_2 + \theta_3)}
\]

The graphs obtained from theoretical and experimental data have the same profile with one spike formation in the positive torque ratio range and another in the negative torque ratio range. This shows that there is significant change in direction of the torque applied. The greatest crushing action is experienced at the two angles where the spike occurs. This is at 90° and 210°.

However there was a difference in the profile of the graph obtained from the theoretical analysis and that of the experimental data. It is evident that the graph obtained theoretically was asymptotic to the x-axis at 1. The experimental graph appeared slightly sinusoidal to the x axis. This is credited to the assumptions made in the theoretical analysis did not take into account all the variables in the practical analysis. One assumption is that the load distribution on the swing jaw is a point load with the torque acting at the center of the arm. The load distribution in the experiment could not be accurately determined. This is why five arbitrary points were taken from the swing jaw to study the displacement, velocity and acceleration of various points of the swing jaw.
5.1.1 **Comparison between the Single Toggle and the Horizontal Pittman Stone Crushers**

From the discussion, it is evident that the single toggle stone crusher has an oscillatory motion of the swing jaw as opposed to the impact crushing of backward and forward in the horizontal Pittman stone crusher. The oscillatory motion in the single toggle jaw crusher introduces a rubbing effect between the swing jaw and the material being crushed thus the teeth of the swing jaw wear out faster as compared to the horizontal Pittman stone crusher.

The magnitudes of the acceleration of the swing jaw in the single toggle jaw crusher are slightly higher as compared to the acceleration magnitudes in the horizontal Pittman stone crusher. Furthermore, these magnitudes are attained faster by the crank in the single toggle jaw crusher. This implies that the crushing speed of the single toggle jaw crusher is better than the horizontal Pittman crusher in cases where crushing time is considered a factor. However, the single toggle jaw crusher has the disadvantage of overheating the joints in its mechanism quicker as compared to the horizontal Pittman jaw crusher. This is because the high velocity that the rocker and crank move at, causing friction between the joints.

Furthermore this increases the cost of maintenance of the single toggle jaw crusher as compared to the horizontal Pittman jaw crusher. The single toggle jaw crusher also has increased maintenance costs due to the rotational movements of the swing jaw meaning it has to be replaced more frequently. Despite this, the Single toggle jaw crusher is less expensive in terms of the initial purchase cost.

If the time taken to begin work is considered, it is evident that the single toggle jaw crusher takes less time to start crushing the fed material as compared to the horizontal Pittman jaw crusher. This is because in the horizontal Pittman stone crusher, the second spike occurs at around 252° as shown in the figure below:
5.2 Conclusion

The main aim of the project was to compare the characteristics of the horizontal Pittman stone crusher and the single toggle stone crusher.

This was done by mainly considering the torque transmission characteristics of the two types of crushers. The graphical representation of these two mechanisms was then done. Experimental data was used to simulate this torque transmission characteristic for a large scale stone crusher using previously established dimensions.

5.3 Recommendations

If maintenance in considered a high priority factor, the stone crusher should secure trouble free operation and increased uptime. For this to be achieved the following factors should be considered:

- Bearings are grease lubricated and should have grease-filled labyrinth seals to prevent entry of dust.
• Automatic lubrication system that can be connected to existing control systems for remote alarm indication ensuring protection of roller bearings.

• Quick and easy installation of jaw breaks by using clamping and support bars to fix the jaw plates to the crusher.

The flywheel guards should be made as compact and service friendly as possible. The flywheel guards should be bolted onto the crusher’s side plates and effectively protects operators from potentially dangerous moving parts. Viewing windows and access doors allow service crews to inspect and service the crusher. Their use also gives greater access to the crusher as the guards are not laying on the service platform.

In conclusion, the feed chute should be custom designed to effectively guide the feed material into the crusher’s cavity. Jaw and cheek plate removal and installation are carried out without having to move or remove the feed chute. The feed chute is bolted to the crusher and can be removed for other maintenance purposes.

5.3.1 Summary of the Advantages of the Recommended Ideas

The recommendations above have the following advantages:

1. Easy automation of the crusher.

2. Full mobility with the lokotrack range.

3. High quality and reliability.


5. Low operating and installation costs.
References


