FME201
Solid & Structural Mechanics I

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Lecture: Mon 11am -1pm (CELT)
Tutorial Tue 12-1pm (E207)

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CHAPTER OBJECTIVES

- Show relationship of stress and strain using experimental methods to determine stress-strain diagram of a specific material
- Discuss the behavior described in the diagram for commonly used engineering materials
- Discuss the mechanical properties and other test related to the development of mechanics of materials

LECTURE OUTLINE

1. Tension and Compression Test
2. Stress-Strain Diagram
3. Stress-Strain Behavior of Ductile and Brittle Materials
4. Hooke's Law
5. Strain Energy
6. Poission's Ratio
7. Shear Stress-Strain Diagram

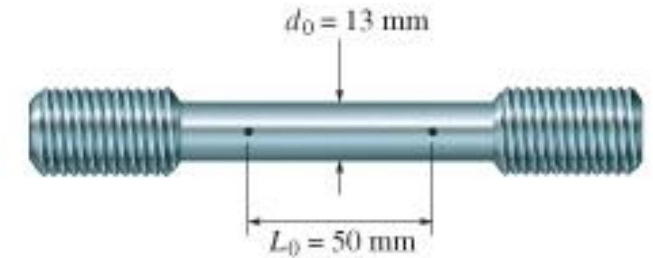
3.1 TENSION & COMPRESSION TEST

- Strength of a material can only be determined by *experiment*
- One test used by engineers is the *tension or compression test*
- This test is used primarily to determine the relationship between the average normal stress and average normal strain in common engineering materials, such as metals, ceramics, polymers and composites

3.1 TENSION & COMPRESSION TEST

Performing the tension or compression test

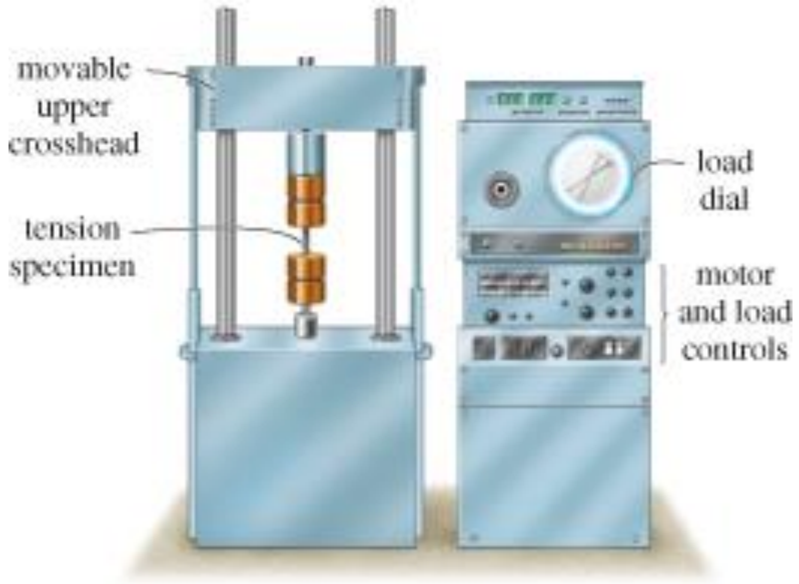
- Specimen of material is made into "standard" shape and size
- Before testing, 2 small punch marks identified along specimen's length
- Measurements are taken of both specimen's initial x-sectional area A_0 and gauge-length distance L_0; between the two marks
- Seat the specimen into a testing machine shown below



3.1 TENSION & COMPRESSION TEST

Performing the tension or compression test

- Seat the specimen into a testing machine shown below

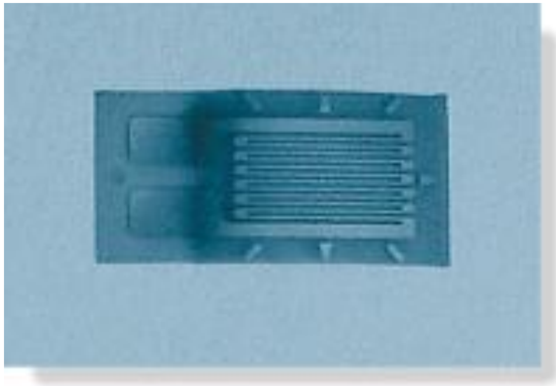


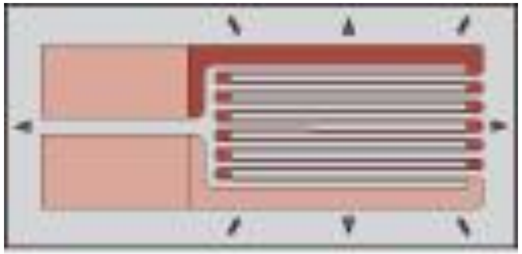
- The machine will stretch specimen at slow constant rate until breaking point
- At frequent intervals during test, data is recorded of the applied load P.

3.1 TENSION & COMPRESSION TEST

Performing the tension or compression test

- Elongation $\delta = L - L_0$ is measured using either a caliper or an extensometer
- δ is used to calculate the normal strain in the specimen
- Sometimes, strain can also be read directly using an *electrical-resistance strain gauge*





Electrical–resistance strain gauge

3.2 STRESS-STRAIN DIAGRAM

- A *stress-strain diagram* is obtained by plotting the various values of the stress and corresponding strain in the specimen

Conventional stress-strain diagram

- Using recorded data, we can determine nominal or engineering stress by

$$\sigma = \frac{P}{A_0}$$

Assumption: Stress is *constant* over the x-section and throughout region between gauge points

3.2 STRESS-STRAIN DIAGRAM

Conventional Stress-Strain Diagram

- Likewise, nominal or engineering strain is found directly from strain gauge reading, or by

$$\varepsilon = \frac{\delta}{L_0}$$

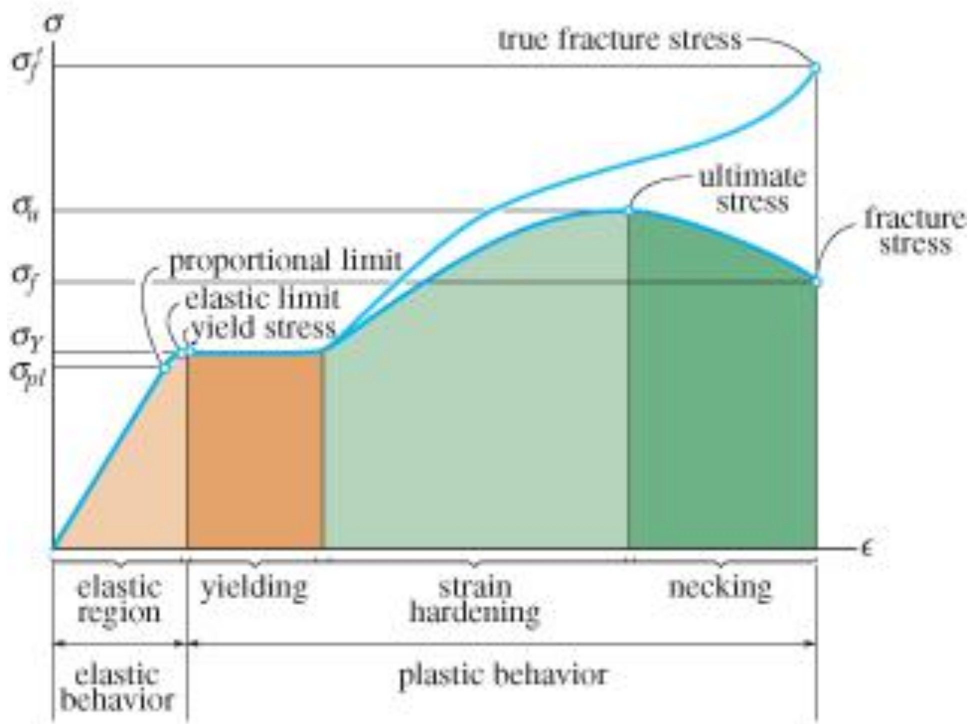
Assumption: Strain is constant throughout region between gauge points

By plotting σ (ordinate) against ε (abscissa), we get a *conventional stress-strain diagram*

3.2 STRESS-STRAIN DIAGRAM

Conventional stress-strain diagram

- Figure shows the characteristic stress-strain diagram for steel, a commonly used material for structural members and mechanical elements



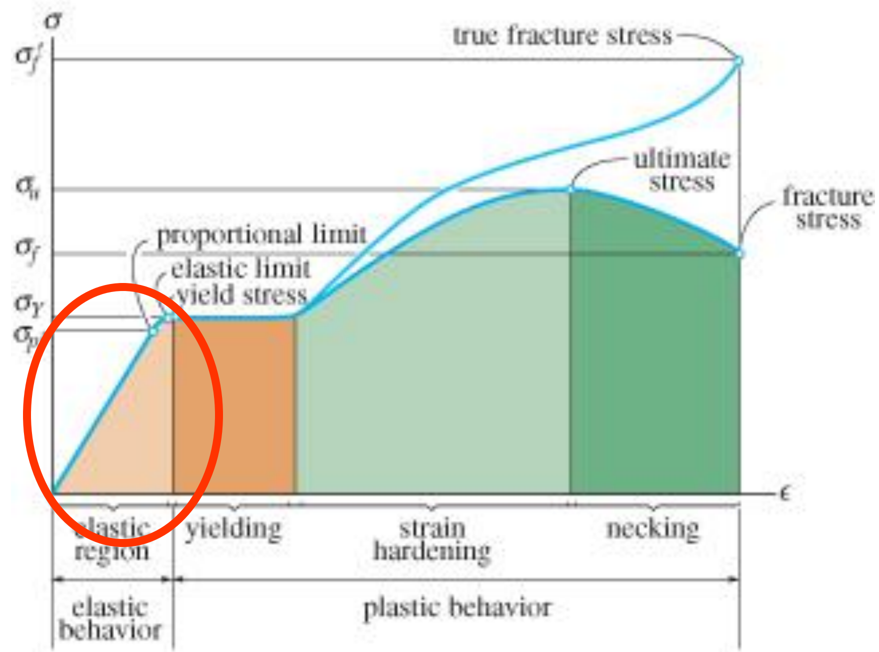
Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

3.2 STRESS-STRAIN DIAGRAM

Conventional stress-strain diagram

Elastic behavior.

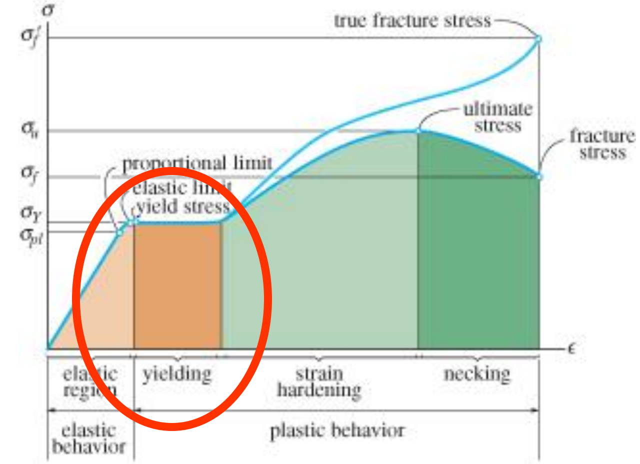
- A straight line
- Stress is proportional to strain, i.e., linearly elastic
- Upper stress limit, or *proportional limit*; σ_{pl}
- If load is removed upon reaching elastic limit, specimen will return to its original shape



Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

3.2 STRESS-STRAIN DIAGRAM

Conventional stress-strain diagram



Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

Yielding.

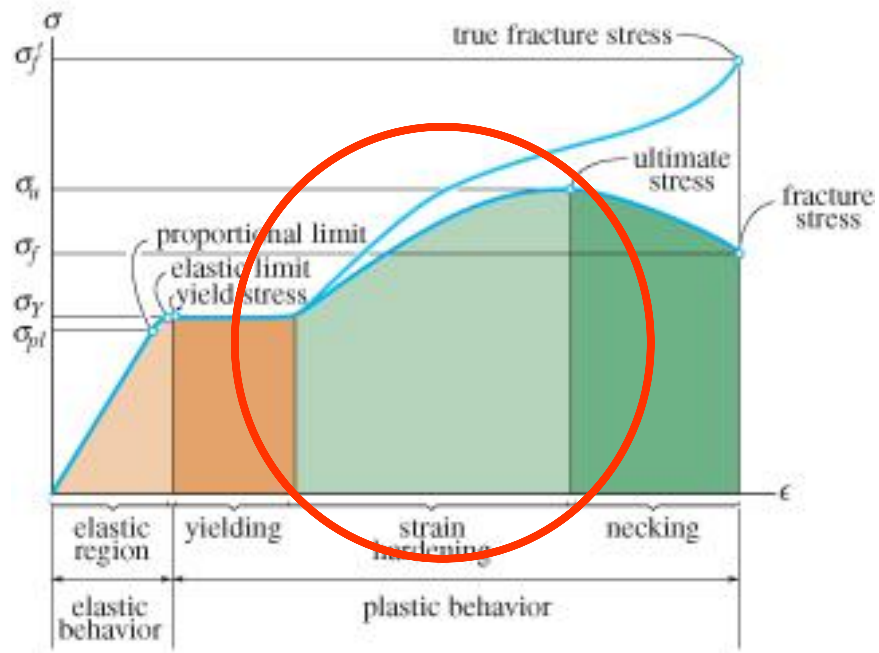
- Material deforms permanently; yielding; plastic deformation
- Yield stress, σ_Y
- Once yield point reached, specimen continues to elongate (strain) *without any increase in load*
- Note figure not drawn to scale, otherwise induced strains is 10-40 times larger than in elastic limit
- Material is referred to as being *perfectly plastic*

3.2 STRESS-STRAIN DIAGRAM

Conventional stress-strain diagram

Strain hardening.

- Ultimate stress, σ_u
- While specimen is elongating, its x-sectional area will decrease
- Decrease in area is fairly uniform over entire gauge length



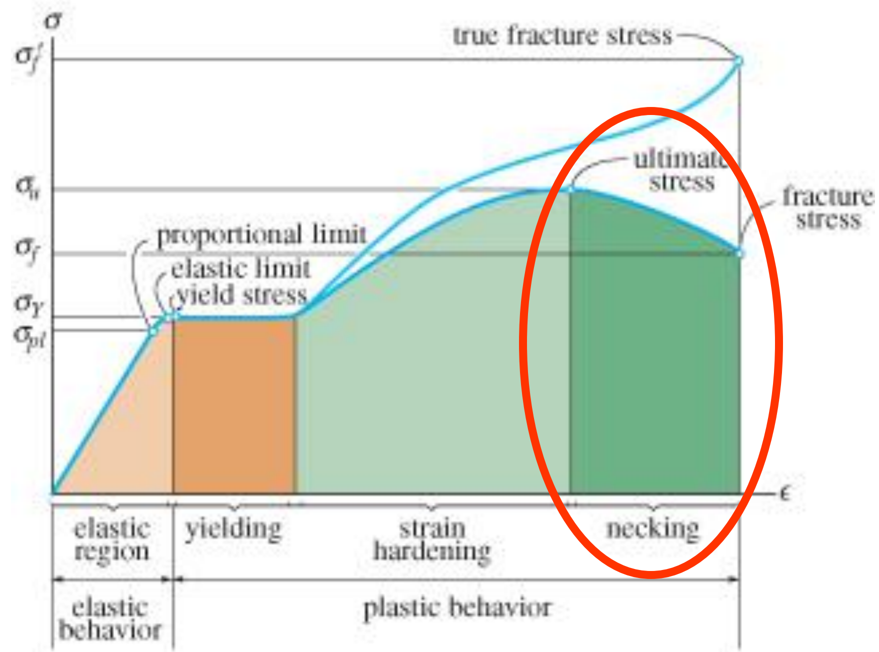
Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

3.2 STRESS-STRAIN DIAGRAM

Conventional stress-strain diagram

Necking.

- At ultimate stress, x-sectional area begins to decrease in a *localized* region
- As a result, a constriction or "neck" tends to form in this region as specimen elongates further



Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

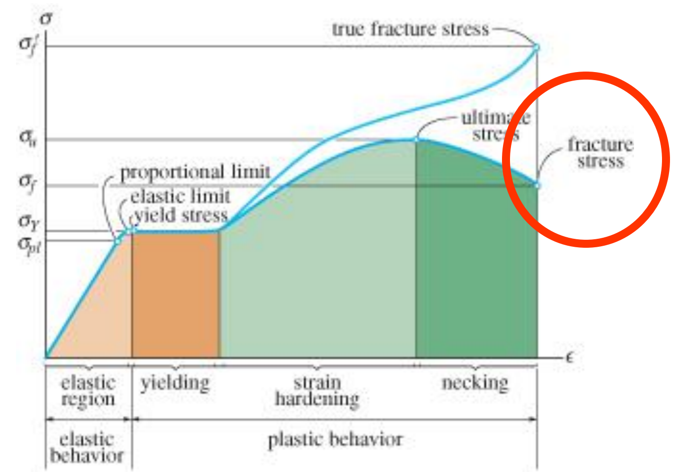
- Specimen finally breaks at fracture stress, σ_f

3.2 STRESS-STRAIN DIAGRAM

Conventional stress-strain diagram

Necking.

- Specimen finally breaks at fracture stress, σ_f



Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

3.2 STRESS-STRAIN DIAGRAM

True stress-strain diagram

- Instead of using *original* cross-sectional area and length, we can use the actual cross-sectional area and length at the *instant* the load is measured
- Values of stress and strain thus calculated are called *true stress* and *true strain*, and a plot of their values is the *true stress-strain diagram*

3.2 STRESS-STRAIN DIAGRAM

True stress-strain diagram

- In strain-hardening range, conventional σ-ε diagram shows specimen supporting *decreasing load*
- While true σ-ε diagram shows material to be sustaining *increasing stress*

3.2 STRESS-STRAIN DIAGRAM

True stress-strain diagram

- Although both diagrams are different, most engineering design is done within elastic range provided
    1. Material is “stiff,” like most metals
    2. Strain to elastic limit remains small
    3. Error in using engineering values of σ and ε is very small (0.1 %) compared to true values

Ductile materials

- Defined as any material that can be subjected to large strains before it ruptures, e.g., mild steel
- Such materials are used because it is capable of absorbing shock or energy, and if before becoming overloaded, will exhibit large deformation before failing
- Ductility of material is to report its percent elongation or percent reduction in area at time of fracture

Ductile materials

- **Percent elongation** is the specimen's fracture strain expressed as a percent

$$\text{Percent elongation} = \frac{L_f - L_0}{L_0}(100\%)$$

- **Percent reduction in area** is defined within necking region as

$$\text{Percent reduction in area} = \frac{A_0 - A_f}{A_0}(100\%)$$

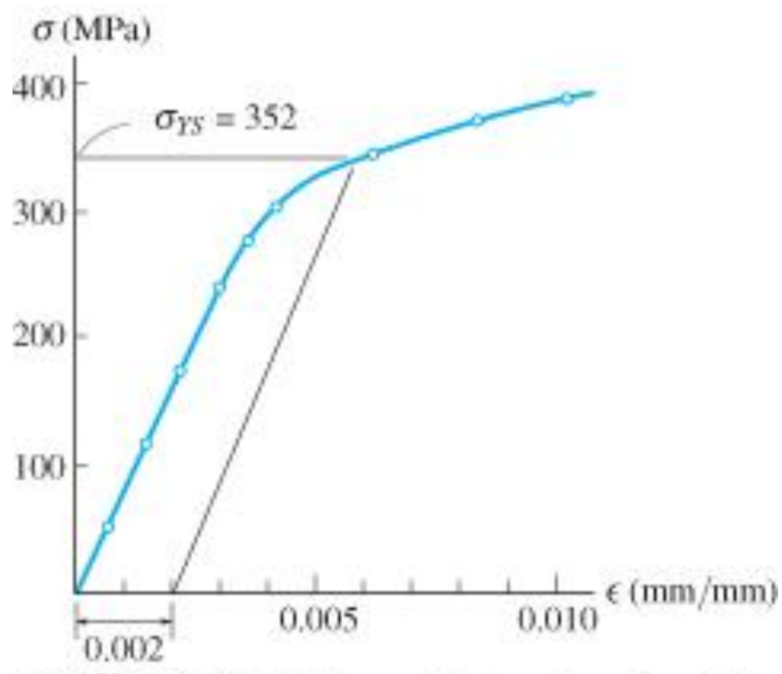
Ductile materials

- Most metals do not exhibit *constant yielding* behavior beyond the elastic range, e.g. aluminum
- It does not have well-defined yield point, thus it is standard practice to define its *yield strength* using a graphical procedure called the offset method

Ductile materials

Offset method to determine yield strength

1. Normally, a 0.2 % strain is chosen.
2. From this point on the ε axis, a line parallel to initial straight-line portion of stress-strain diagram is drawn.
3. The point where this line intersects the curve defines the yield strength.



Yield strength for an aluminum alloy

Brittle Materials

- Material that exhibit little or no yielding before failure are referred to as brittle materials, e.g., gray cast iron
- Brittle materials do not have a well-defined tensile fracture stress, since appearance of initial cracks in a specimen is quite random

Tension failure of a brittle material

(a)

Brittle Materials

- Instead, the *average* fracture stress from a set of observed tests is generally reported

3.4 HOOKE'S LAW

- Most engineering materials exhibit a *linear relationship* between stress and strain with the elastic region
- Discovered by Robert Hooke in 1676 using springs, known as *Hooke's law*

$$\sigma = E\varepsilon$$

- E represents the constant of proportionality, also called the *modulus of elasticity* or *Young's modulus*
- E has units of stress, i.e., pascals, MPa or GPa.

3.4 HOOKE'S LAW

- As shown above, most grades of steel have same modulus of elasticity, E_{st} = 200 GPa
- Modulus of elasticity is a mechanical property that indicates the *stiffness* of a material
- Materials that are still have large E values, while spongy materials (vulcanized rubber) have low values

3.4 HOOKE'S LAW

IMPORTANT

- Modulus of elasticity E, can be used only if a material has linear-elastic behavior.
- Also, if stress in material is greater than the proportional limit, the stress-strain diagram ceases to be a straight line and the equation is not valid

3.4 HOOKE'S LAW

Strain hardening

- If a specimen of ductile material (steel) is loaded into the *plastic region* and then unloaded, *elastic strain* is recovered as material returns to its equilibrium state
- However, *plastic strain* remains, thus material is subjected to a *permanent set*

3.4 HOOKE'S LAW

Strain hardening

- Specimen loaded beyond yield point *A* to *A'*
- Inter-atomic forces have to be overcome to elongate specimen *elastically,* these same forces pull atoms back together when load is removed
- Since E is the same, slope of line $O'A'$ is the same as line OA

3.4 HOOKE'S LAW

Strain hardening

- Load reapplied, atoms will be displaced until yielding occurs at or near A', and stress-strain diagram continues along same path as before
- New stress-strain diagram has *higher* yield point (A'), a result of strain-hardening
- Specimen has a *greater elastic region* and *less ductility*

3.4 HOOKE'S LAW

Strain hardening

- As specimen is unloaded and loaded, heat or *energy* may be *lost*
- Colored area between the curves represents lost energy and is called *mechanical hysteresis*
- It's an important consideration when selecting materials to serve as dampers for vibrating structures and equipment

3.5 STRAIN ENERGY

- When material is deformed by external loading, energy is stored *internally* throughout its volume
- Internal energy is also referred to as strain energy
- Stress develops a force,

$$\Delta F = \sigma \, \Delta A = \sigma \, (\Delta x \, \Delta y)$$

3.5 STRAIN ENERGY

- Strain-energy density is strain energy per unit volume of material

$$u = \frac{\Delta U}{\Delta V} = \frac{\sigma \varepsilon}{2}$$

- If material behavior is linear elastic, Hooke's law applies,

$$u = \frac{\sigma}{2}\left(\frac{\sigma}{\varepsilon}\right) = \frac{\sigma^2}{2E}$$

3.5 STRAIN ENERGY

Modulus of resilience

- When stress reaches proportional limit, strain-energy-energy density is called modulus of resilience

$$u_r = \frac{\sigma_{pl}\,\varepsilon_{pl}}{2} = \frac{\sigma_{pl}^{2}}{2E}$$

- A material's resilience represents its ability to absorb energy without any permanent damage

3.5 STRAIN ENERGY

Modulus of toughness

- Modulus of toughness u_t, indicates the strain-energy density of material before it fractures
- Shaded area under stress-strain diagram is the modulus of toughness
- Used for designing members that may be accidentally overloaded
- Higher u_t is preferable as distortion is noticeable before failure

EXAMPLE 3.1

Tension test for a steel alloy results in the stress-strain diagram below.

Calculate the modulus of elasticity and the yield strength based on a 0.2%.

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EXAMPLE 3.1 (SOLN)

Modulus of elasticity

Calculate the slope of initial straight-line portion of the graph. Use magnified curve and scale shown in light blue, line extends from *O* to *A*, with coordinates (0.0016 mm, 345 MPa)

$$E = \frac{345 \text{ MPa}}{0.0016 \text{ mm/mm}}$$

$$= 215 \text{ GPa}$$

EXAMPLE 3.1 (SOLN)

Yield strength

At 0.2% strain, extrapolate line (dashed) parallel to *OA* till it intersects stress-strain curve at *A'*

$$\sigma_{YS} = 469 \text{ MPa}$$

EXAMPLE 3.1 (SOLN)

Ultimate stress

Defined at peak of graph, point *B*,

$$\sigma_u = 745.2 \text{ MPa}$$

EXAMPLE 3.1 (SOLN)

Fracture stress

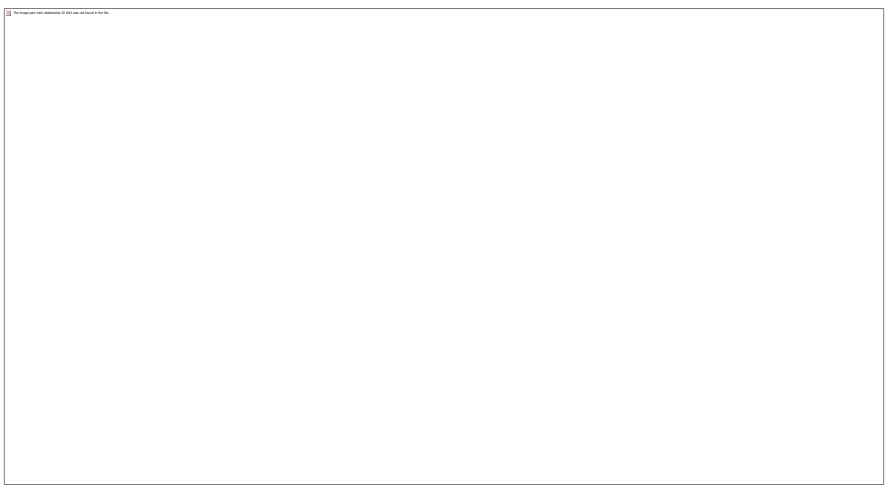
When specimen strained to maximum of $\varepsilon_f = 0.23$ mm/mm, fractures occur at C.

Thus,

$$\sigma_f = 621 \text{ MPa}$$

3.6 POISSON'S RATIO

- When body subjected to axial tensile force, it elongates and contracts laterally
- Similarly, it will contract and its sides expand laterally when subjected to an axial compressive force



3.6 POISSON'S RATIO

- Strains of the bar are:

$$\varepsilon_{\text{long}} = \frac{\delta}{L} \qquad \varepsilon_{\text{lat}} = \frac{\delta'}{r}$$

- Early 1800s, S.D. Poisson realized that within elastic range, ration of the two strains is a constant value, since both are proportional.

$$\textit{Poisson's ratio},\ \nu = -\frac{\varepsilon_{\text{lat}}}{\varepsilon_{\text{long}}}$$

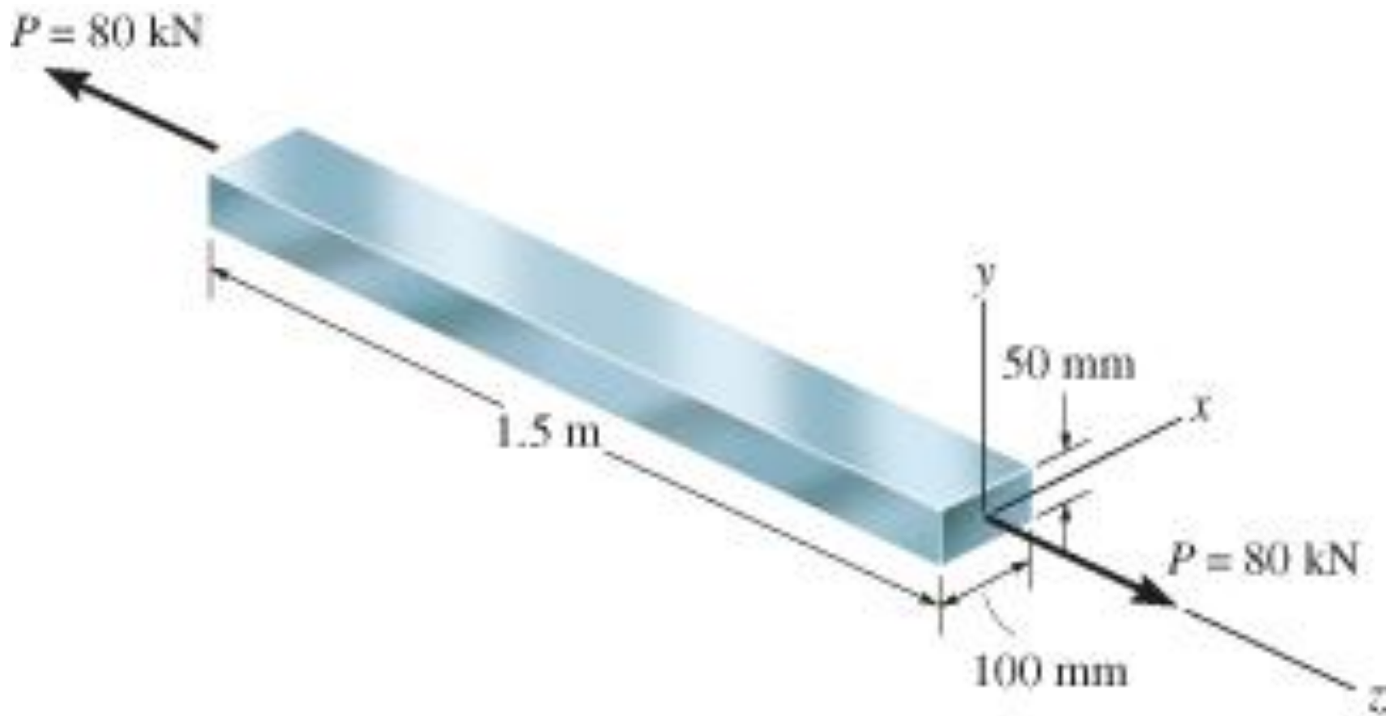
3.6 POISSON'S RATIO

- v is unique for *homogenous* and *isotropic* material
- Why negative sign? Longitudinal elongation cause lateral contraction (-ve strain) and vice versa
- Lateral strain is the same in all lateral (radial) directions
- Poisson's ratio is dimensionless, $0 \leq v \leq 0.5$

EXAMPLE 3.4

Bar is made of A-36 steel and behaves elastically.

Determine change in its length and change in dimensions of its cross section after load is applied.



EXAMPLE 3.4 (SOLN)

Normal stress in the bar is

$$\sigma_z = \frac{P}{A} = 16.0(10^6)\text{ Pa}$$

From tables, E_{st} = 200 GPa, strain in z-direction is

$$\varepsilon_z = \frac{\sigma_z}{E_{st}} = 80(10^{-6})\text{ mm/mm}$$

Axial elongation of the bar is,

$$\delta_z = \varepsilon_z L_z = [80(10^{-6})](1.5\text{ m}) = -25.6\ \mu\text{m/m}$$

EXAMPLE 3.4 (SOLN)

Using v_{st} = 0.32, contraction strains in both *x* and *y* directions are

$$\varepsilon_x = \varepsilon_y = -v_{st}\varepsilon_z = -0.32[80(10^{-6})] = -25.6\ \mu\text{m/m}$$

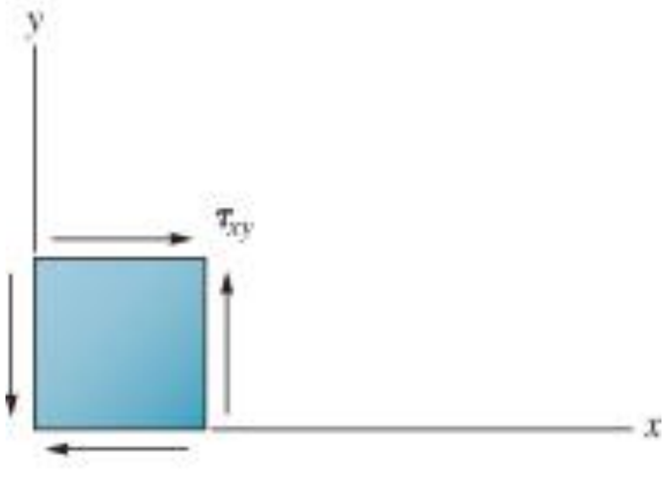
Thus changes in dimensions of cross-section are

$$\delta_x = \varepsilon_x L_x = -[25.6(10^{-6})](0.1\text{ m}) = -25.6\ \mu\text{m}$$

$$\delta_y = \varepsilon_y L_y = -[25.6(10^{-6})](0.05\text{ m}) = -1.28\ \mu\text{m}$$

3.6 SHEAR STRESS-STRAIN DIAGRAM

- Use thin-tube specimens and subject it to torsional loading
- Record measurements of applied torque and resulting angle of twist



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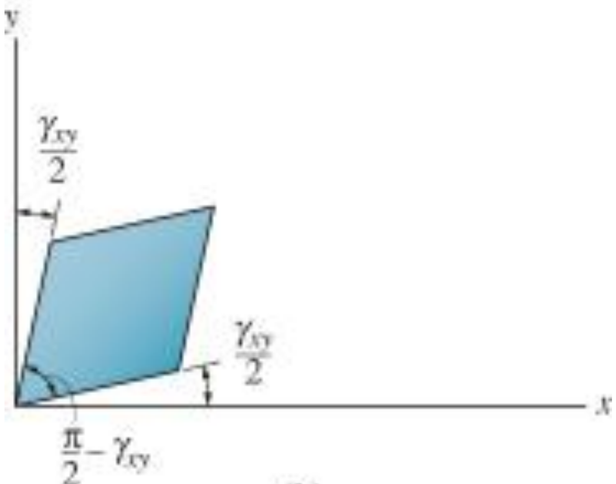
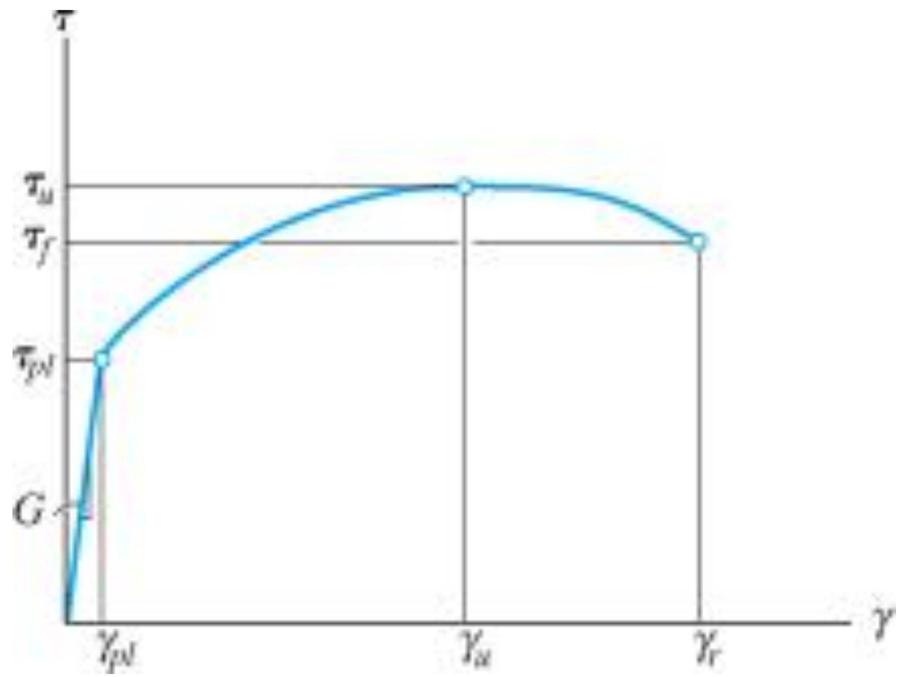


(b)

3.6 SHEAR STRESS-STRAIN DIAGRAM

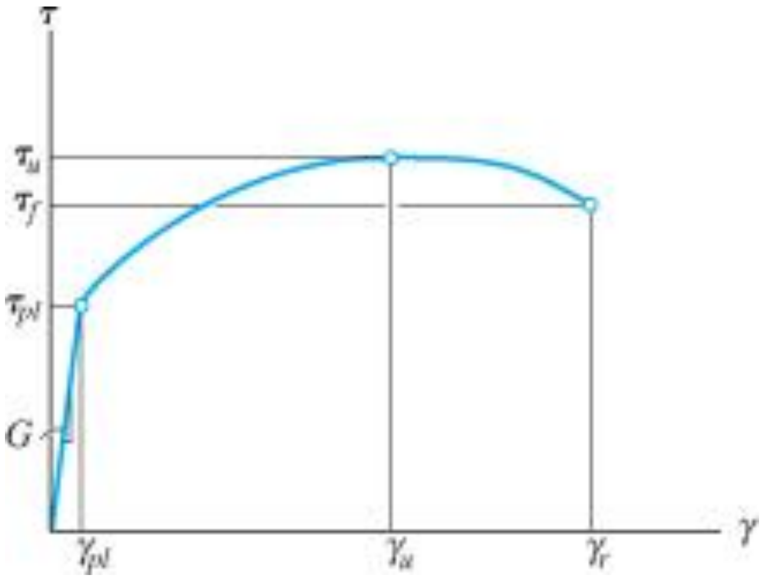
- Material will exhibit linear-elastic behavior till its proportional limit, τ_{pl}
- Strain-hardening continues till it reaches ultimate shear stress, τ_u
- Material loses shear strength till it fractures, at stress of τ_f



3.6 SHEAR STRESS-STRAIN DIAGRAM

- Hooke's law for shear

$$\tau = G\gamma$$



G is shear modulus of elasticity or modulus of rigidity

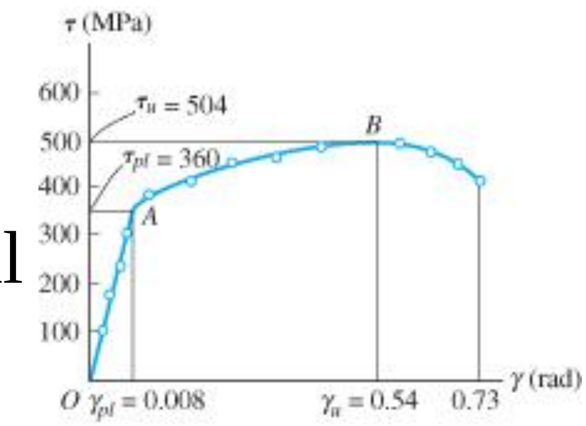
- *G* can be measured as slope of line on τ-γ diagram, $G = \tau_{pl} / \gamma_{pl}$
- The three material constants *E*, *v*, and *G* is related by

$$G = \frac{E}{2(1 + v)}$$

EXAMPLE 3.5

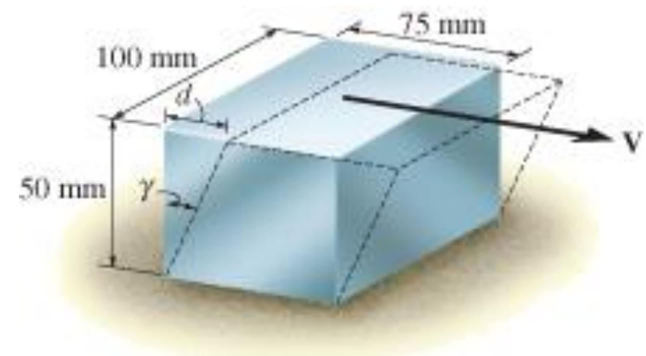
Specimen of titanium alloy tested in torsion & shear stress-strain diagram shown below.

Determine shear modulus *G*, proportional limit, and ultimate shear stress.



(a)

Also, determine the maximum distance *d* that the top of the block shown, could be displaced horizontally if material behaves elastically when acted upon by **V**. Find magnitude of **V** necessary to cause this displacement.

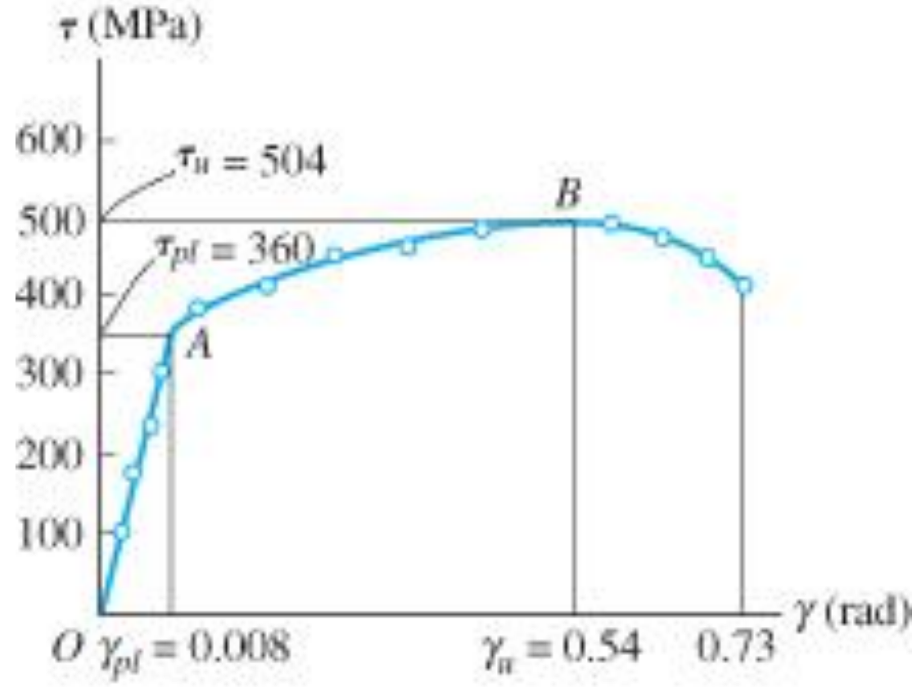


(b)

EXAMPLE 3.5 (SOLN)

Shear modulus

Obtained from the slope of the straight-line portion *OA* of the τ-γ diagram. Coordinates of *A* are (0.008 rad, 360 MPa)



(a)

$$G = \frac{360 \text{ MPa}}{0.008 \text{ rad}}$$

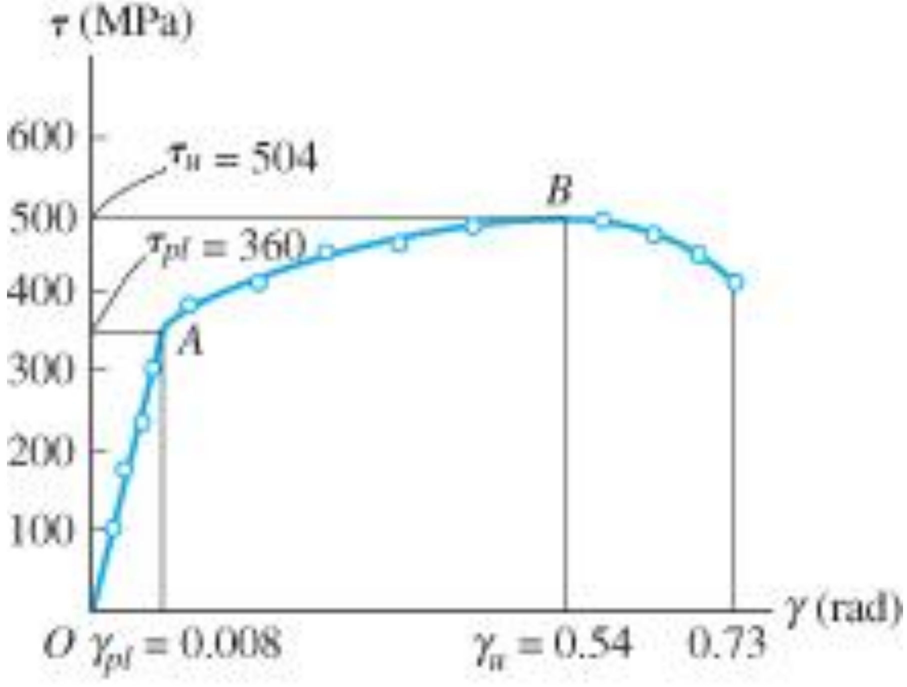
$$= 45(10^3) \text{ MPa}$$

EXAMPLE 3.5 (SOLN)

Proportional limit

By inspection, graph ceases to be linear at point A, thus,

$$\tau_{pl} = 360 \text{ MPa}$$



(a)

Ultimate stress

From graph,

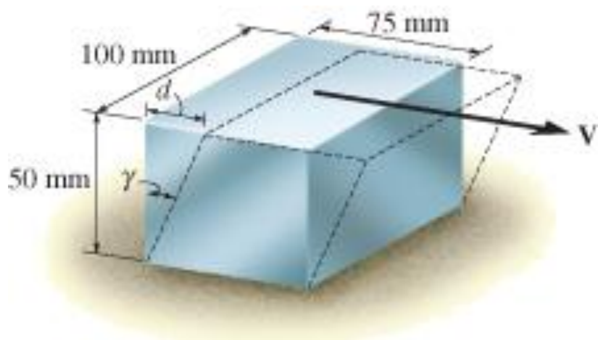
$$\tau_u = 504 \text{ MPa}$$

EXAMPLE 3.5 (SOLN)

Maximum elastic displacement and shear force

By inspection, graph ceases to be linear at point A, thus,

$$\tan(0.008 \text{ rad}) \approx 0.008 \text{ rad} = \frac{d}{50 \text{ mm}}$$



(b)

$$d = 0.4 \text{ mm}$$

$$\tau_{avg} = \frac{V}{A} \qquad 360 \text{ MPa} = \frac{V}{(75 \text{ mm})(100 \text{ mm})}$$

$$V = 2700 \text{ kN}$$

*3.7 FAILURE OF MATERIALS DUE TO CREEP & FATIGUE

Creep

- Occurs when material supports a load for very long period of time, and continues to deform until a sudden fracture or usefulness is impaired
- Is only considered when metals and ceramics are used for structural members or mechanical parts subjected to high temperatures
- Other materials (such as polymers & composites) are also affected by creep without influence of temperature

*3.7 FAILURE OF MATERIALS DUE TO CREEP & FATIGUE

Creep

- Stress and/or temperature significantly affects the rate of creep of a material
- Creep strength represents the *highest initial stress* the material can withstand during *given time* without causing *specified creep strain*

Simple method to determine creep strength

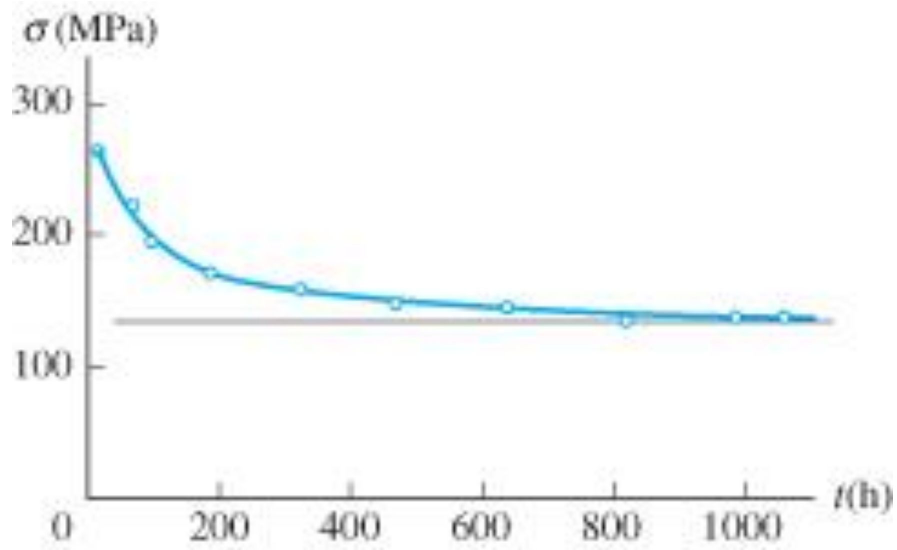
- Test several specimens simultaneously
  - At constant temperature, but
  - Each specimen subjected to different axial stress

*3.7 FAILURE OF MATERIALS DUE TO CREEP & FATIGUE

Creep

Simple method to determine creep strength

- Measure time taken to produce allowable strain or rupture strain for each specimen
- Plot stress vs. strain
- Creep strength inversely proportional to temperature and applied stresses



σ–t diagram for stainless steel at 650°C and creep strain at 1%

*3.7 FAILURE OF MATERIALS DUE TO CREEP & FATIGUE

Fatigue

- Defined as a metal subjected to repeated cycles of stress and strain, breaking down structurally, before fracturing
- Needs to be accounted for in design of connecting rods (e.g. steam/gas turbine blades, connections/supports for bridges, railroad wheels/axles and parts subjected to cyclic loading)
- Fatigue occurs at a stress *lesser* than the material's yield stress

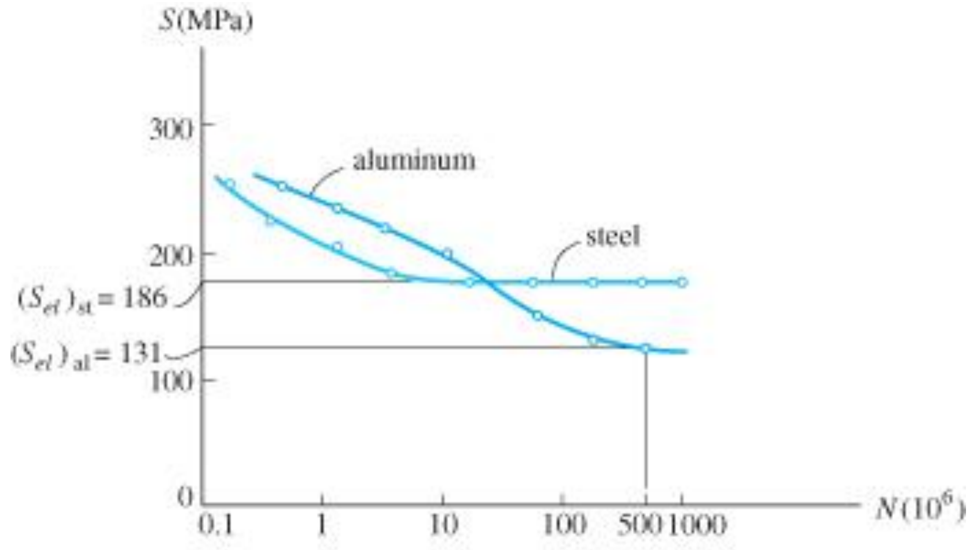
*3.7 FAILURE OF MATERIALS DUE TO CREEP & FATIGUE

Fatigue

- Also referred to as the endurance or fatigue limit

Method to get value of fatigue

- Subject series of specimens to specified stress and cycled to failure
- Plot stress (S) against number of cycles-to-failure N (S-N diagram) on logarithmic scale



S–N diagram for steel and aluminum alloys (N axis has a logarithmic scale)

CHAPTER REVIEW

- *Tension test* is the most important test for determining material strengths. Results of normal stress and normal strain can then be plotted.
- Many engineering materials behave in a *linear-elastic* manner, where stress is proportional to strain, defined by Hooke's law, $\sigma = E\varepsilon$. E is the modulus of elasticity, and is measured from slope of a stress-strain diagram
- When material stressed beyond yield point, permanent deformation will occur.

CHAPTER REVIEW

- Strain hardening causes further yielding of material with increasing stress
- At ultimate stress, localized region on specimen begin to constrict, and starts *"necking"*. Fracture occurs.
- *Ductile* materials exhibit both plastic and elastic behavior. *Ductility* specified by permanent elongation to failure or by the permanent reduction in cross-sectional area
- *Brittle* materials exhibit little or no yielding before failure

CHAPTER REVIEW

- Yield point for material can be increased by *strain hardening*, by applying load great enough to cause increase in stress causing yielding, then releasing the load. The larger stress produced becomes the new yield point for the material
- Deformations of material under load causes *strain energy* to be stored. Strain energy per unit volume/strain energy density is equivalent to area under stress-strain curve.

CHAPTER REVIEW

- The area up to the yield point of stress-strain diagram is referred to as the *modulus of resilience*
- The entire area under the stress-strain diagram is referred to as the *modulus of toughness*
- *Poisson's ratio* (v), a dimensionless property that measures the lateral strain to the longitudinal strain [$0 \leq v \leq 0.5$]
- For shear stress vs. strain diagram: within elastic region, $\tau = G\gamma$, where G is the *shearing modulus*, found from the slope of the line within elastic region

CHAPTER REVIEW

- *G* can also be obtained from the relationship of $G = E/[2(1+ v)]$
- When materials are in service for long periods of time, *creep* and *fatigue* are important.
- Creep is the time rate of deformation, which occurs at high stress and/or high temperature. Design the material not to exceed a predetermined stress called the *creep strength*

CHAPTER REVIEW

- *Fatigue* occur when material undergoes a large number of cycles of loading. Will cause micro-cracks to occur and lead to brittle failure.
- Stress in material must not exceed specified *endurance or fatigue limit*