UNIVERSITY OF NAIROBI

DEPARTMENT OF MECHANICAL AND MANUFACTURING ENGINEERINNG

FME 502: SOLID MECHANICS II

TUTORIAL SHEET NO. 2.

1. A thin steel disc of uniform thickness t has an outside radius of 275 mm and an inside radius of 55 mm. When the disc is rotating at a speed of 5,000 r.p.m. a radial tensile stress of 80 MN/m^2 is applied at the outer circumference due to blading, while the bore is free of stress. Determine:
   a) The value of the maximum tensile stress in the disc, and
   b) The increase inn the inner and outer radii compared with the disc at rest under no stress.

   Take for steel the Young's modulus to be 200 GN/m^2, Poisson's ratio to be 0.3 and the density to be 7.83×10^3 kg/m^3.

   (Ans. a) 302 MN/m^2, b) 0.083 mm, 0.132 mm)

2. A hub-shaft assembly has an external diameter of the hub of 500 mm, and interface diameter of 100 mm. The assembly was made by shrink-fitting the hub onto the solid shaft, with an original interference of an amount, δ. The assembly is to be rotated at a constant angular velocity ω radians/sec about its axis. It is found that at 6,000 r.p.m. rotating speed, the hub just separates from the shaft. Determine:
   a) The value of the original interference, δ, and
   b) The factor of safety of the hub against yielding at the above separation speed.

   Assume that the hub and the shaft are made from the same steel material of Young' modulus of 207 GN/m^2, yield point stress of 210 MN/m^2, Poisson's ratio of 0.33 and density of 7.8×10^3 kg/m^3. You may also assume that the material yields according to Tresca's yield criterion.

   (Ans. a) $\delta = 0.039$ mm, b) $u_o = 1.3$)

3.
   a) A thin circular solid disc of diameter 950 mm and of uniform thickness rotates about its axis at a constant angular velocity ω. The disc is made from a stainless steel material whose maximum allowable stress is 650 MN/m^2. Starting from the general stress distribution equations for rotating discs, determine:
      i) The maximum rotating speed for the disc, and
      j) The radial shift of the disc at the outer radius if the maximum allowable stress is not to be exceeded.

   b) A ring made from same stainless steel material, as the solid disc in a) above of nominal internal diameter 950 mm and outer diameter 1250 mm and of uniform thickness equal to that of the disc is shrink-fitted onto the disc in a) above. The disc-ring assembly has an original interference between the common diameters of 7.5 mm as predetermined for ultimate design. Working from the general stress distribution equations, determine:

i) The initial contact pressure between the disc and the ring interface at standstill, and

j) The maximum rotational velocity at which the ring would just "free" itself from the disc.

Take for the stainless steel material Young's modulus to be 195 GN/m^2, Poisson's ratio to be 0.28 and density to be 7.91×10^3 kg/m^3.

(Ans. a) i) 942.5 rad/sec. j) 0.691 mm; b) i) 650.27 MN/m^2, j) 10,527 r.p.m.)

4. A flat thin circular plate of radius 950 mm and thickness 12.5 mm is clamped (or built-in) around its periphery. The plate is subjected to a uniform pressure p per unit area over its entire surface. Working from the general plate-deflection and moments equations and assuming that the plate maximum deflection is not to exceed 7.5 mm, determine:
    a) The maximum allowable pressure that can be applied over the plate surface, and
    b) The radial and the circumferential stresses at the centre and outer radius of the plate, and hence sketch the stress distributions across the plate.

    Take Young's modulus and Poisson's ratio for the plate material to be 200 GN/m^2 and 0.29 respectively.

    (Ans. a) p_A = 20.945 kN/m^2, b) Radial stresses = 58. 52 MN/m^2 & 90.73 MN/m^2; Circumferential stresses = 58.52 MN/m^2 & 26.31 MN/m^2)

5. A circular plate of radius R and uniform thickness t is made from a material of Young's modulus E and Poisson's ration ν. The plate is simply-supported along its edges and carries a total load P which is uniformly distributed over its entire surface. Working from the general plate-deflection and moments equations, derive expressions for the elastic deflection, w, and the circumferential stress at any radius r of the plate. If the plate has dimensions R = 150 mm and t = 10 mm, and is made from a material whose yield stress is 81 MN/m^2, E = 210 GN/m^2, ν = 0.33, evaluate the maximum deflection of the plate for a condition of no yielding anywhere on the plate as per von Mises criterion.

    (Ans. w_{max} = 0.463 mm)

6.

a) A thin rectangular plate of length L, width b (L>b), and thickness t is built-in around its entire periphery. A uniformly distributed load, p per unit area, is applied over the entire plate surface. Using an approximate method with adequate reasoning, derive expressions for the maximum deflection and the maximum stress in the plate. If the plate dimensions are L = 1.2 m, b = 900 mm, and t = 20 mm, and the maximum allowable stress for the plate material is 225 MN/m^2, determine the maximum distributed load that the plate can carry and the maximum deflection that load causes on the plate. Take Young's modulus for the plate material to be 200 GN/m^2.

b) A thin square plate of sides a and thickness t is simply-supported around its periphery. A uniform pressure p per unit area is applied over the plate's entire surface. Using an approximate solution with sound explanation, derive a simple expression for the maximum bending stress in the plate.

(Ans. a) p_{max} = 585.1 kN/m^2, δ_{max} = 5.696 mm; $\sigma_{max} = pa^2/4t^2$)

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