FME201
Solid & Structural Mechanics I

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Lecture: Mon 11am -1pm (E207)
Tutorial Tue 12-1pm (E207)

Outline

- This lecture is based on chapter 1 of Hibbeler's book – Mechanics of materials
- 1.1 Introduction
- 1.2 Equilibrium of deformable body
- 1.3 Stress
- 1.4 Average normal stress in an axially loaded bar
- 1.5 Average shear stress
- 1.6 Allowable stress

Introduction

- Mechanics of materials – study the relationship between *external* loads applied to a deformable body and the intensity of the *internal* forces acting within the body.
- Also a study of the calculation of *deformations* of the body and it provides the body's *stability* when the body is subjected to external forces.
- We use principles of statics to determine on and within the member. The size of the member, its deflection and stability depends on both the internal loadings and material behaviour.

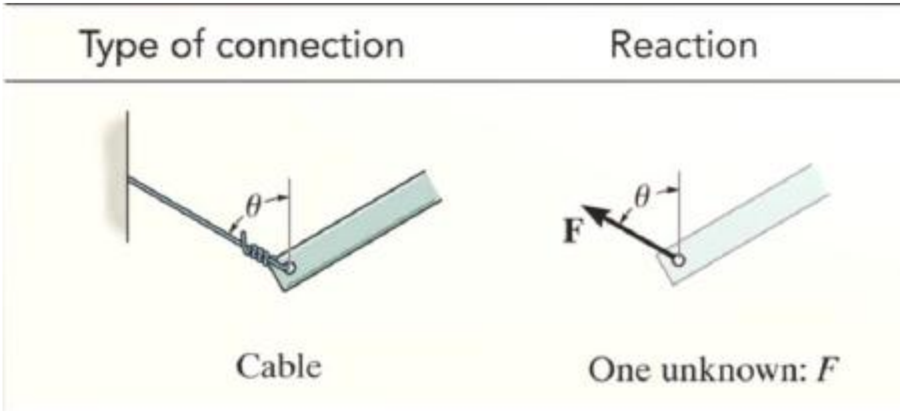
Chapter 1 Stress: objectives

- Review important principles of statics and show how they are used to determine the internal resultant loadings in a body
- Understand the concepts of normal and shear stress
- Analysis and design of connections subjected to an axial load or direct shear

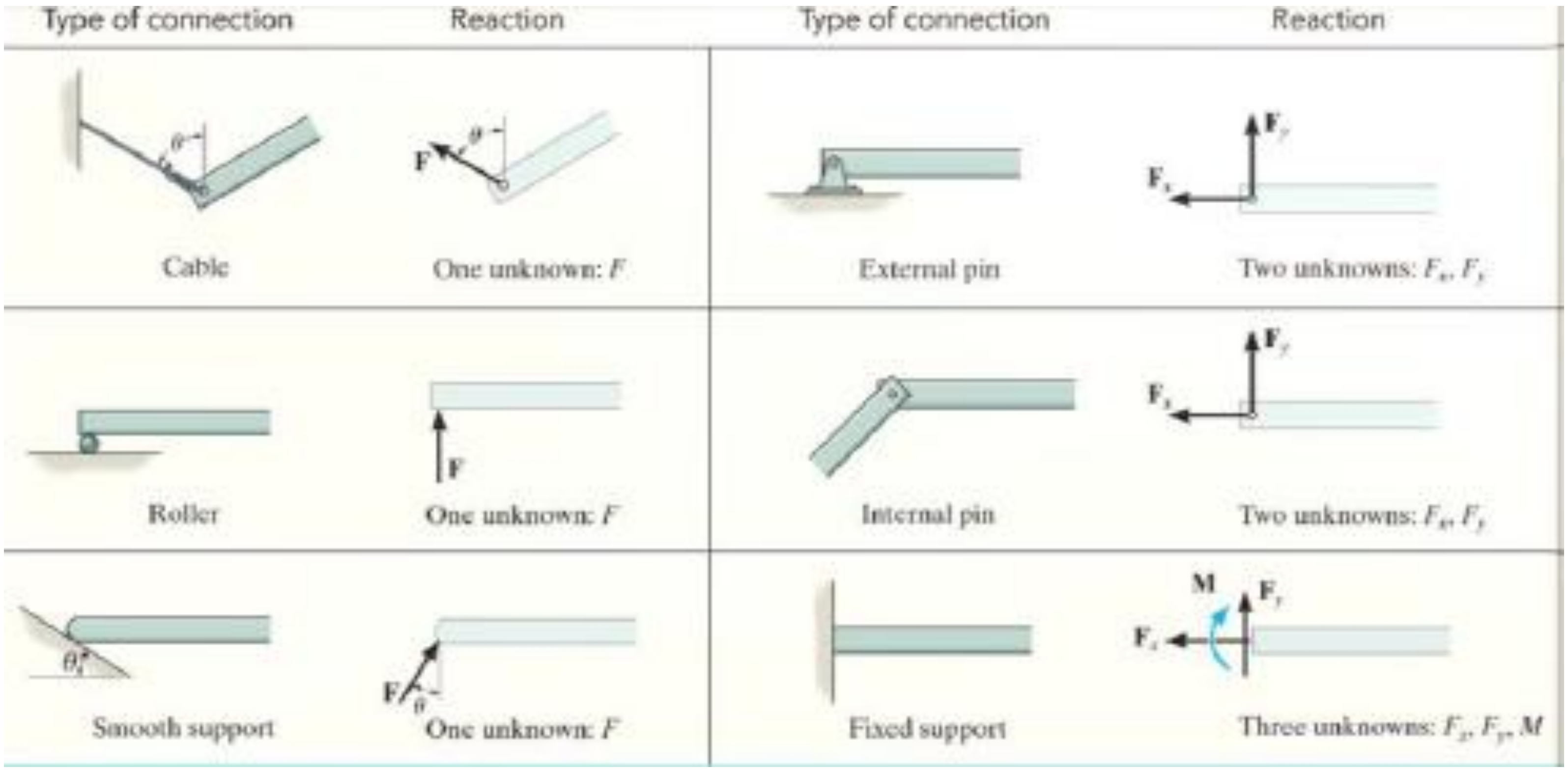
Equilibrium of a deformable body cont..

Support reactions

- If the support prevents translation in a given direction, then a force must be developed on the member in that direction.
- If rotation is prevented, a couple moment must be exerted on the member



Idealisation of support reactions



Equations of Equilibrium

$$\sum \mathbf{F} = 0; \quad \sum \mathbf{M}_O = 0$$

In three dimensional space

$$\sum F_x = 0; \quad \sum F_y = 0; \quad \sum F_z = 0$$
$$\sum M_x = 0; \quad \sum M_y = 0; \quad \sum M_z = 0$$

In two dimensions

$$\sum F_x = 0; \quad \sum F_y = 0$$
$$\sum M_O = 0$$

In this course (FME 201) we will mainly with structures in two dimensions

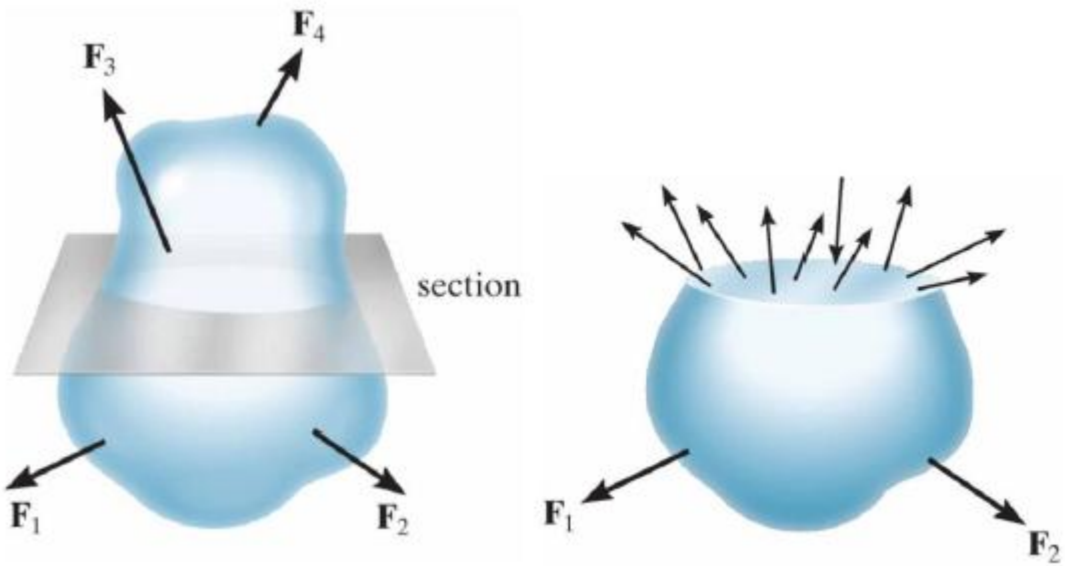
- **The best way to account for these forces is to draw the body's free-body diagram**

Pre-requisite knowledge -statics

- You should have successfully completed FME 173 to do this course.
- This means you should know
  - **Force vectors** (scalars, vectors, vector operations, cartesian vectors)
  - **Moment of a force**, cross product, moment of a force couple, etc.
  - **Equilibrium** of a particle and of a rigid body
- We will cover plane frame analysis in this course

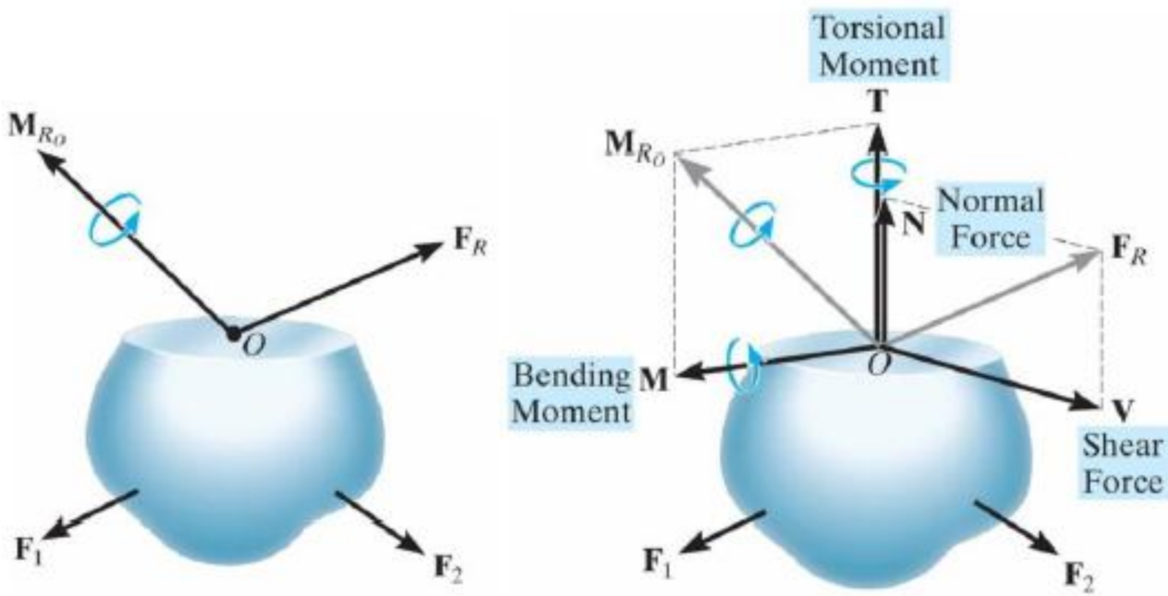
Internal resultant loadings

- Conception of a section



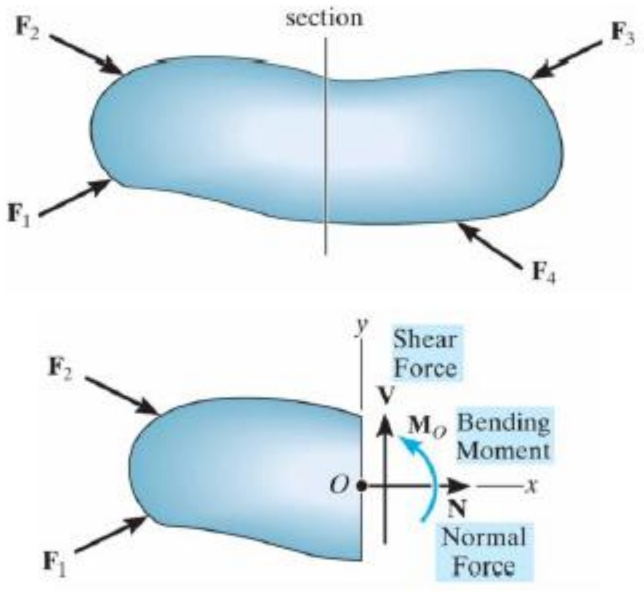
- Note that the weight of the member is not shown (why???)

Internal resultant loadings cont..



- Normal force (N)
- Shear force (V)
- Torsional moment (T)
- Bending moment (M)

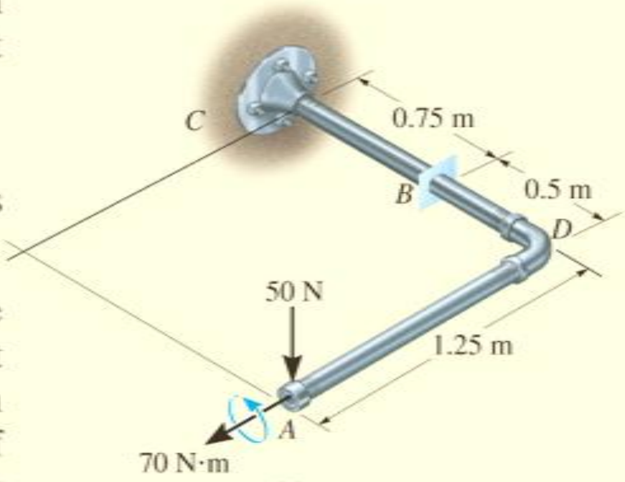
Co-planar loading



Example 1.5 (page 15)

EXAMPLE 1.5

Determine the resultant internal loadings acting on the cross section at B of the pipe shown in Fig. 1–8*a*. The pipe has a mass of 2 kg/m and is subjected to both a vertical force of 50 N and a couple moment of 70 N · m at its end A. It is fixed to the wall at C.



(a)

Solution

The problem can be solved by considering segment AB, which does *not* involve the support reactions at C.

Free-Body Diagram. The x, y, z axes are established at B and the free-body diagram of segment AB is shown in Fig. 1–8*b*. The resultant force and moment components at the section are assumed to act in the positive coordinate directions and to pass through the *centroid* of the cross-sectional area at B. The weight of each segment of pipe is calculated as follows:

$$W_{BD} = (2\text{ kg/m})(0.5\text{ m})(9.81\text{ N/kg}) = 9.81\text{ N}$$

$$W_{AD} = (2\text{ kg/m})(1.25\text{ m})(9.81\text{ N/kg}) = 24.525\text{ N}$$

These forces act through the center of gravity of each segment.

Example 1.5

Equations of Equilibrium. Applying the six scalar equations of equilibrium, we have*

$\Sigma F_x = 0;$ $\quad (F_B)_x = 0$ *Ans.*

$\Sigma F_y = 0;$ $\quad (F_B)_y = 0$ *Ans.*

$\Sigma F_z = 0;$ $\quad (F_B)_z - 9.81\text{ N} - 24.525\text{ N} - 50\text{ N} = 0$

$(F_B)_z = 84.3\text{ N}$ *Ans.*

$\Sigma (M_B)_x = 0;$ $\quad (M_B)_x + 70\text{ N}\cdot\text{m} - 50\text{ N}\,(0.5\text{ m}) - 24.525\text{ N}\,(0.5\text{ m}) - 9.81\text{ N}\,(0.25\text{ m}) = 0$

$(M_B)_x = -30.3\text{ N}\cdot\text{m}$ *Ans.*

$\Sigma (M_B)_y = 0;$ $\quad (M_B)_y + 24.525\text{ N}\,(0.625\text{ m}) + 50\text{ N}\,(1.25\text{ m}) = 0$

$(M_B)_y = -77.8\text{ N}\cdot\text{m}$ *Ans.*

$\Sigma (M_B)_z = 0;$ $\quad (M_B)_z = 0$ *Ans.*

What do the negative signs for $(M_B)_x$ and $(M_B)_y$ indicate? Note that the normal force $N_B = (F_B)_y = 0$, whereas the shear force is $V_B = \sqrt{(0)^2 + (84.3)^2} = 84.3\text{ N}$. Also, the torsional moment is $T_B = (M_B)_y = 77.8\text{ N}\cdot\text{m}$ and the bending moment is $M_B = \sqrt{(30.3)^2 + (0)} = 30.3\text{ N}\cdot\text{m}$.

*The *magnitude* of each moment about an axis is equal to the magnitude of each force times the perpendicular distance from the axis to the line of action of the force. The *direction* of each moment is determined using the right-hand rule, with positive moments (thumb) directed along the positive coordinate axes.

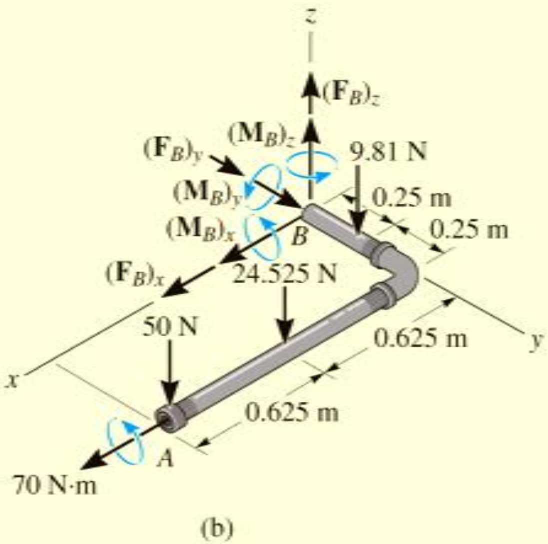
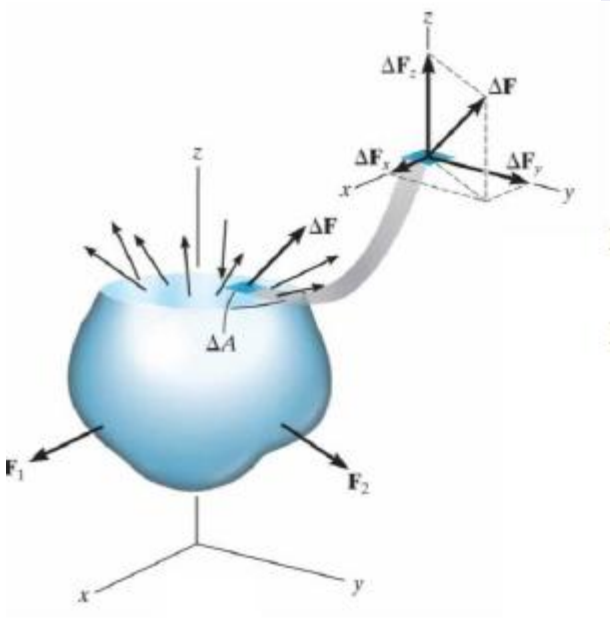


Fig. 1–8

Stress

- Stress is defined as the intensity of a force per unit area



- Normal stress

$$\sigma_z = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$

pulls on $\Delta A \Rightarrow$ tensile

pushes on $\Delta A \Rightarrow$ compressive

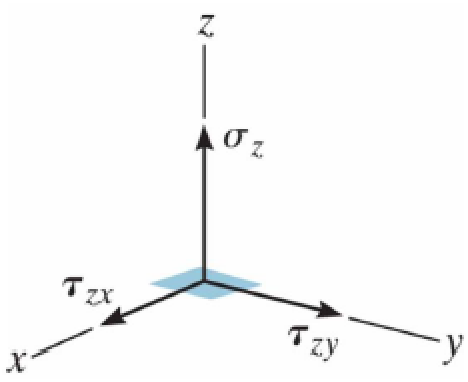
- Shear stress

$$\tau_{zx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}$$

$$\tau_{zy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A}$$

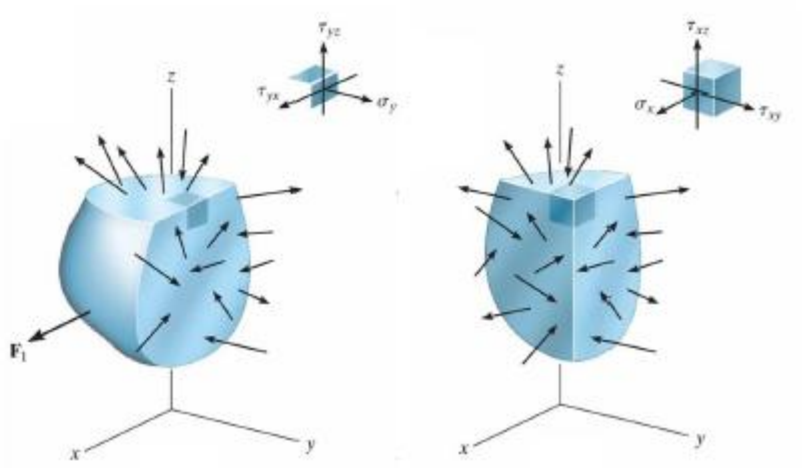
Stress cont..

- Z species the orientation of the area

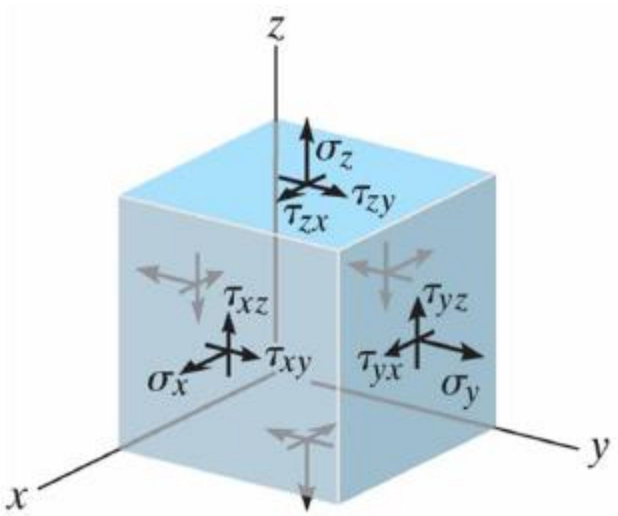


- x and y indicate the axes along which the shear stress acts

- Shear stress



General state of stress



Unit of Stress (SI)

$F \rightarrow \mathrm{N}, \quad A \rightarrow \mathrm{m}^2$

$(\sigma, \tau) = F/A = \mathrm{N/m}^2 = \mathrm{Pa}$

10^3 Pa = 1 kPa (k ≡ kilo)

10^6 Pa = 1 MPa (M ≡ mega)

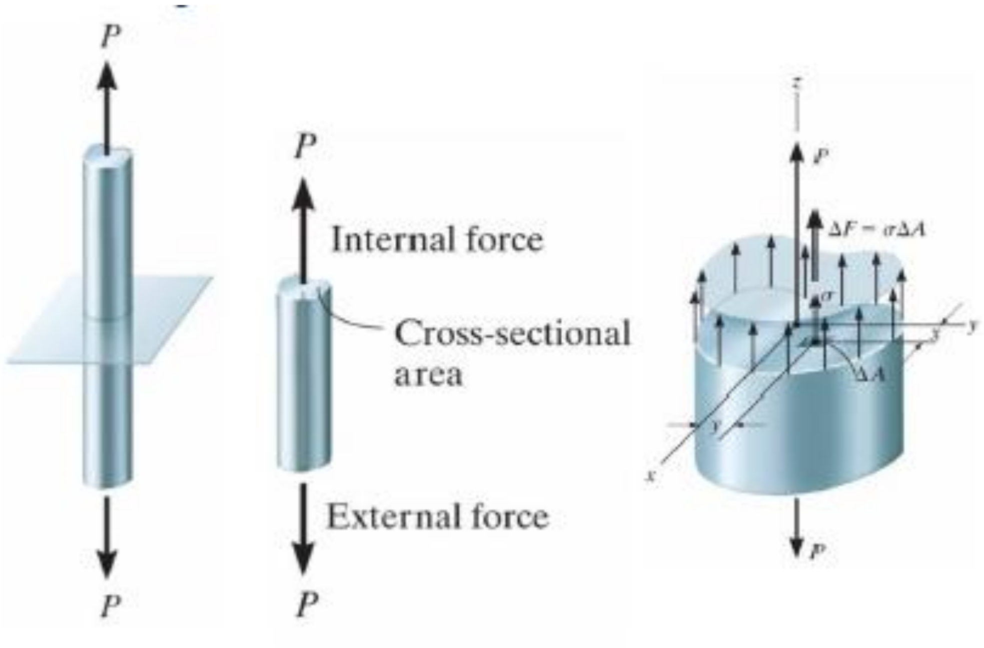
10^9 Pa = 1 GPa (G ≡ giga)

Use Newtons for force and mm^2 for area and you will get stress in MegaPascals

Average normal stress in an axially loaded bar

- Assumptions
  - Homogenous material – same physical and mechanical properties throughout the material
  - Isotopric material – same physical and material properties in all directions

Average normal stress in an axially loaded bar



Average normal stress in an axially loaded bar

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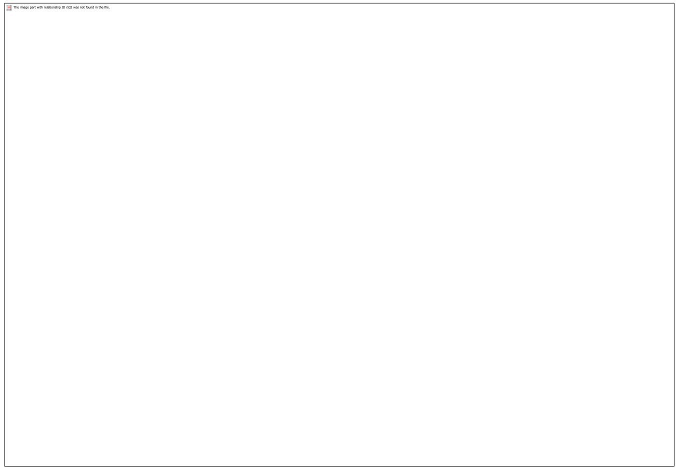
Average normal stress in an axially loaded bar

- Uni-axial force
- Uni-axial stress



Example 1.6

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Determine internal forces at all points between A and B, B and C & C and D.

How?

Cut at any point between **A and B** and then draw FBD.

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Cut at any point between **B and C** and then draw FBD.

Cut at any point between **C and D** and then draw FBD.

Question: Can we draw the internal force diagram?

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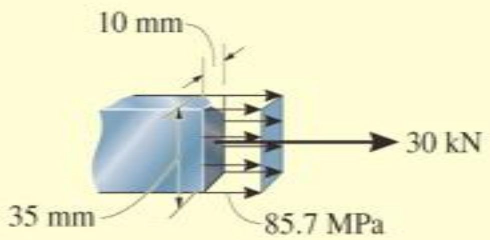
Question: What is the largest average normal stress in the bar?



(d)

Fig. 1–16

Solution

Internal Loading. By inspection, the internal axial forces in regions *AB*, *BC*, and *CD* are all constant yet have different magnitudes. Using the method of sections, these loadings are determined in Fig. 1–16*b*; and the normal force diagram which represents these results graphically is shown in Fig. 1–16*c*. By inspection, the largest loading is in region *BC*, where $P_{BC} = 30$ kN. Since the cross-sectional area of the bar is *constant*, the largest average normal stress also occurs within this region of the bar.

Average Normal Stress. Applying Eq. 1–6, we have

$$\sigma_{BC} = \frac{P_{BC}}{A} = \frac{30(10^3)\text{N}}{(0.035\text{ m})(0.010\text{ m})} = 85.7\text{ MPa} \qquad \textit{Ans.}$$

The stress distribution acting on an arbitrary cross section of the bar within region *BC* is shown in Fig. 1–16*d*. Graphically the *volume* (or "block") represented by this distribution of stress is equivalent to

Example 1.7

EXAMPLE 1.7

The 80-kg lamp is supported by two rods AB and BC as shown in Fig. 1–17*a*. If AB has a diameter of 10 mm and BC has a diameter of 8 mm, determine the average normal stress in each rod.

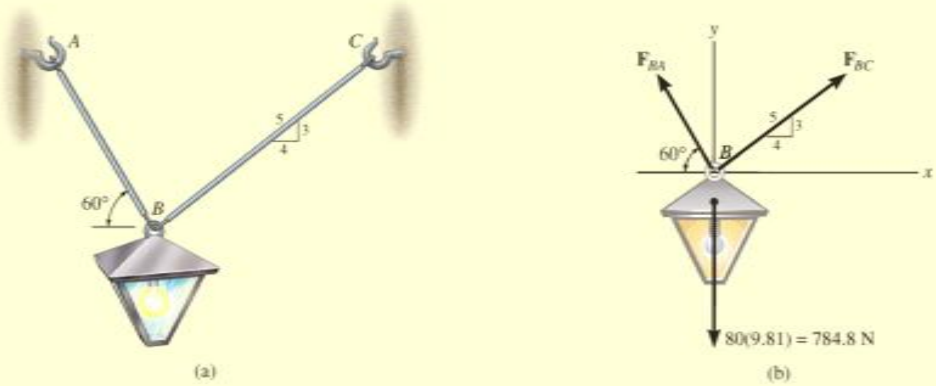


Fig. 1–17

Example 1.7 cont…

Solution

Internal Loading. We must first determine the axial force in each rod. A free-body diagram of the lamp is shown in Fig. 1–17*b*. Applying the equations of force equilibrium yields

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{BC}\left(\tfrac{4}{5}\right) - F_{BA}\cos 60^\circ = 0$$

$$+\uparrow \Sigma F_y = 0; \quad F_{BC}\left(\tfrac{3}{5}\right) + F_{BA}\sin 60^\circ - 784.8\text{ N} = 0$$

$$F_{BC} = 395.2\text{ N}, \qquad F_{BA} = 632.4\text{ N}$$

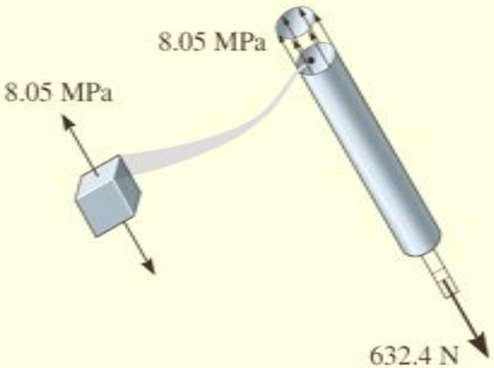
By Newton's third law of action, equal but opposite reaction, these forces subject the rods to tension throughout their length.

Average Normal Stress. Applying Eq. 1–6, we have

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{395.2\text{ N}}{\pi(0.004\text{ m})^2} = 7.86\text{ MPa} \qquad \textit{Ans.}$$

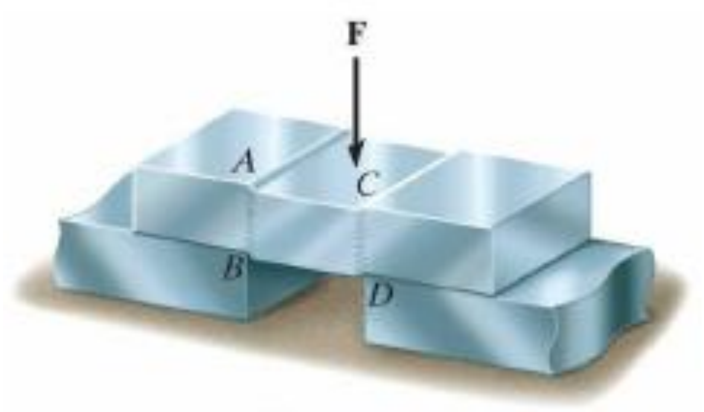
$$\sigma_{BA} = \frac{F_{BA}}{A_{BA}} = \frac{632.4\text{ N}}{\pi(0.005\text{ m})^2} = 8.05\text{ MPa} \qquad \textit{Ans.}$$

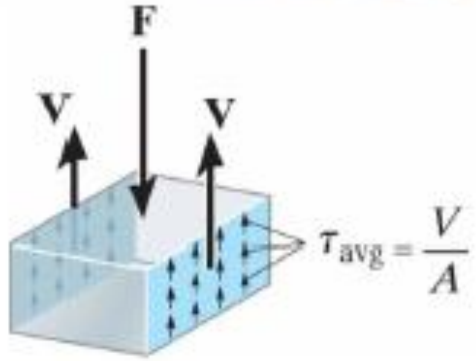
The average normal stress distribution acting over a cross section of rod AB is shown in Fig. 1–17*c*, and at a point on this cross section, an element of material is stressed as shown in Fig. 1–17*d*.



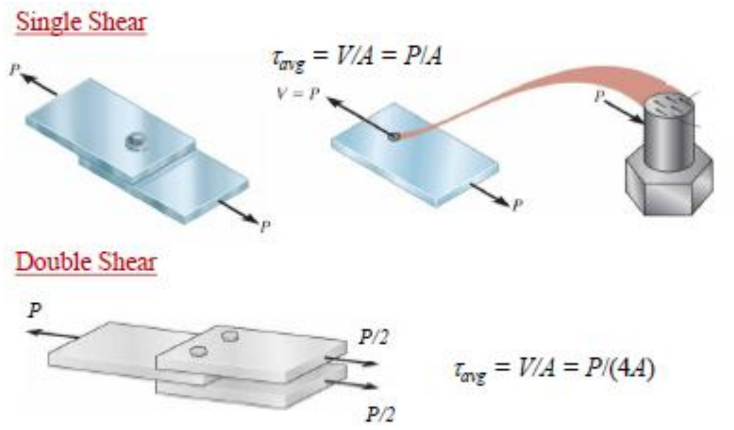
(d) (c)

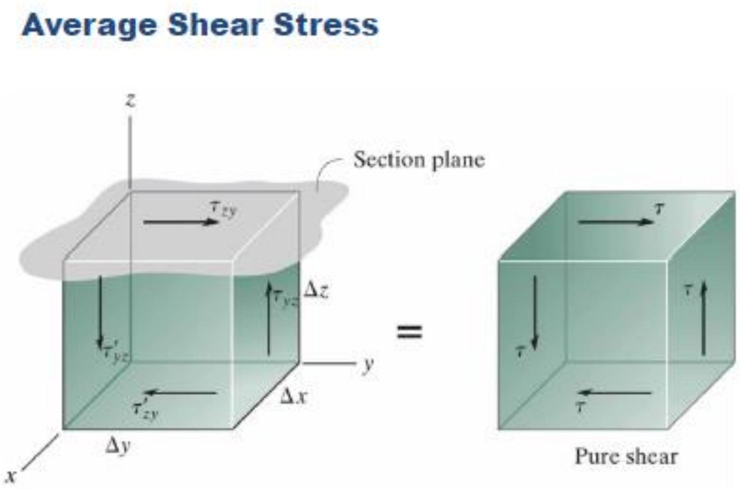
Average Shear Stress





Average shear stress





Allowable stress

Factor of Safety: $\text{F.S.} = \dfrac{F_{fail}}{F_{allow}}$

If there is a linear relationship between the applied load and the corresponding stress: $\sigma = P/A$ or $\tau_{avg} = V/A$

$$\text{F.S.} = \frac{\sigma_{fail}}{\sigma_{allow}} \qquad \text{F.S.} = \frac{\tau_{fail}}{\tau_{allow}}$$

Example 1.10

EXAMPLE 1.10

The bar shown in Fig. 1–24*a* has a square cross section for which the depth and thickness are 40 mm. If an axial force of 800 N is applied along the centroidal axis of the bar's cross-sectional area, determine the average normal stress and average shear stress acting on the material along (a) section plane *a–a* and (b) section plane *b–b*.

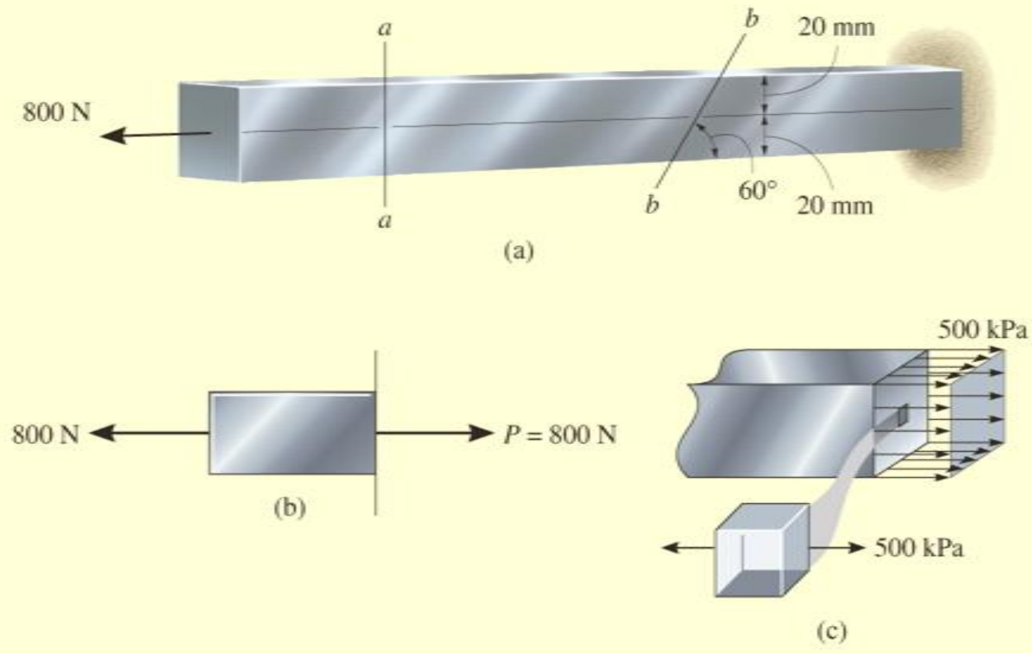


Fig. 1–24

Example 1.10 cont…

Solution

Part (a)

Internal Loading. The bar is sectioned, Fig. 1–24*b*, and the internal resultant loading consists only of an axial force for which $P = 800$ N.

Average Stress. The average normal stress is determined from Eq.1–6.

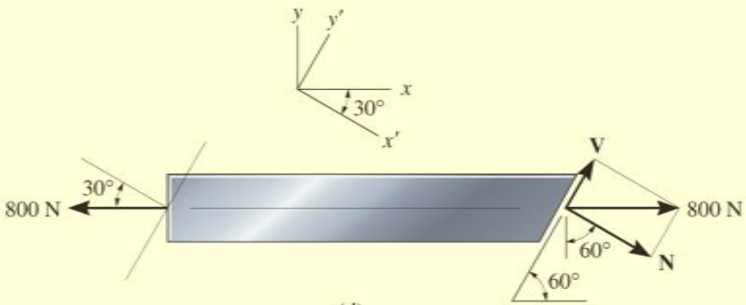
$$\sigma = \frac{P}{A} = \frac{800\text{ N}}{(0.04\text{ m})(0.04\text{ m})} = 500\text{ kPa} \qquad \textit{Ans.}$$

No shear stress exists on the section, since the shear force at the section is zero.

$$\tau_{avg} = 0 \qquad \textit{Ans.}$$

The distribution of average normal stress over the cross section is shown in Fig. 1–24*c*.

Example 1.10 cont..



(d)

Part (b)

Internal Loading. If the bar is sectioned along b–b, the free-body diagram of the left segment is shown in Fig. 1–24*d*. Here both a normal force (**N**) and shear force (**V**) act on the sectioned area. Using x, y axes, we require

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -800\text{ N} + N \sin 60^\circ + V \cos 60^\circ = 0$$
$$+\uparrow \Sigma F_y = 0; \qquad V \sin 60^\circ - N \cos 60^\circ = 0$$

or, more directly, using x', y' axes,

$$+\searrow \Sigma F_{x'} = 0; \qquad N - 800\text{ N} \cos 30^\circ = 0$$
$$+\nearrow \Sigma F_{y'} = 0; \qquad V - 800\text{ N} \sin 30^\circ = 0$$

Example 1.10 cont…

Solving either set of equations,

$$N = 692.8\ \text{N}$$
$$V = 400\ \text{N}$$

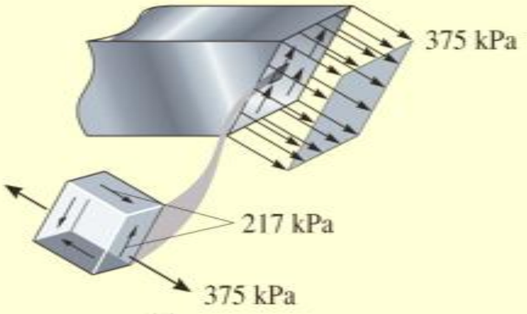
Average Stresses. In this case the sectioned area has a thickness and depth of 40 mm and 40 mm/sin 60° = 46.19 mm, respectively, Fig. 1–24*a*. Thus the average normal stress is

$$\sigma = \frac{N}{A} = \frac{692.8\ \text{N}}{(0.04\ \text{m})(0.04619\ \text{m})} = 375\ \text{kPa} \qquad \textit{Ans.}$$

and the average shear stress is

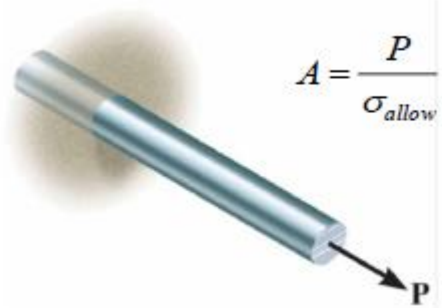
$$\tau_{\text{avg}} = \frac{V}{A} = \frac{400\ \text{N}}{(0.04\ \text{m})(0.04619\ \text{m})} = 217\ \text{kPa} \qquad \textit{Ans.}$$

The stress distribution is shown in Fig. 1–24*e*.



(e)

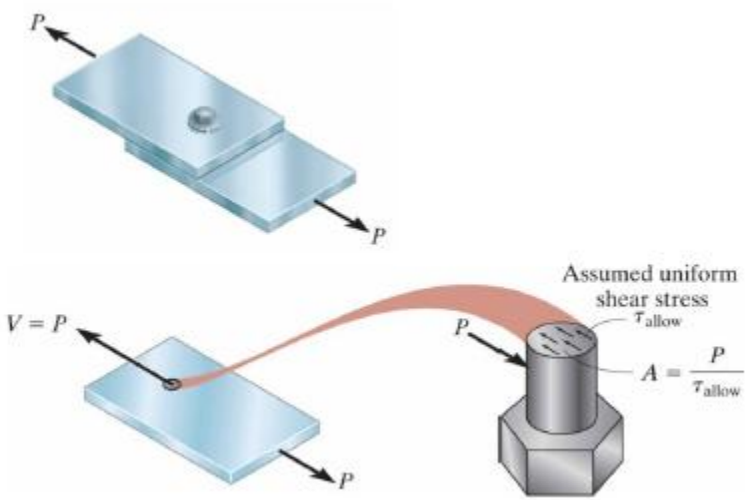
Design of simple connections



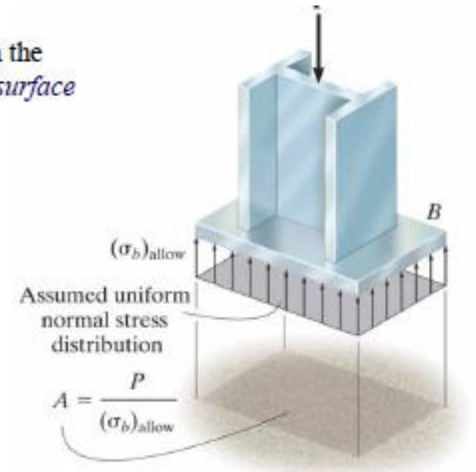
$$A = \frac{P}{\sigma_{allow}} = \frac{(F.S.)P}{\sigma_{fail}}$$

The embedded length l of this rod in concrete can be determined using the allowable shear stress of the bonding glue.

Design of simple connections cont..



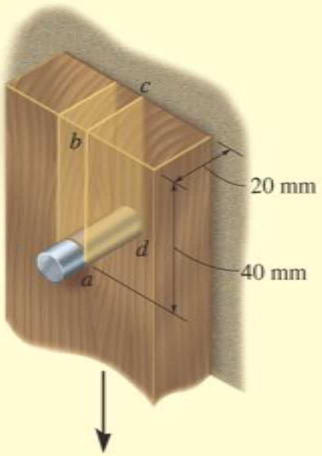
Bearing stress: stress on the *bearing surface*, or the *surface of contact*



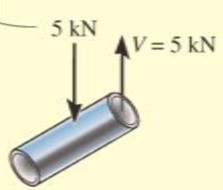
The area of the column base plate B is determined from the allowable bearing stress for the concrete.

Example 1.11

EXAMPLE 1.11



(a)



(b)

The wooden strut shown in Fig. 1–25*a* is suspended from a 10-mm-diameter steel rod, which is fastened to the wall. If the strut supports a vertical load of 5 kN, compute the average shear stress in the rod at the wall and along the two shaded planes of the strut, one of which is indicated as *abcd*.

Solution

Internal Shear. As shown on the free-body diagram in Fig. 1–25*b*, the rod resists a shear force of 5 kN where it is fastened to the wall. A free-body diagram of the sectioned segment of the strut that is in contact with the rod is shown in Fig. 1–25*c*. Here the shear force acting along each shaded plane is 2.5 kN.

Average Shear Stress. For the rod,

$$\tau_{avg} = \frac{V}{A} = \frac{5000\text{ N}}{\pi(0.005\text{ m})^2} = 63.7\text{ MPa} \qquad \textit{Ans.}$$

For the strut,

$$\tau_{avg} = \frac{V}{A} = \frac{2500\text{ N}}{(0.04\text{ m})(0.02\text{ m})} = 3.12\text{ MPa} \qquad \textit{Ans.}$$

Example 1.11 cont..

The average-shear-stress distribution on the sectioned rod and strut segment is shown in Figs. 1–25*d* and 1–25*e*, respectively. Also shown with these figures is a typical volume element of the material taken at a point located on the surface of each section. Note carefully how the shear stress must act on each shaded face of these elements and then on the adjacent faces of the elements.

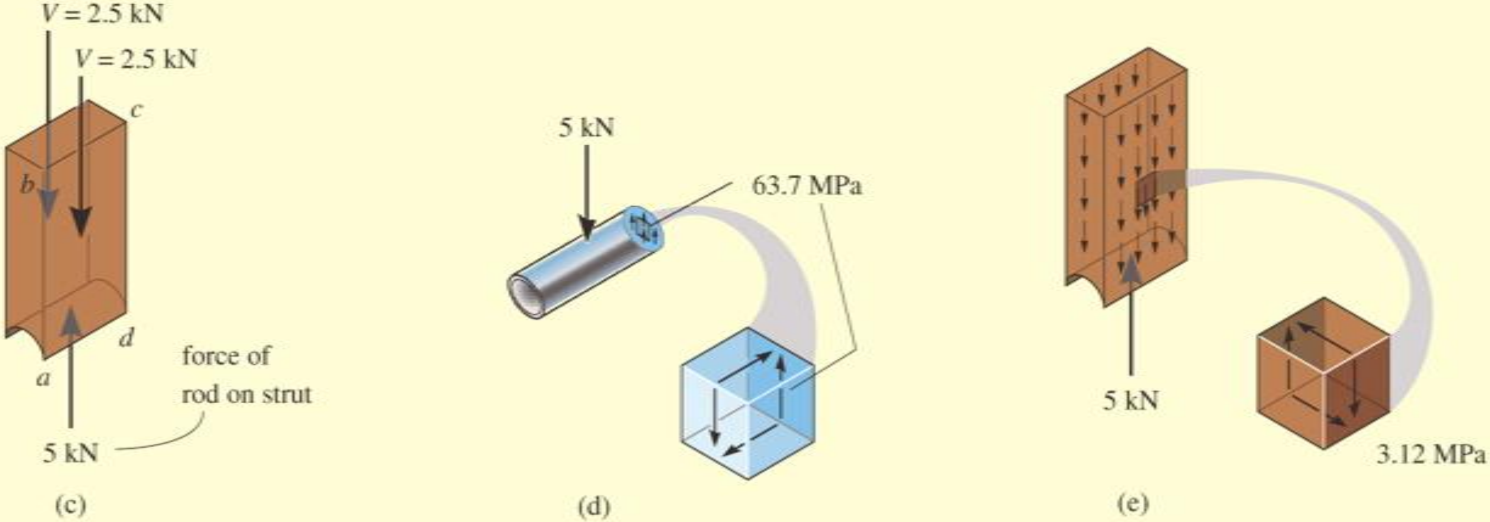


Fig. 1–25

Homework

- 1.82
- 1.83
- 1.86
- 1.93
- 1.111

Recommended Texts

- Mechanics of Materials – 2nd Edition, Madhukar Vable – available online **FREE**
- Engineering Mechanics – Statics, R.C. Hibbler,
- Engineering Mechanics – Statics, D.J. McGill & W.W. King
- Mechanics of Materials , J.M. Gere & S.P. Timoshenko
- Mechanics of solids, Abdul Mubeen, Pearson Education Asia

Expectations

- I expect students to be active learners. There will be **<u>no passengers</u>** in my class.
- Learning is an active process and it has been shown that a variety of factors influence learning such as educational experience, age, abilities, motivation, expectations etc.
- I will treat you as adults. I expect respect and maturity from you.
- All laboratory assignments must be typed and printed.

My teaching style

- Not didactic- I don’t posses all the knowledge to be imparted to the learners.
- Collaborative learning- I give the outlearn and you fill in the gaps. Lectures are used to give the outline and the theory.
- More like a coach. Tutorials where we will solve problems together