

UNIVERSITY OF NAIROBI

DEPARTMENT OF MECHANICAL AND MANUFACTURING
ENGINEERING

**DESIGN OF A WIND TURBINE SYSTEM FOR ELECTRICITY
GENERATION**

FINAL YEAR PROJECT REPORT

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A FINAL YEAR PROJECT IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR
THE AWARD OF THE DEGREE IN BACHELOR OF SCIENCE IN MECHANICAL AND
MANUFACTURING ENGINEERING.

DECLARATION

We declare that this is our original work and has not been published or presented for a degree award in any other university/college/institution there before.

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SIGNATURE	DATE
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DEDICATION

This project is dedicated to our beloved parents for their continuous support in bringing us this far academically and morally to the University of Nairobi, Our supervisor Mr. Misango who guided us through and to the Mechanical and manufacturing Engineering Department.

ACKNOWLEDGEMENT

We thank the Lord God Almighty for giving u strength and sustaining us during the course of this project

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We also acknowledge the Kenya Meteorological Department and Ngong wing generation project for its support in giving us information regarding wind data pattern in Kenya. We also thank the ministry of energy, Kenya Generating Company KenGen) and Kenya Power for the data provided unto us in the course of undertaking our project.

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LIST OF FIGURES NUMBER	PAGE
Figure 1.0: Components of a conventional wind turbine.....	4
Figure 1.1: Detailed view of a wind turbine.....	5
Figure 1.2: Vertical and Horizontal wind turbines.....	8
Figure 2.1: Actuator disc.....	18
Figure 2.2: Stream boundary at rotor plane.....	22
Figure 2.3: Sketch of wake.....	23
Figure 2.4: An airfoil.....	26
Figure 2.5: An aerofoil.....	27
Figure 2.6: Aerofoil shape of a bladder.....	28
Figure 2.7: Sketch of a 3-blade wind turbine.....	29
Figure 3.0: Diagram of forces acting on aerofoil.....	38
Figure 3.1: Pattern of wind distribution in Kenya.....	39
Figure 3.2: Diurnal wind speed over Marsabit.....	39
Figure 3.3: Diurnal variation of the wind power variation over Marsabit.....	40
Figure 3.4: Graph of wind speed against Days of Year.....	40
Figure 3.5: Graph of power against Days of the Year.....	41
Figure 3.6: Major parts of a permanent magnet generator.....	42
Figure 3.7: A 6KW permanent magnet generator.....	43
Figure 3.8: Dimensions of generator.....	43
Figure 4.0: Types of blades.....	48
Figure 4.1: Variation of stress verses Time.....	49

LIST OF FIGURES	PAGE NUMBER
Figure 4.2: Blade manufacturing industry (http://www.google.com/3905714943.html).....	84
Figure 5.0: A graph of coefficient of performance verses Tip speed Ratio for 17% thick LS (1) 4/7 aerofoil (http://www.google.com/876678669.html).....	60
Figure 5.1: A graph of coefficient of lift, coefficient of drag verses Angle of attack for 17% thick LS (1) 4/7 aerofoil (http://www.google.com/876678668.html).....	61
Figure 5.2: A cross section view of an aerofoil.....	84
Figure 5.3: A hand drawn cross section view of an aerofoil.....	84
Figure 5.4: variation of bending moment with radius.....	85
Figure 5.5: A sketch of blade elements arrangement.....	88
Figure 5.6: A sketch of inner and outer diameters.....	94
Figure 5.7: parts of the 6KW wind turbine.....	102
Figure 5.8: Front view of the 6KW wind turbine.....	103
Figure 5.9: 3-blade rotor of the 6KW wind turbine.....	104

LIST OF TABLES	PAGE NUMBER
Table 1.0: Terrain verses Z_0 (mm) where Z_0 is the relative surface roughness.....	16
Table 2.0: Tip speed ration verses number of blades.....	35
Table 3.0: Tip speed ration verses number of blades.....	59
Table 4.0: Blade elements chord length, twist distribution and pitch (summary).....	71
Table 5.0: Material selection index.....	79
Table 6.0: Summary of material selection coefficient.....	81
Table 7.0: Blade elements chord length and thickness.....	92
Table 8.0: Blade element and corresponding bending moment.....	93
Table 9.0: Blade elements characteristics.....	97

TABLE OF CONTENTS	PAGE NUMBER
--------------------------	--------------------

Declaration.....	ii
Dedication.....	iii
Acknowledgement.....	iv

CHAPTER ONE: GENERATION OF ELECTRICAL POWER USING WIND ENERGY

1.0.0 Objective.....	2
1.0.1 Introduction.....	2
1.0.2 Literature review.....	3
1.0.3 Components of nacelle.....	4
1.0.4 Low speed and high speed shaft.....	5
1.0.5 The gearbox.....	6
1.0.6 Coupling.....	6
1.0.7 Bearings.....	6
1.0.8 Mechanical brakes.....	6
1.0.9 The generator.....	6
1.1.0 Power electronics.....	6
1.1.1 Cooling unit.....	7
1.1.2 Yaw mechanism and four points bearing.....	7
1.1.3 Electronic controllers.....	7
1.1.4 Sensors.....	8
1.1.5 Types of turbines.....	8
1.1.6 Orientation of axis.....	8
1.1.7 Wind turbine classifications.....	9
1.1.8 Power in wind.....	10

CHAPTER TWO: DESIGN OF WIND TURBINE SYSTEM AND THEORY

2.0.0 Theory.....	11
2.0.1 Statistical methods of determining wind distribution.....	12

2.0.2 Wind variation with height.....	14
2.0.3 Logarithmic profile.....	14
2.0.4 Values of surface roughness length for various types of terrain.....	16
2.0.5 Effects of terrain.....	17
2.0.6 Depression.....	17
2.0.7 One dimensional momentum theory and Betz limit.....	18
2.0.8 The actuator disc.....	18
2.0.9 Ideal horizontal axis wind turbine with ware rotation.....	21
2.1.0 Airfoils and general concepts of aerodynamics.....	26
2.1.1 General rotor blade performance prediction.....	26
2.1.2 Blade element theory.....	29
2.1.3 Blade element momentum theory.....	30
2.1.4 Calculation of power coefficient.....	31
2.1.5 Tip loss: effect on power coefficient of number of blades.....	32
2.1.6 Blade shape for optimum rotor with wake.....	33

CHAPTER 3: ROTOR DESIGN PROCEDURE

3.0.0 General rotor design procedure.....	35
3.1.1 Wind density and distribution in Kenya.....	38
3.1.2 Case study: Marsabit region.....	41
3.1.3 Generator specifications.....	41
3.1.4 Permanent magnet generators.....	42
3.1.5 Generator weight.....	43
3.1.6 Generator dimensions.....	43
3.1.7 Generator number of poles.....	44
3.1.8 Power output.....	44
3.1.9 Power storage.....	44
3.2.0 Height of the tower.....	44

3.2.1 Tower construction.....	45
3.2.2 The foundation of the tower.....	45
3.2.3 Power storage system.....	45

CHAPTER FOUR: MECHANICS AND DYNAMICS OF WIND TURBINE

4.0.0 Mechanics and dynamics of wind turbine.....	46
4.0.1 Wind turbine loads.....	46
4.0.2 Steady loads.....	46
4.0.3 Cyclic loads.....	46
4.0.4 Transient loads.....	47
4.0.5 Stochastic loads.....	47
4.0.6 Resonance induced load.....	47
4.0.7 Sources of loads.....	47
4.0.8 Effects of loads.....	47
4.0.9 Tower loads.....	47
4.1.0 Type of common blades.....	48
4.1.1 Wind turbine materials and components.....	48
4.1.2 Material fatigue.....	49
4.1.3 Fatigue in wind turbines.....	49
4.1.4 Assessment of fatigue.....	49
4.1.5 Goodman diagram.....	50
4.1.6 Effect of operating conditions.....	50
4.1.7 Wind turbine materials.....	50
4.1.8 Review of basic mechanical properties of materials.....	51
4.1.9 Machine elements.....	51
4.2.0 Gear dynamic loading.....	52
4.2.1 Principal components.....	53
4.2.2 Blade manufacturing.....	54

4.2.3 Wind turbine design and testing.....	57
--	----

4.2.4 Some important parameters.....	57
--------------------------------------	----

CHAPTER FIVE: THE 6KW ROTOR DESIGN PROCEDURE

5.0.0 The 6KW rotor design procedure.....	58
---	----

5.0.1 Rotor design.....	58
-------------------------	----

5.0.2 Calculation of rotor performance and modification of design.....	71
--	----

5.0.3 Material selection of wind turbine.....	79
---	----

5.0.4 Mechanical design of the blades.....	82
--	----

5.0.5 Structural design of the wind turbine blades.....	83
---	----

5.0.6 The flap wise bending moments of various blade elements.....	87
--	----

5.0.7 Stress analysis of the blades.....	94
--	----

5.0.8 Shear strength analysis.....	98
------------------------------------	----

5.0.9 Test of whether the blade would rotate.....	99
---	----

5.1.0 Shaft design.....	100
-------------------------	-----

5.1.1 The 6KW wind turbine drawings.....	102
--	-----

Conclusion.....	104
-----------------	-----

References.....	106
-----------------	-----

CHAPTER ONE: GENERATION OF ELECTRICAL POWER USING WIND ENERGY

ABSTRACT

The aim of this project is to design a wind turbine energy system to produce electricity while working on an optimum rotor. In Kenya, energy is classified as a prime mover for many industries and factories. In a country where both income and energy are both tragically low, renewable energy source will be the ultimate solution to these problems.

Kenya is in the midst of unresolved debate over energy policy. Most important part is on the development of renewable clean sources of energy like the wind power. It is in this light that this project looks at most suitable design and structural adjustments that can be made or done on the wind turbine to make it most efficient for electricity generation in the country.

To realize this objective, we made visits to the Ngong wind farm to understand turbine operation and the barriers to absolute conversion and also obtained data to be used in the analysis of the design procedure.

We carried out analysis using data from various reliable wind energy associations. We also visited the Kenya Meteorological Department of Kenya to obtain wind data which indicated the great potential for clean source of energy in Kenya.

1.0.0 OBJECTIVE

The objective of this project is to work on an optimum wind turbine design using available analysis of the already designed wind turbines in order to create most efficient wind power harnessing wind turbine to produce cheapest and clean source of energy for Marsabit region.

1.0.1 INTRODUCTION

Wind is defined as air in motion. It is caused by different pressure zones. During daytime, the air above land heats up quickly than air that is found over water. Warm air over the land expands and rises and the heavier, cooler air rushes in to take its place hence creating wind.

Wind power is sourced as a result of conversion of wind energy into electricity using wind turbine. Worldwide wind generated power is totalling more than 74223 megawatts. Denmark tops in wind power generation at 18%, Spain at 9%, Germany at 7% this information is according to wind energy council.

In ancient times, wind power was only used as a source of mechanical power to pump water and grind cereals. It was one of the earliest sources of energy with its use dating back to early civilization mainly in the banks of river Euphrates and Tigris.

In Kenya wind power is exploited in certain areas where wind velocity is at least constant for a prolonged period of time across the year. In areas like Ngong, six wind turbines have been installed and are connected to national grid. Other areas like Turkana and Marsabit Counties where wind velocities on average range from 11-20 m /s according to data from meteorological department of Kenya, a wind farm is in the process of development and will have a collective output of 735 Megawatts. Wind farms are large scale establishments of wind turbines in a certain location.

1.0.2: LITERATURE REVIEW

In Kenya three organisations are providing funding and research for more sustainable wind power production in Kenya. The Industrial Development Corporation, KP&P Africa B.V and Norfund and Wind power Invest A.S this information is according to ministry of energy.

Wind power has grown steadily over the last decade at an average of 30% per year. A total of 120 GW are produced annually from wind power. The largest wind turbine is the Emerson, located in Texas USA with an output of 6MW. The largest wind farm is also in Texas with a capacity of 420 turbines and produces a total of 735 MW of energy this information can be obtained directly from the internet. With such energy production from wind over 160 million tonnes of carbon dioxide are cut hence reduced emission of greenhouse gases.

The U.S leads in the wind energy production with an output of over 25 GW according to the wind energy council. Several developing nations are turning towards this clean cheap source of energy which is easy to harness compared with other sources of energy like fossil fuels and hydroelectric power.



Figure 1.0: Components of a conventional wind turbine (<http://www.flickr.com/photos/pjh/185488411/>.)

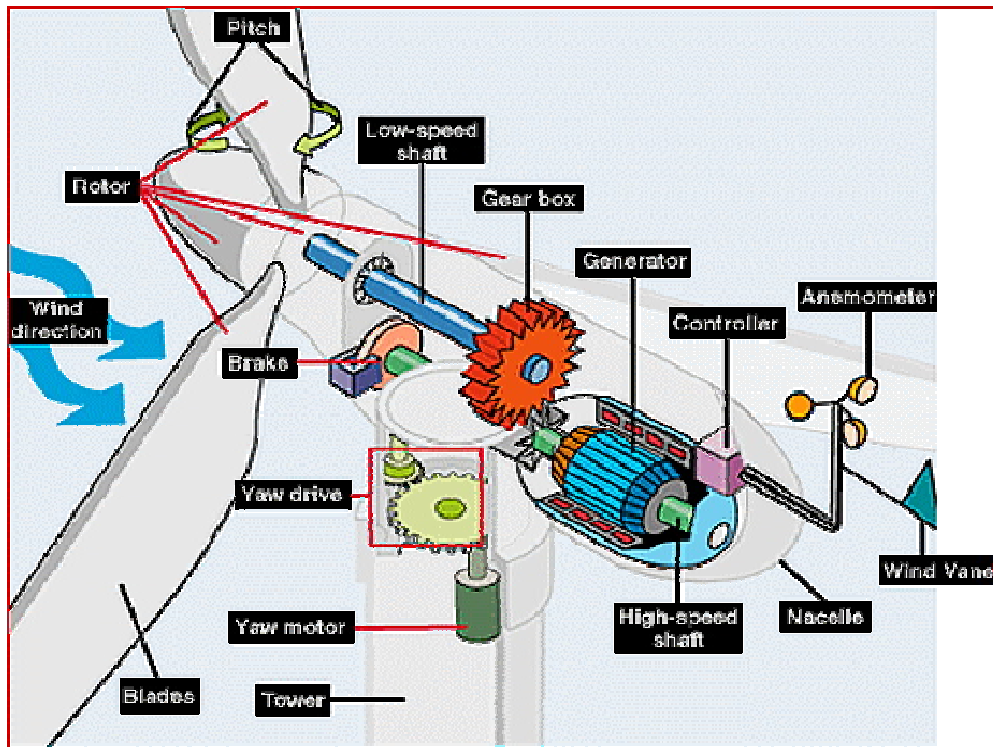


Figure 1.1: Detailed view of a wind turbine (<http://www.flickr.com/photos/pjh/185488415/>.)

1.0.3 Components of Nacelle

This is the wind turbine housing in many cases made of aluminium metal because aluminium is cheap and corrosion resistant, it consists of the following components

- Outer frame protecting machinery from external environment
- Internal frame supporting and distributing weight of the machinery
- A power train to transmit energy and to increase speed
- A generator to convert mechanical energy to electricity
- A yaw drive to rotate the nacelle on tower
- Electronics to control and monitor operation of the wind turbine.

1.0.4 Low speed and high speed shaft

This transmits rotational work from rotor to the gear box where the high speed shaft transmits the rotary motion to the generator the high speed shaft transmits motion from the high speed gear drive while the low speed shaft transmits motion from turbine rotor.

1.0.5 The gearbox

Converts low speed rotations from input shaft to high speed output shaft. This is a speed multiplication component of the wind turbine system which increases low rotary speed from the turbine to high rotary speed of the generator.

1.0.6 Coupling

Couplings join a component to another. In a wind turbine couplings attach shaft to the generator. There are different types of couplings but most commonly used type of couplings in wind turbines are flexible couplings that have the advantage of reducing oscillatory loads that otherwise would damage some components like the generator.

1.0.7 Bearing

The role bearings are to reduce friction. In wind turbines bearings are used to support the rotating shaft and in other wind turbines that do not have a yawing mechanism operated by a motor i.e. the free yaw wind turbines to yaw.

1.0.8 Mechanical brakes

The role of mechanical blades is to stop motion. Different types of brakes in a wind turbine the most commonly used type of brakes are disk brakes because they occupy a small volume and are very efficient and the assembly is easy. The system of brakes is used to stop rotation of blades during regular maintenance practice.

1.0.9 The generator

The component that converts rotary motion to electrical energy. Induction generators are most commonly used type of generators because they do not require an exciting field current. Generators are continuously being developed for wind turbines with research centred to producing high electrical power at low speed.

1.1.0 Power electronics

Electrical output from the generator is fed to the step-up transformer, where voltage is stepped up for transmission purposes.

1.1.1 Cooling unit

The purpose of the fan is to drive in cool air to the generator and gearbox and exhaust hot air from the nacelle assembly this is achieved with suitable ducting which directs cool air to and from the generator.

1.1.2 Yaw mechanism and four point bearing

This ensures that the turbine faces the most suitable wind direction. It also turns the blades 90 degrees from prevailing winds under high winds to reduce stress on blades and internal components and avoid over-speed conditions which can lead to overloading of the generator which can lead to damage of generator windings.

1.1.3 Electronic controllers

Electronic controllers are used to control functionality of the wind turbine to ensure that design limits are not exceeded to prevent damage to the wind turbine system and ensure maximum power production.

This includes:

- Base controller located at the base of the tower connected to computer and stores and monitors the data of the turbine. It also facilitates data communications between the sub-controllers and the external components.
- Nacelle controller that monitors activity within the nacelle cavity for example draining of lubricant in the gearbox.
- Hub controller that is usually in direct communication with the nacelle controller to more precisely move the blades.
- Rotor activity. They control the speed of revolution of the rotor and the pitch angles.

1.1.4 Sensors

Sensors collect information and change it to electrical signals which are relayed to actuators that give appropriate response. The sensors in wind turbines include; the anemometer to measure wind speed, wind vane to indicate wind direction, thermocouple to indicate temperature changes and cable twist counter monitors within the tower to determine if the turbine has been yawing in one direction for prolonged period of time.

The rotors are normally vertically or horizontally mounted on the tower. A shaft is connected from the rotor to the gearbox. In the gear box the speed is multiplied by means of suitable gears with minimal power loss and minimal noise. A shaft is introduced from the gear box to the generator that produces power. A synchronous generator is mostly used due to its minimal energy loss and its ability to control voltage.

1.1.5 Types of Turbines

Wind turbines are classified by the orientation of its axis to the direction of wind and power output. The axis can be horizontally or vertically oriented, which have various affects on the design of the overall structure. The power output of the turbine can be selected based on the energy needs of the site and affect the overall size of the turbine.

1.1.6 Orientation of Axis

There are two main designs of wind turbines, the horizontal axis wind turbines (HAWT) and the vertical axis wind turbines (VAWT). Horizontal axis wind turbines are more common and widely available because they are the more efficient than vertical axis wind turbine. For these reasons more information is available on the construction of horizontal axis wind turbines than the vertical axis wind turbine. The HAWT model will be used in this project. Horizontal axis wind turbines usually have two or three tapered fibre glass-reinforced blades or any other composite material which are operated facing the wind direction. The turbine unit, which includes the blades, rotor, nacelle, and additional mechanical equipment, is attached to a tubular steel tower which supports a main rotor shaft and electrical generator at the top. Most horizontal axis wind turbines are equipped with furling and shutdown systems to protect the turbines from damaging at high-speed winds. If the wind speeds are too high for the designed critical strength of the blades, a furling system will turn the wind turbine generator rotor away from the direct wind force while still producing maximum output. A shutdown system can be

electrical or mechanical and is designed to shut down the generator completely and also stop the blade from spinning due to a braking system installed in the shutting system

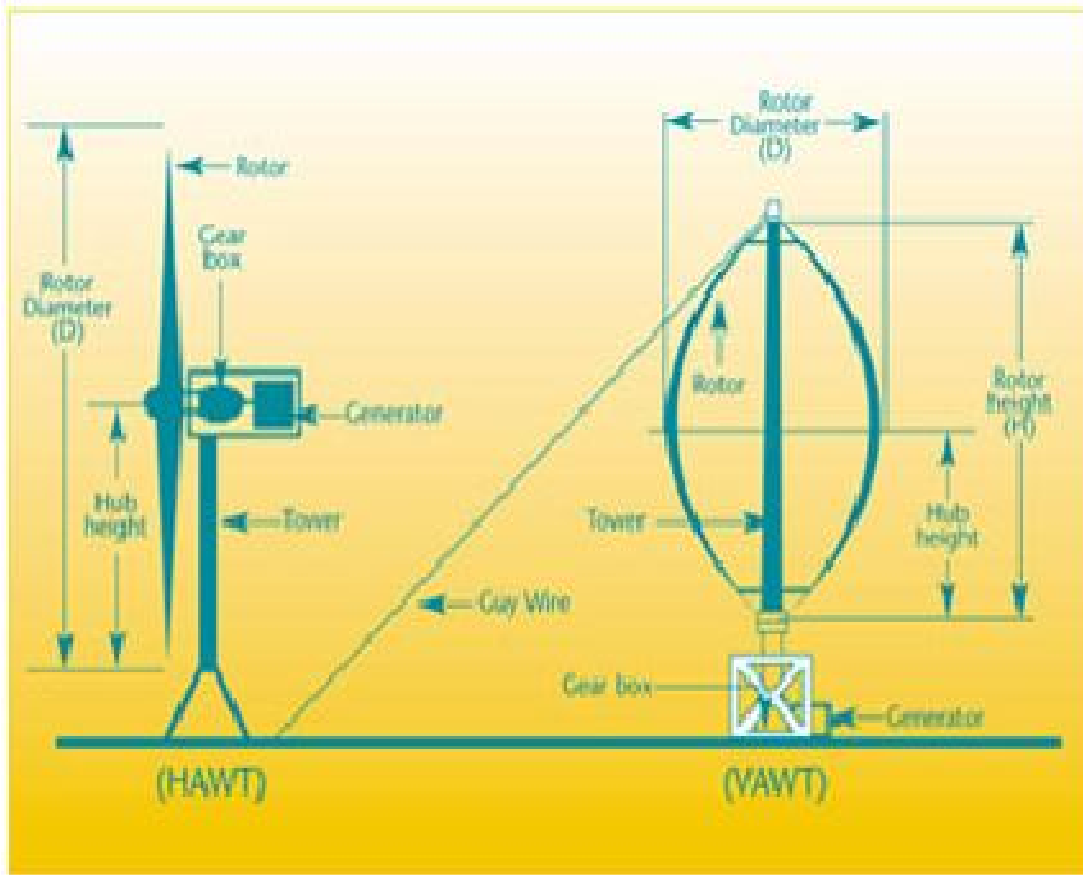


Figure 1.2: Vertical and horizontal wind turbines (Jakob Vernersten & Søren Krag, December 2010)

1.1.7 Wind Turbine Classifications

Wind turbines are classified in 3 classes the classes are industrial, residential and utility

Utility 900 kW - 2 MW

Generate bulk energy for sale in power markets commonly used in wind farm.

Industrial 50 kW - 250 kW

Remote grid production, Reduce consumption of higher cost grid power, May be sold if permitted by state regulations

Residential 400 W to 50 kW

Remote power, Battery charging, Net-metering type generation mostly used in residential areas.

1.1.8 POWER IN WIND

The power available in wind is given as follows

$$\text{Power} = 0.5 * \text{density of wind} * \text{swept area} * (\text{velocity of wind})^3 \dots\dots\dots (1.0)$$

(Wind Energy Explained, pg 53)

But the maximum efficiency of a wind turbine theoretically is $(16/57)$ which is about 59%.

The actual velocity in practical turbines lies in the range of 35-45% this will be illustrated in section 2.0.8

In design of turbines various factors are considered. They include:

- Lift and drag forces
- Twist and taper
- Tip speed ratio
- Betz limit
- Rotor solidity
- Shaft design

All these factors are discussed in section 2.0.8.

CHAPTER TWO: DESIGN OF WIND TURBINE SYSTEM AND THEORY

2.0.0 THEORY

Wind is the movement of air caused by the uneven heating of the earth surface, pressure forces of the air and rotation of the earth on its axis. The relevant information for the design of wind power systems is as follows;

- 1) Wind source information e.g. the wind speed and frequency of the wind flowing
- 2) Sitting requirements that include assessment and prediction of relative desirability
- 3) Variations of time short term variations in wind direction need to be considered in wind turbine design and sitting.

There are two types of wind turbine namely;

- 1) Horizontal axis wind turbine (HAWT)
- 2) Vertical axis wind turbine (VAWT)

For the horizontal axis wind turbine due to the fluctuation of wind direction there is the need for yawing. Yawing is the action of rotating a wind turbine to orient it towards the direction of the oncoming wind.

Short term wind direction variations and the associated motion affect fatigue life of components such as blades and yaw drives

The horizontal axis wind turbine is the most efficient wind turbine used so far. It overpowers other forms of turbines e.g. the Savonious rotor, the Darrieus, the Paddles and the sail wing turbines which have been manufactured and tested in regards to efficiency (wind energy explained, pg 9 2nd edition 2010)

The process of designing of a wind turbine begins by estimation of the wind resource. The estimation of wind resource is done using statistical methods to find frequent wind speed and law of conservation of energy which states that energy cannot be created or destroyed it can only be changed from one form to another to find available power in wind. Combining this law with the continuity equation of fluid mechanics the mass flow rate is a function of air density δ , air velocity U and swept area A and is given by

$$\frac{dm}{dt} = \delta AU \dots\dots\dots (2.10) \text{(Wind energy explained, pg 33 2010)}$$

Where $\frac{dm}{dt}$ is the elemental mass flow rate

Therefore the K.E or kinetic energy per unit flow is given as $\frac{1}{2}m^0v^2 \dots\dots\dots(2.11)$

Where m^0 is the mass and v is the velocity

$$\therefore \frac{1}{2} \times \frac{dm}{dt} \times U^2 = K.E \text{ per unit of flow } \dots\dots\dots(2.12)$$

$$\text{Substituting } \frac{dm}{dt} = \frac{1}{2} \times A\delta U \times U^2$$

$$= \frac{1}{2}AU^3\delta\dots\dots\dots(1.13)$$

$$\text{Wind power per unit area} = \frac{K.E}{A} = \frac{\frac{1}{2}SAU^3}{A} = \frac{1}{2}U^3\delta\dots\dots\dots(2.14)$$

Therefore the following conclusions are made;

- 1) Wind power density is proportional to the density of air
- 2) Power of the wind is proportional to the area swept by the rotor
- 3) Wind power density is proportional to the cube of wind velocity

Efficiency of rotor generator combination also shall be considered when estimating the power. Annuals wind speeds are known for certain regions one can develop maps that show average wind power density over regions. Wind maps are diagrams that show wind distributions in region. In most cases meteorological departments in various countries including Kenya research and record wind speed data but in cases where the data is not available, statistical methods are used to determining annual wind velocity distributions.

2.0.1 STATISTICAL METHODS OF DETERMINING WIND DISTRIBUTION

Most wind distribution data shows either the weibull distribution function or the Rayleigh distribution for more information on these distributions see (wind energy explained, pg 41) functions. Probability density functions show the probability of a wind speed occurring between two interval wind speeds the wind speed with the highest probability of occurring is selected for design. The normal probability distribution function for continuous data in terms of variables used is given by;

$$P(u) = \frac{1}{\sigma_u \sqrt{2\pi}} \exp \left[\frac{-(u-U)^2}{2\sigma^2} \right] \dots\dots\dots (2.15)$$

(Wind Energy Explained, 2010)

This normal distribution profile helps to determine the mean wind speed from sampled data. By the knowledge of the mean wind speed the probability of any other wind speed can be shown by the Rayleigh distribution, where, U is the mean of distribution and u is the sample mean and σ is the sample standard deviation of sample data (wind energy explained pg 46 2010 edition)

$$P(u) = \frac{\pi}{2} \left(\frac{U}{U^2} \right) \exp \left[\frac{-\pi}{4} \left(\frac{U}{U^2} \right)^2 \right] \dots\dots\dots (2.16)$$

The weibull distribution requires knowledge of two parameters K a shape factor and C a scale factor. Both parameters are functions of \bar{U} and σ . The weibull probability density function is given by

$$P(U) = \left(\frac{K}{C} \right) \left(\frac{U}{C} \right)^{K-1} \exp \left[- \left(\frac{U}{C} \right)^K \right] \dots\dots\dots (2.17)$$

Cumulative distributions function represents the time fraction or probability that the wind speed is smaller than or equal to a given wind speed and is given as

$$F(U) = 1 - \exp \left[- \left(\frac{U}{C} \right)^K \right]$$

In cases where the distribution of the wind are not clear the mean wind speed for design can be calculated by averaging regular wind speeds be shown as

$$U = \sum_{i=1}^{Ns} U_i \dots\dots\dots (2.18)$$

Where N , is a given series of wind observations averaged over a certain time interval time interval.

U = mean velocity

U_i = varying wind speeds

2.0.2 WIND VARIATION WITH HEIGHT

The mean wind speed increases with height. This phenomenon is known as wind shear. In the design of a wind turbine one requires assessment of wind resources over a wide area by collection of anemometer data .Rotor blade fatigue life will be influenced by cyclic loads resulting from rotation through the wind fields . In wind energy studies two mathematical models or laws have generally been used to model the vertical profile of wind velocities

Model laws have been used to model the vertical profile of wind speeds over regions of homogeneous flat terrain. They are based on boundary layer flow in fluid mechanics and atmospheric research. The two main models are

- i) The logarithmic profile
- ii) Power law profile

2.0.3 Logarithmic profile

Was developed by mixing length analysis given by Wortman (1982) and (Wind Energy, Explained pg 45, 2010). Near the earth surface the momentum equation reduces to

$$\frac{dp}{dx} = \frac{d}{dz} T_{xz} \dots \dots \dots (2.19)$$

X and Z are horizontal and vertical components P is the pressure and T_{xz} is the shear stress in the direction of x whose normal coincides with z .In this region pressure is independent of Z and the integration lead to

$$T_{xz} = t_0 + z \frac{dp}{dx} \dots \dots \dots (2.20)$$

Where t_0 is the surface value of the share stress, near the surface the pressure gradient is small so the second term on the right hand side may be neglected

Using prandtl mixing length theory the shear stress may be expressed

$$\tau_{xz} = \delta l^2 \left(\frac{du}{dz} \right)^2 \dots \dots \dots (2.21)$$

Where δ is the density of air.

U is the horizontal component of velocity and L the mixing length is used here to signify the effects of turbulence have been averaged out

Combining equation (2.20) and (2.21) gives

$$\frac{du}{dz} = \frac{i}{l} \sqrt{\frac{t_0}{\rho}} = \frac{U^*}{L} \dots \dots \dots (2.22)$$

Where $U^* = \sqrt{\frac{t_0}{\rho}}$ defined as friction velocity by prandtl (fluid mechanics by Prof K.L. Kumar, 2009)

If one assumes a smooth surface $L=KZ$ with $K=0.4$ which is the (von Karman's constant)

Equation (14) can be integrated directly from Z_0 to Z where Z is surface roughness length which characterizes the roughness of ground terrain this yields to

$$U(z) = \frac{U^*}{K} \ln \left(\frac{z}{Z_0} \right) \dots \dots \dots (2.23)$$

This is known as logarithmic wind profile.

The integrating is from lower limit of Z_0 instead of zero because natural surfaces are never uniform and smooth

Equation (2.23) can be written as;

$$\ln(z) = \left(\frac{K}{U^*} \right) U(z) + \ln(Z_0) \dots \dots \dots (2.24)$$

This equation can be plotted as a straight line on a semi log graph which resembles the straight line equation $Y = MX + C \dots \dots \dots (2.25)$

The gradient of the graph M is equal to $\left(\frac{K}{U^*} \right)$

From a graph of experimental data U^* and Z_0 can be calculated. The log law is often used to extract wind speed from a reference height Z_r to another level using the following relationship.

$$\frac{U(z)}{U(z_r)} = \frac{\ln\left(\frac{z}{Z_0}\right)}{\ln\left(\frac{z_r}{Z_0}\right)} \dots\dots\dots (2.26)$$

Sometimes the log law is modified to consider mixing length at the earth's surface by expressing the mixing length as $l = k(z + z_0)$ when this is used the log profile becomes

$$U(z) = \frac{U^*}{K} \ln\left(\frac{z+z_0}{z_0}\right) \dots\dots\dots (2.27)$$

Experimental values from various agencies like NASA and different researcher's around the world gives approximate surface roughness lengths for various terrain types.

2.0.4 Values (approximate) of surface roughness length for various types of terrain (Wind Energy Explained pg 46, 2nd Edition 2010)

Table 1: Terrain versus Z_0 (mm) where Z_0 is the relative surface roughness.

TERRAIN	Z_0 (mm)
Very smooth ice or mud	0.01
Calm open sea	0.20
Blown sea	0.50
Snow surface	3.00
Length grass	8.00
Rough pasture	10.00
Fallow field	30.00
Crops	50.00
Few trees	100.00
Many trees, hedges, few building	250.00
Forest and woodlands	500.00
Suburbs	1500.00
Centers with cities and buildings	3000.00

This height in Table 1, distribution helps in setting up of the tower height of the wind turbine during the design. Most designers the set tower height at about 1 to 1.5 times the size of the rotor and a height of at least 20M high for noise control and safety of people and animals near the wind turbine.(wind energy explained, pg 48)

2.0.5 EFFECTS OF TERRAIN

The logarithm power laws were developed for smooth surfaces and can be effective for flat terrain. The terrain affects the applicability of the tools in table 2.0.4 .Effects of terrain includes velocity deficits, unusual winds shear and wind acceleration.

For a terrain to be regarded as flat, the surrounding terrain are not greater than 60m anywhere in about 11km diameter circle around the turbine site, No hill has on aspect ratio greater than $1/50$ within 4km upstream and downstream the site, elevation difference between lower end of the rotor disc and the lowest elevation on the terrain is greater than three times the maximum elevation difference (wind energy explained, pg 50). Generally we can summarize the terrain effect on wind speed in the following ways.

- Obstacles reduce wind speed
- Terrains with changing roughness affects the wind profile
- In sitting wind turbines in ridges the prevailing wind direction should be perpendicular to the ridge axis if the ridge is not a suitable site
- Steep slopes give high wind velocity but on the leeward ridge they give rise to high turbulence
- Flat topped ridge creates a region of high wind shear due to flow separation
- depressions can channel the wind effectively they cause the change of wind velocity by increasing it so it is favourable to locate the wind turbine at the edges of depression

This information helps in the appropriate sitting of a wind turbine or a wind farm wind energy explained, pg 52)

2.0.7 ONE DIMENSIONAL MOMENTUM THEORY AND BETZ LIMIT

A sample model generally attributed to Betz can be used to determine the power from an ideal rotor, the thrust of wind on the ideal rotor and the effect of rotor operation. The turbine

is represented in form of a uniform actuator disc which creates discontinuity of pressure in a stream tube flowing through it. This model is based on linear momentum theory of physics.

2.0.8 THE ACTUATOR DISC

This is a device used in the analysis of the aerodynamic behaviour of wind turbines by taking into account the energy extraction process. Airstream tube containing the disc has a cross sectional area smaller than that of the disc upstream and an area larger than the disc downstream. This is to ensure that the mass flow rate is constant according to the continuity equation

$$P_1 A_1 V_1 = P_2 A_2 V_2 = P_3 A_3 V_3 \dots \dots \dots (2.28)$$

Where **P** is the pressure, **A** is the cross section area at various positions of the of the disc and **V** is the velocity of the airstream. Wind.

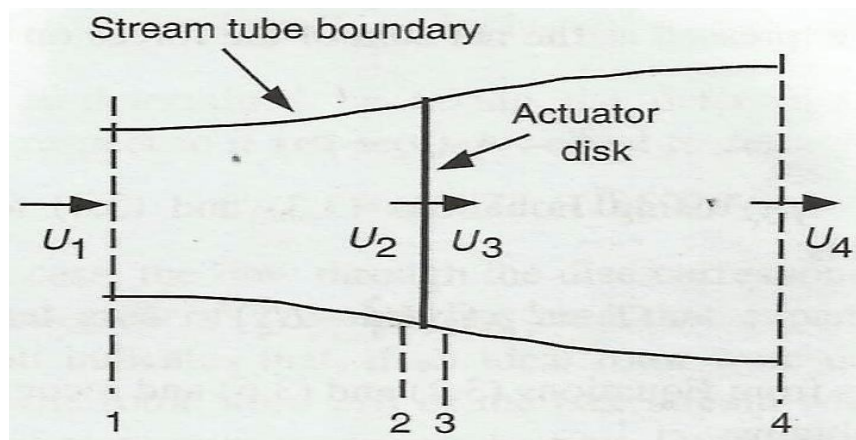


Figure 2.1: Actuator disc

The analysis of the actuator disc carries the following assumptions

- Homogeneous incompressible fluid flow
- No frictional drag
- An infinite number of blades
- Uniform thrust over the disc rotor area
- Non rotating wake

- The static pressure far upstream and far downstream of the rotor is equal to undisturbed ambient pressure.

Applying the conservation of linear momentum to the control volume enclosing the whole system one can find the net force equal and opposite to thrust T which is the force acting on wind turbine

$$T = U_1(A\delta U)_1 - U_4(A\delta U)_4 \dots \dots \dots (2.29)$$

Where δ is the air density, A is cross sectional area and U is the air velocity and subscripts indicate values numbered in the cross section figure 2.1.

For steady flow

$(A\delta U)_1 = (A\delta U)_4 = M$ Where M is the mass flow rate therefore

$$T = M(u_1 - u_4) \dots \dots \dots (2.30)$$

The thrust is positive so velocity behind the rotor u_4 is less than the free velocity u_1 . Bernoulli's function can be used in the above control volume since there is no work done on either side of the turbine rotor. Applying Bernoulli's theorem in the upstream of the disc

$$P_1 + \frac{1}{2}U\delta_1^2 = P_2 + \frac{1}{2}U\delta_2^2 \dots \dots \dots (2.31)$$

In the downstream of the disc

$$P_3 + \frac{1}{2}U\delta_3^2 = P_4 + \frac{1}{2}U\delta_4^2 \dots \dots \dots (2.32)$$

If the upstream and downstream pressures are equal and the velocity across the disc remains same thrust can be expressed as the net sum of forces on each side of actuator disk.

$$T = A_2(P_2 - P_3) \dots \dots \dots (2.33)$$

If one solves for $(P_2 - P_3)$ using (2.31) and (2.32) one obtains

$$T = \frac{1}{2}A\delta_2(U_1^2 - U_4^2) \dots \dots \dots (2.34)$$

Equating (2.33) and (2.34) one obtains

$$U_2 = \frac{U_1 + U_4}{2} \dots \dots \dots (2.35)$$

Thus wind velocity at rotor plane using a simple model is the average of the upstream and downstream wind speeds

If one defines the axial induction factor as a functional decrease in wind velocity between the stream and rotor plane then

$$a = \frac{U_1 - U_2}{U_1} \dots \dots \dots (2.36)$$

$$U_2 = U_1(1 - a) \dots \dots \dots (2.37)$$

$$U_4 = U_1(1 - 2a) \dots \dots \dots (2.38)$$

The quantity U_{1a} is often referred to as induced velocity at the rotor

The power out P is equal to thrust times velocity at disc therefore

$$P = \frac{1}{2} A \delta_2 (U_1^2 - U_4^2) U_2 = \frac{1}{2} A \delta_2 U_2 (U_1 + U_4) (U_1 - U_4)$$

Substituting equations U_2 and U_4 from equations 2.37 and 2.38 gives

$$P = \frac{1}{2} A \delta_2 U^3 4a(1 - a)^2 \dots \dots \dots (2.39)$$

Where the control volume area at rotor A_2 is replaced by A. the rotor area and the free stream velocity U_1 is replaced by U

Wind turbine rotor performance is usually characterized by its power coefficient C_p where,

$$C_p = \frac{p}{\frac{1}{2} \rho U^3 A} = \frac{\text{rotor power}}{\text{power of the wind}} \dots \dots \dots (2.40)$$

$$C_p = 4a(1 - a)^2 \dots \dots \dots (2.41)$$

The maximum C_p is obtained by taking the derivative of power coefficient with respect to a and equating it to 0 thus

$$C_p \text{ max} = \frac{16}{22} = 0.5926.$$

This occurs when a is equal to a third, for this case the flow through the disk corresponds to stream tube with an upstream cross-section area of two thirds the disk area that expands to twice the disk area downstream. This result shows that if an ideal rotor were designed and operated such that wind speed at the rotor were two thirds of the free stream wind then it would be operating at a point of maximum power production this is the maximum power production according to the laws of physics.

Thrust is characterized by coefficient of thrust T , from equations 2.37, 2.38 and 2.39 axial thrust can be given as.

$$T = \frac{1}{2} A \delta U^2 4a(1-a) \dots \dots \dots (2.42)$$

Coefficient of thrust can be given as

$$C_T = \frac{T}{\frac{1}{2} U \delta^2 A} = \frac{\text{thrust force}}{\text{dynamic force}} \dots \dots \dots (2.43)$$

$$\text{hence } C_T = 4a(1-a) \dots \dots \dots (2.44)$$

$$\text{At maximum power output } a = \frac{1}{3} \qquad \max C_T = \frac{8}{9}$$

The maximum possible thrust for wind turbine is $\frac{8}{9}$

We note that the overall turbine efficiency is found to be a function of both rotor power coefficient and mechanical (including electrical) efficiency

$$\eta_{\text{overall}} = \frac{\text{power}}{\frac{1}{2} \delta A U^3} = \eta_{\text{mech}} C_p \dots \dots \dots (2.45)$$

$$\text{power} = \frac{1}{2} A \delta U^3 (\eta_{\text{mech}} C_p)$$

2.0.9 IDEAL HORIZONTAL AXIS WIND TURBINE WITH WAKE ROTATION

In the previous analysis of linear momentum it was assumed that no rotation was imparted to the flow the previous analysis can be extended to the case where rotating rotor generates angular momentum which can be related to rotor torque. In the case of a rotating wind turbine rotor the flow behind the rotor rotates in opposite direction to the rotor in reaction to torque exerted by flow of the rotor. An annular stream tube model of this flow illustrating wake

rotation is shown in figure 1.5 the generation of rotational kinetic energy in the wake results in less energy extracted by the rotor than would be expected.

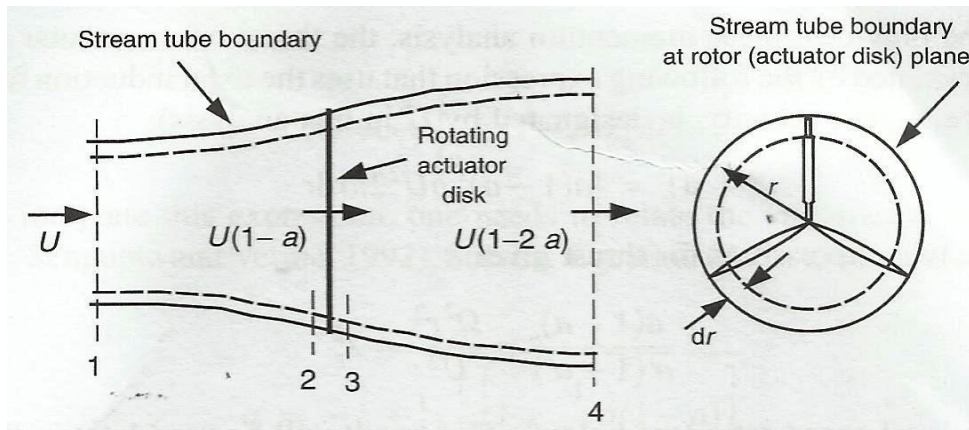


Figure 2.2: Stream boundary at rotor plane

Fig 2.2 shows the schematics of the parameters involved in this analysis. Subscripts to denote values at the cross-sections are denoted. It is assumed that the angular velocity imparted to the flow stream is small compared to the angular velocity of the wind turbine rotor; then it can be assumed that the pressure in the far wake is equal to the pressure in the free stream. The analysis that follows is based on an annular stream tube with radius r and thickness dr .

The pressure difference across the blades is shown in equation (2.46)

$$P_2 - P_3 = P \left(\Omega + \frac{1}{2} w \right) w r^2 \dots\dots\dots (2.46)$$

Where Ω is the angular velocity of the wind turbine, and w is the angular velocity imparted to the flow. The resulting thrust on the annular element denoted as dT is

$$dT = (P_2 - P_3) dA = \left[\delta \left(\Omega + \frac{1}{2} w \right) w r^2 \right] 2\pi r dr \dots\dots\dots (2.47)$$

An angular induction factor a^1 is then defined as

$$a^1 = \frac{w}{2\Omega} \dots\dots\dots (2.48)$$

When wake rotation is included in the analysis, the induced velocity at the rotor consists of not only axial component U_a but also component in the rotor plane $r\Omega a^1$. The Thrust expression becomes

$$dT = 4a^1(1 + a^1)\frac{1}{2}\delta\Omega^2r^22\pi r dr \dots\dots\dots (2.49)$$

Wake is the rotation of wind after turning the turbine as illustrated in figure 1.5

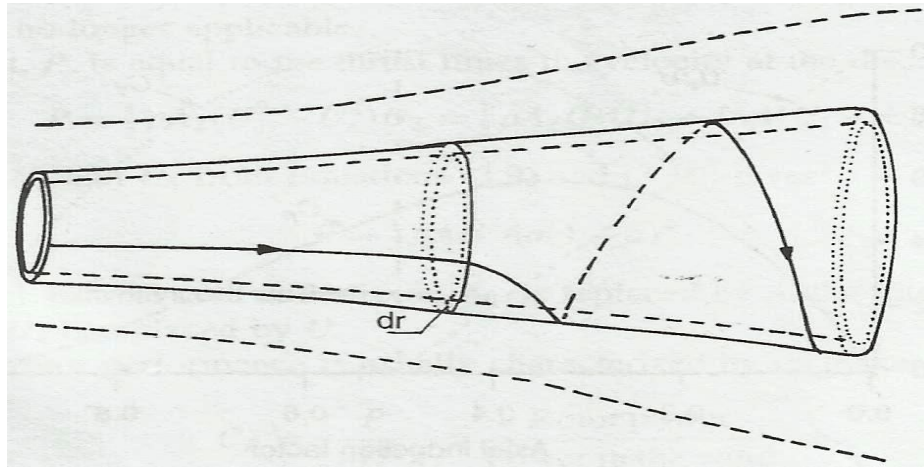


Figure 2.3: Sketch of wake

Following the previous linear momentum analysis the thrust on an annular cross section can be determined by the following expression in equation (2.50) that uses axial induction factor denoted as a

$$dT = 4a(1 - a)\frac{1}{2}\delta U^22\pi r dr \dots\dots\dots (2.50)$$

Equating the two equations i.e. (2.50 and 2.51) of thrust gives

$$\frac{a(1-a)}{a^1(1+a^1)} = \frac{\Omega^2r^2}{U^2} = \lambda_r^2 \dots\dots\dots (2.51)$$

The tip speed ratio λ is defined as the ratio of blade tip speed to the free wind speed is given as

$$\lambda = \frac{\Omega R}{U} \dots\dots\dots (2.52)$$

The local speed ratio occurs in the aerodynamic equations of the rotor. The local speed ratio is the rotor speed at some intermediate radius of the rotor to the wind speed and is given as

$$\lambda_r = \frac{\Omega r}{U} = \frac{\lambda_r}{R} \dots\dots\dots (2.53)$$

Where λ_r is the local tip speed ratio and r is the local radius.

Next, one can derive an expression for the torque on the rotor using conservation of angular momentum which must equal the change in angular momentum of the wake. On an increment annular area element this gives

$$dQ = d\dot{m}(wr)r = \delta U_2 2\pi r dr (wr)r \dots\dots\dots (2.54)$$

Where dQ is the incremental torque.

Since $U_2 = U(1 - a)$ and $a^1 = \frac{w}{2\Omega}$ the expression reduces to

$$dQ = 4a^1(1 - a) \frac{1}{2} \delta \Omega r^2 2\pi r dr \dots\dots\dots (2.55)$$

The power generated at each element dp is given by

$$dp = \Omega dQ$$

So the expression of power generated by each element becomes

$$dP = \frac{1}{2} \delta A U^3 \left[\frac{8}{\lambda^2} a^1 (1 - a) \lambda_r^3 d\lambda_r \right] \dots\dots\dots (2.56)$$

The incremental contribution to the power coefficient dC_p from each annular ring is given by $dC_p = dP / 0.5 \delta A U^3 \dots\dots\dots (2.57)$

Integrating

$$C_p = \frac{8}{\lambda^2} \int_0^\lambda a^1 (1 - a) \lambda_r^3 d\lambda_r \dots\dots\dots (2.58)$$

In order to integrate this expression one needs to relate variables a , a^1 and λ_r solving equation (2.55) to express a^1 in terms of a , one gets

$$a^1 = \frac{-1}{2} + \frac{1}{2} \sqrt{\left[1 + \frac{4}{\lambda_r^2} a(1 - a) \right]} \dots\dots\dots (2.59)$$

Aerodynamic conditions for maximum possible power production occur when the term

$a^1(1 - a)$ in equation (2.58) is at its greatest value. Substituting the value for a^1 from equation (2.59) into $a^1(1 - a)$ and setting the derivative with respect to a equal zero yields

$$\lambda_r^2 = \frac{(1-a)(4a-1)^2}{1-3} \dots\dots\dots (2.60)$$

Substituting this value in equation (2.51) one finds that for maximum power in each annular ring

$$a^1 = \frac{1-3a}{4a-1} \dots \dots \dots (2.61)$$

If equation (2.58) is differentiated with respect to a one obtains relationship between $d\lambda_r$ to da at those conditions that result in maximum power production

$$2\lambda_r \delta \lambda_r = \left[\frac{6(4a-1)(1-2a)^2}{(1-3a)^2} \right] da \dots \dots \dots (2.62)$$

Substituting equations (2.64) and (2.68) into the expression for power coefficient equation (44) gives

$$Cp_{max} = \frac{24}{\lambda^2} \int_{a_1}^{a_2} \left[\frac{(1-a)(1-2a)(1-4a)}{(1-3a)} \right]^2 da \dots \dots \dots (2.63)$$

Here the lower limit of integration a^1 corresponds to axial induction factor for $\lambda_r = 0$ and upper limit a_2 , corresponds to the axial induction factor $\lambda_r = \lambda$ from eqn (2.64) $a=0.25$ gives λ_r a value of zero. From eqn. (2.64) $a_2 = 1/3$ the upper limit of axial induction factor giving infinitely large tip speed ratio.

$$\lambda^2 = (1-a_2)(1-4a_2)^2/(1-3a_2) \dots \dots \dots (2.64)$$

A definite integral can be evaluated by changing variables x for $(1-3a_2)$ in equation (2.63) this gives

$$Cp_{max} = \frac{8}{729\lambda^2} \left\{ \frac{64}{5}x^5 + 72x^4 + 124x^3 + 38x^2 - 63x - 12[\ln(x)] - 4x^{-1} \right\}_{x=(1-3a_2)}^{x=0.25} \dots \dots \dots (2.65)$$

2.1.0 AIRFOILS AND GENERAL CONCEPTS OF AERODYNAMICS

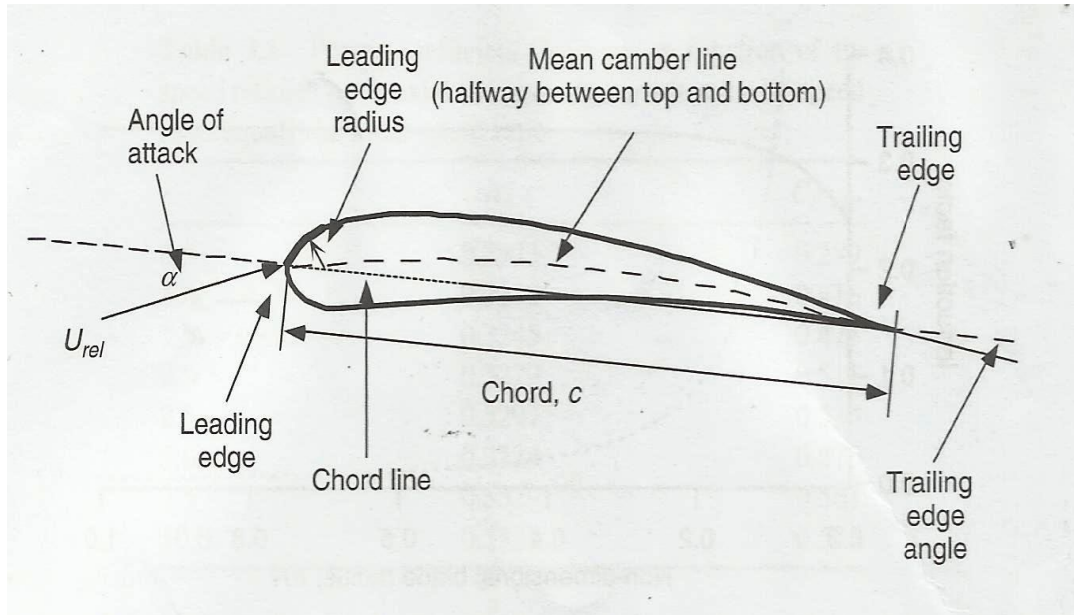


Figure 2.4: airfoil parameters

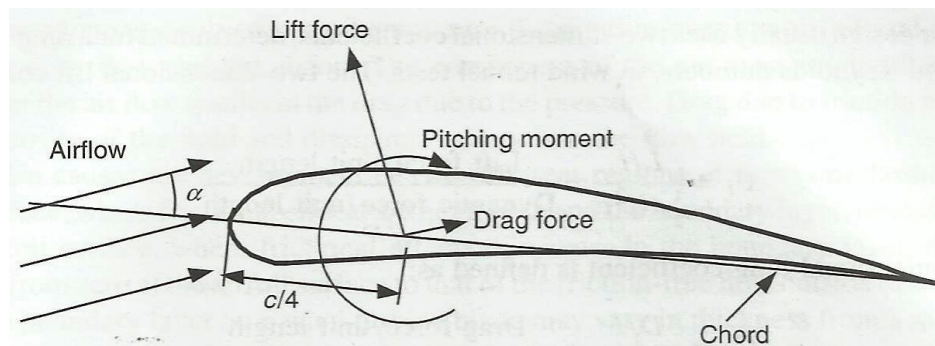


Figure 1.7: An aerofoil force diagram.

This section deals with calculation of rotor performance and aerodynamically effective blade shape. As presented in the section the analysis builds on foundation presented in the previous sections. A wind turbine rotor consists of airfoils that generate lift by virtue of pressure difference across the airfoil producing the same step change in pressure seen in the actuator disc determined using conservation of linear and angular momentum. The flow field characterized by axial and angular induction factors that are a function of rotor power extraction and thrust will be used to define airflows at rotor aerofoils. This can be used to determine rotor shape if certain performance parameters are known or blade performance if

the blade shape has been defined. The analysis uses forces on blades and is based on conservation of linear and angular momentum. Blade element theory refers to analysis of forces at a section of the blades, the results are combined to what is known as blade element momentum theory used to relate blade shape to rotor ability to extract power. Generally research in aeronautical engineering has shown that lift airfoils are much more efficient than drag airfoils (wind energy explained pg108). The main method used in analysis of airfoils is momentum theory. The forces on blade can be expressed as a function of lift and drag coefficients and angle of attack. For this analysis the blade is assumed to be divided into (N) sections.

The following assumptions are made;

- 1) No aerodynamic interaction between elements
- 2) The forces on the blades are determined solely by the lift and drag characteristics of the airfoil shape of the bladder

2.1.2 BLADE ELEMENT THEORY

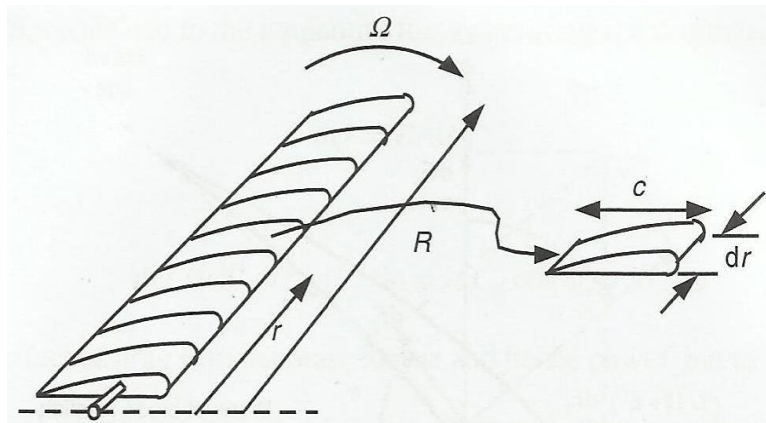


Figure 2.5: Aerofoil shape of a bladder

The relative wind is vector sum of the wind velocity at the rotor $U(1 - a)$ and the wind velocity due to rotation of the blade, this rotational component is the vector sum of the blade section velocity Ωr and induced angular velocity at the blades from conservation of angular momentum $\frac{wr}{2}$ or

$$\Omega r + \left(\frac{w}{2}\right) r = \Omega r + \Omega a^1 r = \Omega r(1 + a^1) \dots \dots \dots (2.66)$$

The overall flow situation is shown in figure 1.9 and the relationship of various forces, angles and velocities are shown fig 2.5

Here θ_p is the section pitch angle which is the angle between the chord line and the plane of rotation $\theta_{p,0}$ is the blade pitch angle at the tip. θ_T is the blade twist angle, α is the angle of attack. dF_L is the incremental lift force, dF_d is the incremental drag force, dF_N is the incremental force normal to the plane of rotation (this contributes to thrust) and dF_t is the incremental force tangential to the circle swept by rotor. This is the force that creates useful torque; finally U_{rel} is the relative wind velocity.

$$\theta_T = \theta_p - \theta_{p,0} \dots \dots \dots (k)$$

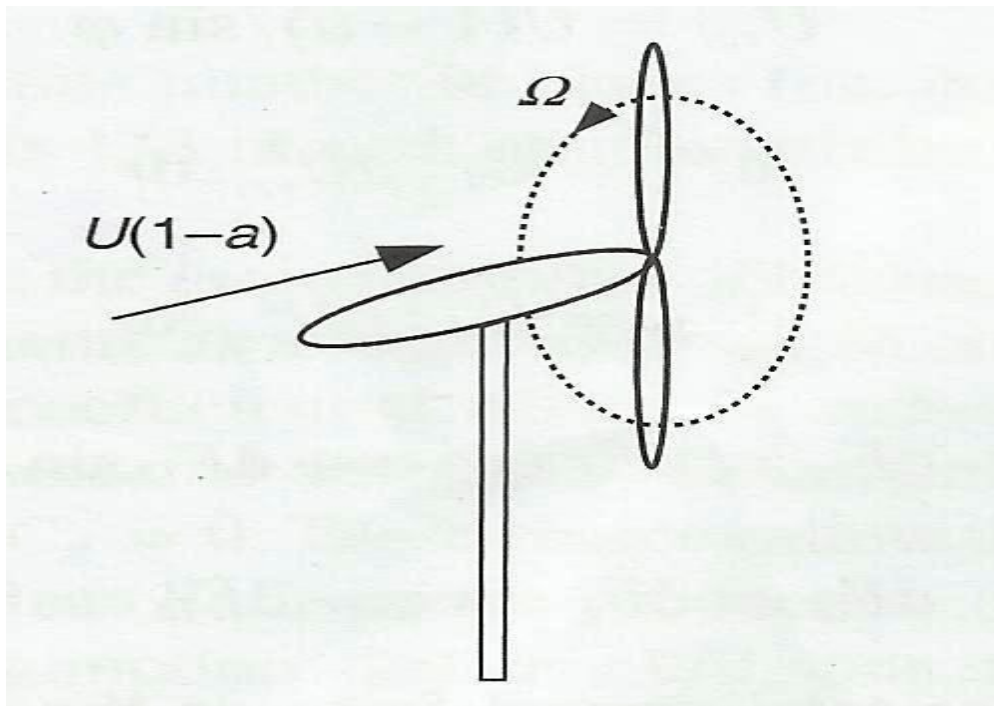


Figure 2.6: Sketch of overall for a downwind horizontal wind turbine showing velocity parameters.

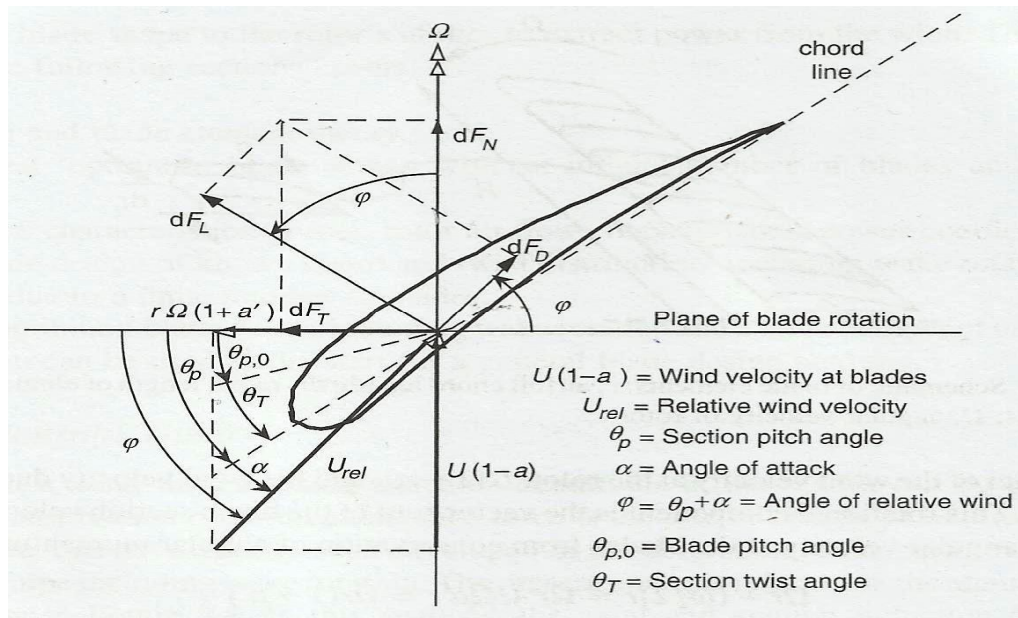


Figure 2.7: Diagram of forces acting on aerofoil

From the diagram above one can determine the following relationships the angle of relative wind can be shown as

$$\varphi = \theta_p + \alpha \dots \dots \dots (2.67)$$

$$\tan \varphi = \frac{U(1-a)}{\Omega r(1+a')} = \frac{1-a}{(1+a')\lambda_r} \dots \dots \dots (2.68)$$

$$U_{rel} = \frac{U(1-a)}{\sin \varphi} \dots \dots \dots (2.69)$$

$$dfl = C_l \frac{1}{2} \delta U_{rel}^2 C dr \dots \dots \dots (2.70)$$

$$dfd = C_d \frac{1}{2} \delta U_{rel}^2 C dr \dots \dots \dots (2.71)$$

$$dFN = dfl \cos \varphi + dfd \sin \varphi \dots \dots \dots (2.72)$$

$$dFT = dfl \sin \varphi + dfd \cos \varphi \dots \dots \dots (2.73)$$

If the rotor has B blades the total normal force on the section at a distance r from the centre is

$$dFN = B \frac{1}{2} \delta U_{rel}^2 (C_l \cos \varphi + C_d \sin \varphi) c dr \dots \dots \dots (2.74)$$

The differential torque due to the tangential force operating at a distance from the centre is given by

$$dQ = BrdFT \dots\dots\dots (2.75)$$

$$dQ = B \frac{1}{2} \delta U^2 \text{rel} (C_1 \sin\varphi - C_d \cos\varphi) crdr \dots\dots\dots (2.76)$$

2.1.1 GENERAL ROTOR BLADE PERFORMANCE PREDICTION

There are two theories used in the prediction of blade performance the two theories are discussed in the previous sections.

i) Momentum theory

ii) Blade element theory

A combination of these two theories by equating some relations from these two theories can be used to predict rotor performance

From blade element theory we have;

$$dFN = B \frac{1}{2} \delta U^2 \text{rel} (C_1 \cos\varphi + C_d \sin\varphi) cdr \dots\dots\dots (2.77)$$

$$dQ = B \frac{1}{2} \delta U^2 \text{rel} (C_1 \sin\varphi - C_d \cos\varphi) crdr \dots\dots\dots (2.78)$$

Equations 2.77 and 2.78 can also be written as

$$dFN = \frac{\sigma^1 \pi \delta U^2 (1-a)^2}{\sin^2 \varphi} (C_1 \cos\varphi + C_d \sin\varphi) r dr \dots\dots\dots (2.79)$$

$$dQ = \frac{\sigma^1 \pi \delta U^2 (1-a)^2}{\sin^2 \varphi} (C_1 \sin\varphi - C_d \cos\varphi) r^2 dr \dots\dots\dots (2.80)$$

In this theory dFN is differential the thrust force

dQ Is the torque

σ^1 Is the local solidity defined by;

$$\sigma^1 = \frac{BC}{2\pi r} \dots\dots\dots (2.81)$$

2.1.3 BLADE ELEMENT MOMENTUM THEORY

In the calculation of induction factors a and a^1 the accepted practice is to set C_d equal to zero (wind energy explained pg 125) so when torque equations from momentum and blade element theory are equated with $C_d = 0$ one gets

$$a^1/(1-a) = \sigma^1 C_1 / (4\lambda_r \sin\varphi) \dots\dots\dots (2.82)$$

By equating the normal force equations from momentum and blade element theory equations one obtains

$$a/(1-a) = \sigma^1 C_1 \cos\varphi / (4\sin^2\varphi) \dots\dots\dots (2.83)$$

After some algebraic manipulation using equation (2.83) and (2.84)

Which relates $a, a^1\varphi$ and λ based on geometric considerations and equations (2.83) and (2.84). The following useful relationships result

$$Cl = 4\sin\varphi \frac{(\cos\varphi - \lambda_r \sin\varphi)}{\sigma^1(\sin\varphi + \lambda_r \cos\varphi)} \dots\dots\dots (2.84)$$

$$a^1/(1-a^1) = \sigma^1 C_1 / (4\cos\varphi) \dots\dots\dots (2.85)$$

Other useful relationships that may be derived are

$$\frac{a}{a^1} = \frac{\lambda_r}{\tan\varphi} \dots\dots\dots (2.86)$$

$$a = \left[\frac{1}{1 + 4\sin^2\varphi / (\sigma^1 C_1 \cos\varphi)} \right] \dots\dots\dots (2.87)$$

$$a^1 = \left[\frac{1}{4\cos\varphi / (\sigma^1 Cl) - 1} \right] \dots\dots\dots (2.88)$$

Solution for C_1 and a can be done with either iteration or use of empirical curves empirical curves are plots of characteristics of aerofoils obtained by tests in wind tunnels for the aerofoils. The steps of iteration are as follows

1. Guess values of a and a^1
2. Calculate the angle of the relative wind from following equation

$$\tan\varphi = \frac{U(1-a)}{\Omega r(1+a^1)} = \frac{1-a}{(1+a^1)\lambda r}$$

3. Calculate the angle of attack from $\theta = a + \theta p$ and then C_l and C_d
4. Update a and a^1 from equations 2.87, 2.88 and 2.89
5. Repeat the process until newly calculated factors are within tolerance of previously calculated ones

2.1.5 TIP LOSS: EFFECT ON POWER COEFFICIENT OF NUMBER OF BLADES

Because the pressure on the suction side of a blade is lower than that on the pressure side air tends to flow around the tip, this leads to the reduction of coefficient of lift. This effect is most noticeable with fewer wider blades.

A number of methods have been suggested for including the effect of tip loss. The most straight forward approach is one developed by Prandtl (Wind Energy Explained, pg 127). According to this method a correction factor F must be introduced into previously discussed equations. The correction factor is a function of the number of blades

Based on prandtl method the tip loss factor is given as

$$F = \left(\frac{2}{\pi}\right) \cos^{-1} \left[\exp \left(- \left\{ \frac{\left(\frac{B}{2}\right) \left[1 - \left(\frac{r}{R}\right)\right]}{\left(\frac{r}{R}\right) \sin\varphi} \right\} \right) \right] \dots\dots\dots (2.89)$$

Where the angle resulting from the inverse cosine function is assumed to be in radians

If the inverse cosine function is in degrees then the initial factor $\frac{2}{\pi}$ is replaced by $\frac{1}{90}$. F is between 0 and 1

The tip loss correction factor characterizes the reduction in the forces at a radius r along the blade

Due to tip loss the thrust from momentum theory becomes

$$\text{Thrust } dT = F \delta U^2 4a(1-a)\pi r dr \dots\dots\dots (2.90)$$

Torque becomes $dQ = Fa^1(1-a)\delta U\pi r^3\Omega dr$ (2.91)

Carrying the tip loss factor through calculation one finds the following changes

$$a^1/(1-a) = \sigma^1 C1 / (4F\lambda_r \sin\varphi) \dots\dots\dots (2.92)$$

$$a^1/1-a = \sigma^1 C1 \cos\varphi / 4F \sin^2\varphi \dots\dots\dots (2.93)$$

$$Cl = 4F \sin\varphi \frac{(\cos\varphi - \lambda_r \sin\varphi)}{\sigma^1 (\sin\varphi + \lambda_r \cos\varphi)} \dots\dots\dots (2.94)$$

$$a^1/(1+a^1) = \sigma^1 C1 / (4F \cos\varphi) \dots\dots\dots (2.95)$$

$$a = \frac{1}{\left[1 + 4F \sin^2\varphi / \sigma^1 C1 \cos\varphi \right]} \dots\dots\dots (2.96)$$

$$U_{rel} = \frac{U(1-a)}{\sin\varphi} = \frac{U}{(\sigma^1 C1 / 4F) \cos\varphi + \sin\varphi} \dots\dots\dots (2.97)$$

The equation of power coefficient can be calculated from

$$Cp = \frac{8}{\lambda^2} \int_{\lambda_h}^{\lambda} F \lambda_r^3 a^1 (1-a) [1 - (C_d/C_1) \cot\varphi] d\lambda_r \dots\dots\dots (2.98)$$

(Wind Energy Explained, pg 127).

$$\text{Or } Cp = \frac{8}{\lambda^2} \int_{\lambda_h}^{\lambda} F \sin^2\varphi (\cos\varphi - \lambda_r \sin\varphi) (\sin\varphi + \lambda_r \cos\varphi) [1 - (C_d/C_1) \cot\varphi] \lambda_r^2 d\lambda_r \dots\dots\dots (2.99)$$

2.1.6 BLADE SHAPE FOR OPTIMUM ROTOR WITH WAKE

Generally it can be concluded that all wind turbines rotate with wake, the analysis of blade shape without wake can be used as a basis and in it the effects of wake can be included, the analysis without wake is done in (wind energy explained, pg 121) for design purpose the analysis with wake is most appropriate because it gives more accurate test results. The blade shape for an ideal rotor that includes the effects of wake rotation can be determined using the analysis developed for a general rotor. This optimization includes wake rotation but ignores the drag $Cd = 0$ and tip losses, $F=1$. One can perform the optimization by taking the partial

derivative of that part of the integrals C_p which is a function of the angle of relative wind and setting it equal to zero.

$$\frac{d}{d\varphi} [\sin^2 \varphi (\cos \varphi - \lambda_r \sin \varphi) (\sin \varphi + \lambda_r \cos \varphi)] = 0 \dots \dots \dots (2.99a)$$

$$\lambda_r = \frac{\sin \varphi (2 \cos \varphi - 1)}{(1 - \cos \varphi) (2 \cos \varphi + 1)} \dots \dots \dots (2.99b)$$

$$\text{More algebra reveals that } \varphi = \frac{2}{3} \tan^{-1} \left(\frac{1}{\lambda_r} \right) \dots \dots \dots (2.99c)$$

$$C = \frac{8\pi r}{BC1} (1 - \cos \varphi) \dots \dots \dots (2.99d)$$

C = aerofoil chord length

$$a = \frac{1}{\left[1 + 4 \sin^2 \varphi / (\sigma^2 C1 \cos \varphi) \right]} \dots \dots \dots (2.99e)$$

$$a^1 = \frac{1-3a}{4a-1} \dots \dots \dots (2.99f)$$

The result can be compared with the result for an ideal blade without wake for which are given as (wind energy explained pg 122, 2010) for chord length and relative wind angle which are.

$$\varphi = \tan^{-1} \left(\frac{2}{3\lambda_r} \right) \dots \dots \dots (2.99g)$$

$$C = \frac{8\pi r}{BC1} \left(\frac{\sin \theta}{3\lambda_r} \right) \dots \dots \dots (2.99h)$$

Solidity is the ratio of the platform area of blades to the swept area

$$\sigma = \frac{1}{\pi R^2} \int_{8r}^R C dr \dots \dots \dots (2.99i)$$

The optimum blade rotor solidity can be found from methods discussed above. When the blade is modelled as a set of N blades the solidity can be calculated from

$$\sigma \cong \frac{B}{N\pi} (\sum_{i=1}^N c_i / R) \dots \dots \dots (2.99j)$$

BIs the number of blades

CHAPTER 3: ROTOR DESIGN PROCEDURE

3.0.0 GENERAL ROTOR DESIGN PROCEDURE

The design of rotor is basically a trial and method which involves the following steps

Step 1

Determine basic rotor parameters

- a) The first step involves deciding what power is needed and the available wind velocity

The radius of the rotor is estimated from

$$p = C_p \Omega \left(\frac{1}{2} \right) \delta \pi R^2 U^3 \dots\dots\dots (3.00) \text{ (Wind energy explained, pg 133)}$$

- b) Choose a tip speed ratio for power production. The recommended tip speed ratios by most designers for electricity generating machines are between $4 < \lambda < 10$ to avoid turbulent wakes which reduce machine efficiency. Higher speed machines use less material in the blades and have smaller gearboxes but require more sophisticated air foils.
- c) Choose number of blades from a table recommended by researchers from the university of Massachusetts

Table 2.0: Tip speed ration verses number of blades. (Wind energy explained, pg 133)

λ	B
1	8-24
2	6-12
3	3-6
4	3-4
> 4	1-3

- d) Select an airfoil if $\lambda < 3$ curved plates can be used if $\lambda > 3$ use more aerodynamic shape is used

STEP 2: DEFINE BLADE SHAPE

- a) Obtain and examine the empirical curves for the aerodynamic properties of the airfoil at each section (the airfoil may vary from root to tip) i.e. C_l coefficient of lift verses α (angle of attack)

Choose the design aerodynamic conditions C_l design and design such that C_d design/ C_l design is at a minimum

- b) Divide the blade into N elements about (10-20) and use the optimum rotor theory to estimate the shape of the i^{th} blade with a midpoint radius r

$$\lambda_{ri} = \lambda \left(\frac{r_i}{R} \right) \text{ (Wind Energy Explained, pg 134)}$$

$$\theta = \left(\frac{2}{3} \right) \tan^{-1} \left(\frac{1}{\lambda_{ri}} \right)$$

$$C_i = \frac{8\pi r_i}{BC1 \text{ design}} (1 - \cos \phi_i) \dots \dots \dots (3.01)$$

$$\theta_{Ti} = \theta_{pi} - \theta_{po} \dots \dots \dots (k)$$

$$\theta_i = \theta_{pi} + \alpha_{designi} \cdot \text{(from fig 2.7)}$$

Using the optimum blade shape as a guide selects a blade shape that promises to be good approximation for ease of fabrication linear variations of chord thickness and twist might be chosen. For example if a_1, b_1 and a_2 are coefficients for a chosen chord and twist distribution then the chord and twist can be expressed as.

$$C_1 = a_1 r_i + b_1$$

$$\theta_{Ti} = a_2 (R - r_i)$$

Because the sophisticated shape of an optimum rotor is difficult to fabricate.

STEP 3: CALCULATE ROTOR PERFORMANCE AND MODIFY BLADE DESIGN PERFORMANCE

Solving for C_l and a

Find the actual angle of attack and lift coefficient for centre of each element using the following equations

$$C_{li} = 4F \sin \varphi_i \frac{(\cos \varphi_i + \lambda_{ri} \sin \varphi_i)}{\sigma^1 (\sin \varphi_i + \lambda_{ri} \cos \varphi_i)} \dots \dots \dots (2.94)$$

$$\sigma^1 = \frac{BC_i}{2\pi r_i} \dots \dots \dots (2.81)$$

$$F_i = \left(\frac{2}{\pi}\right) \cos^{-1} \left[\exp \left(- \left\{ \frac{\left(\frac{B}{2}\right) \left[1 - \left(\frac{r_i}{R}\right) \right]}{\left(\frac{r_i}{R}\right) \sin \varphi} \right\} \right) \right] \dots \dots \dots (2.89)$$

after calculating this parameters you can check whether the wake is turbulent or not. This is done by calculating the axial induction factor if the axial induction factor is more than 0.4 the wake is turbulent if the axial induction factor is less than 0.4 the wake is laminar. Blade evaluation for a turbulent wake cannot be done by use of empirical curves it is done by calculation of axial induction factor by iteration discussed in section 2.1.3 then use the axial induction factor to calculate blade performance. The axial induction factor can be calculated as.

$$a = \frac{1}{\left[1 + 4F \sin^2 \varphi / \sigma^1 C_l \cos \varphi \right]} \dots \dots \dots (2.94)$$

After calculating all the coefficients of lift and checking actual angle of attack from empirical curves one calculates coefficient of power using

$$C_p = \frac{8}{\lambda^2} \int_{\lambda_h}^{\lambda} F \sin^2 \varphi (\cos \varphi - \lambda_r \sin \varphi) (\sin \varphi + \lambda_r \cos \varphi) [1 - (C_d/C_l) \cot \varphi] \lambda_r^2 d\lambda_r \dots \dots (2.99)$$

The process of evaluation continues until the calculated power coefficient is within tolerance limits with the probable power coefficient estimated in the beginning of the design.

3.1.1 Wind density and distribution in Kenya

The wind distribution and density in Kenya is shown in figure 3.0 . Wind velocities are high in the northern part of Kenya mainly due to plains of land without trees. Wind in this region blows out through the year hence this region is best suited for tapping of wind power. Wind power is a clean source of energy that is renewable and inexpensive.

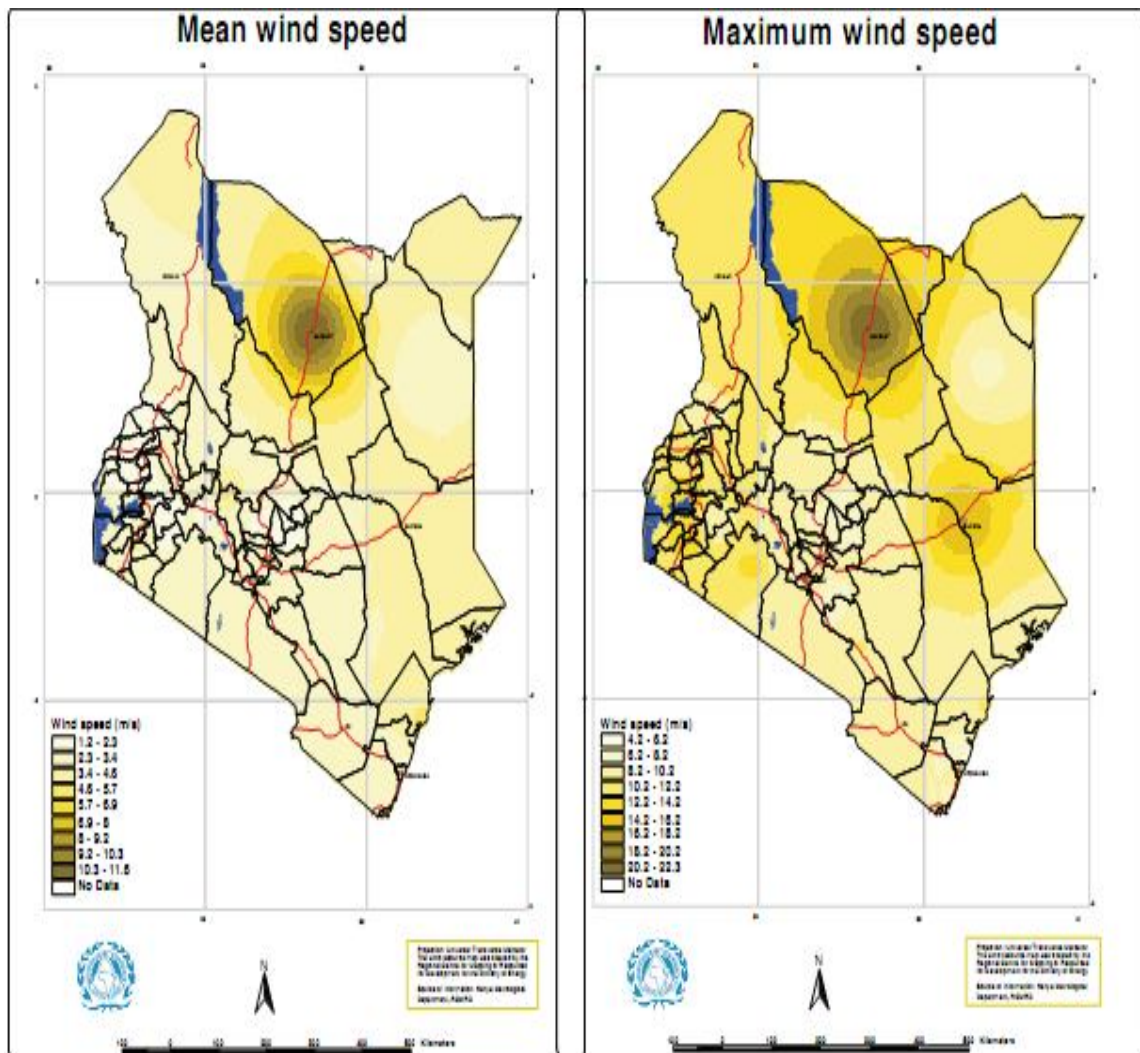


Figure 3.0: pattern of wind distribution in Kenya

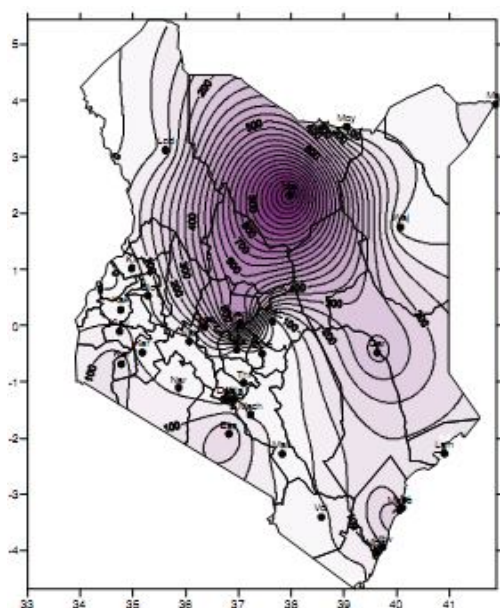


Figure 6b: Spatial patterns of the wind power density (Wm^{-2}) over Kenya

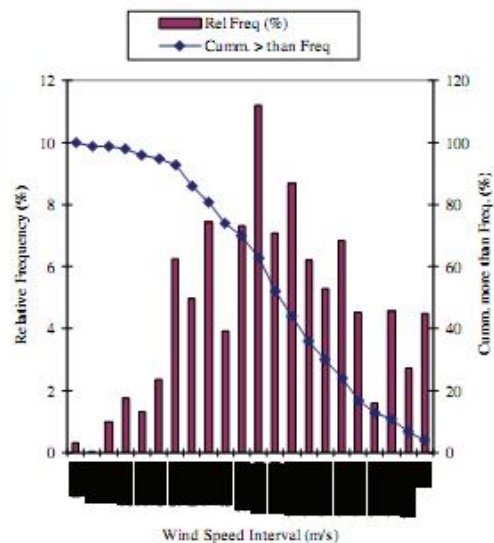


Figure 7a: Relative Frequency Distribution of the wind speeds over Marsabit

Figure 3.1: wind distribution in Kenya and on (right) wind distribution in Marsabit

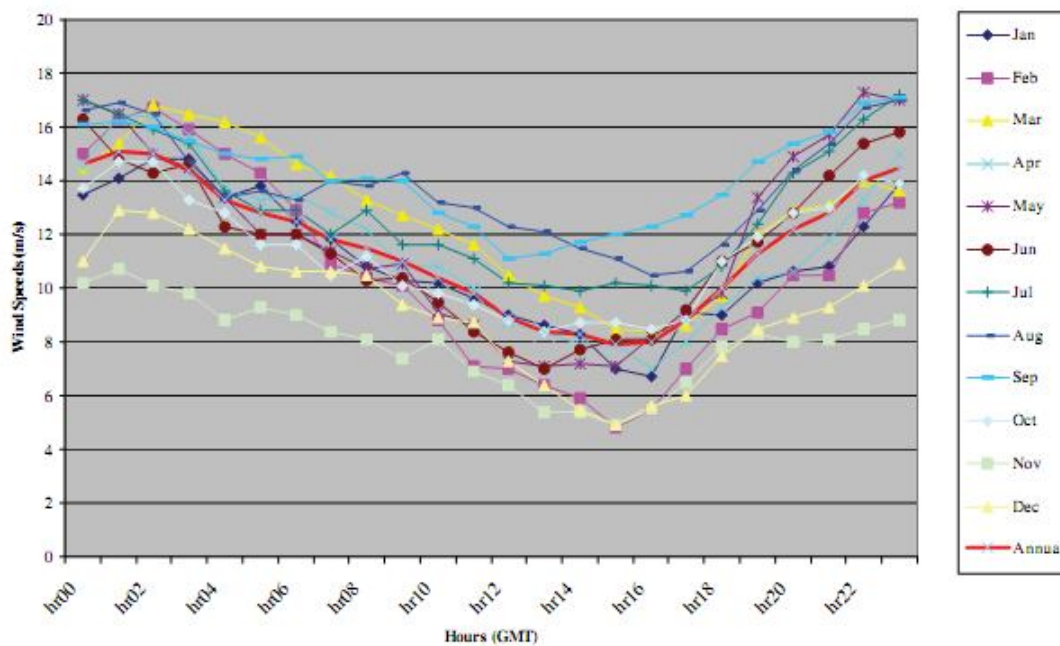


Figure3.2: Diurnal wind speed variation marsabit.

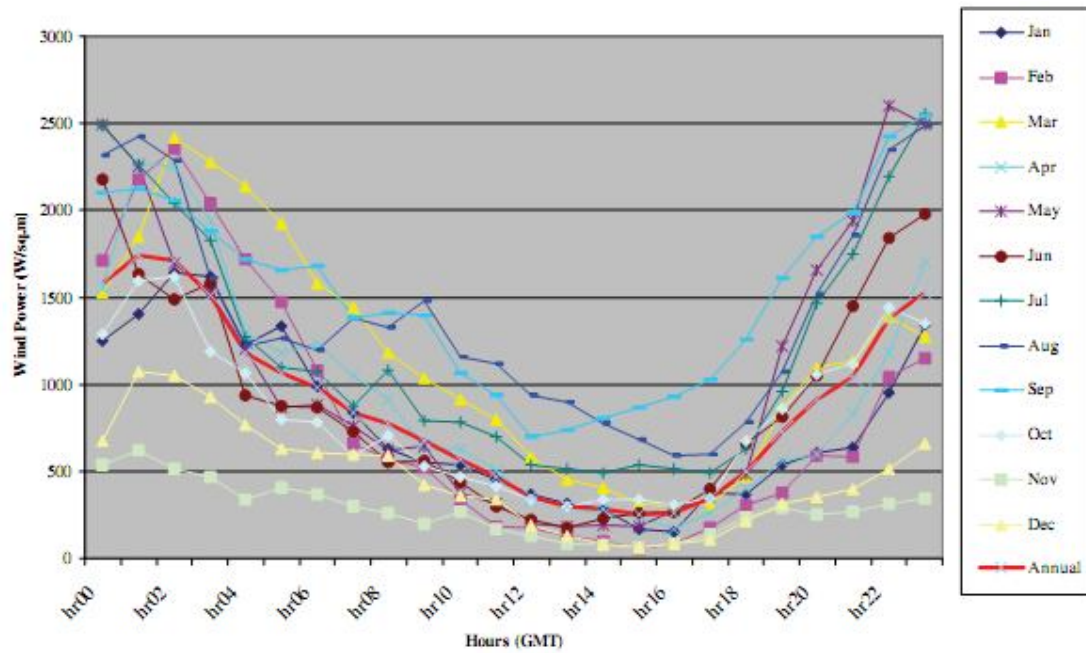


Figure 7f: Diurnal variation of the wind power variation over Marsabit

Figure 3.3: Diurnal variation of the wind power variation over Marsabit

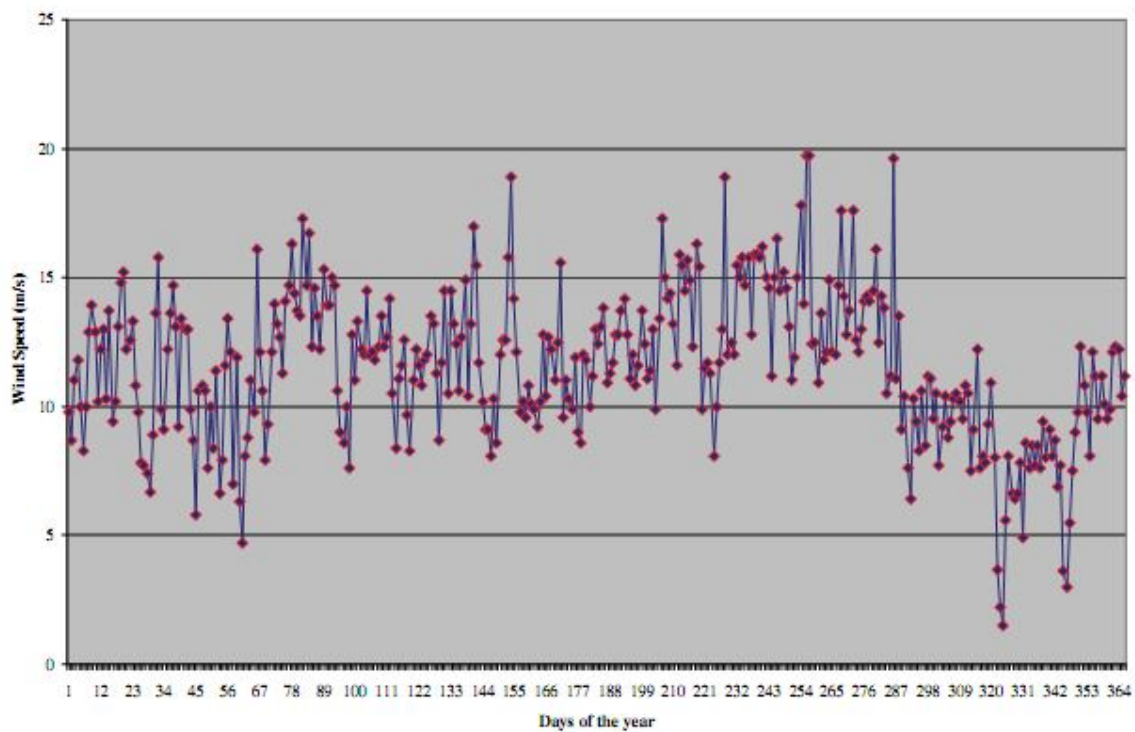


Figure 3.4: Graph of wind speed against Days of Year

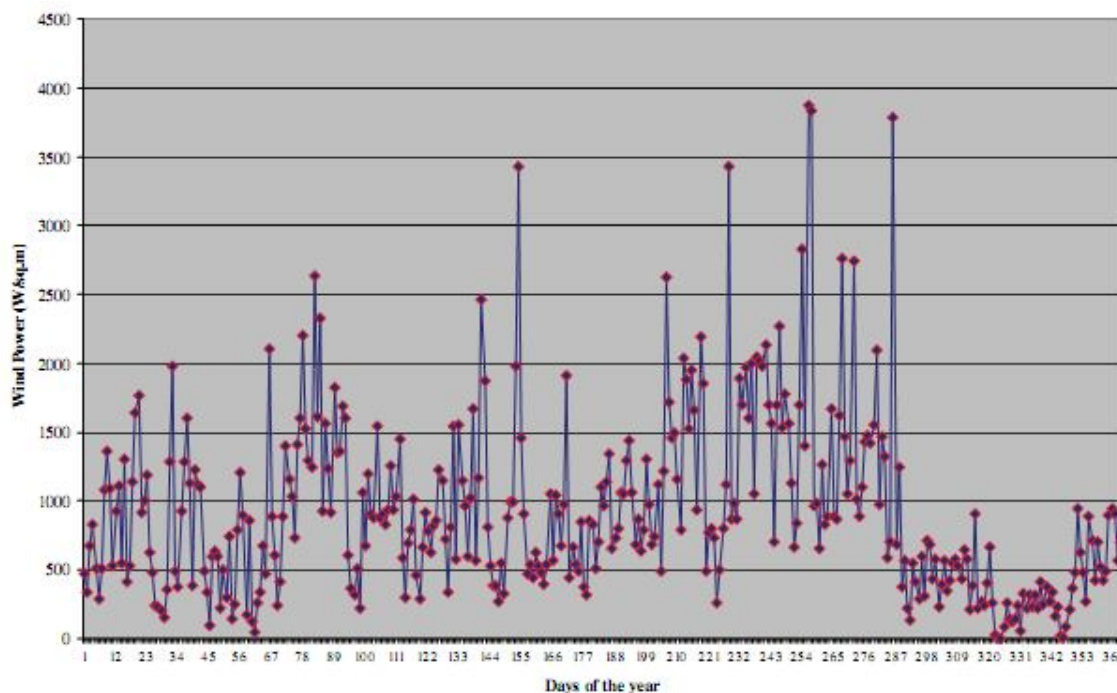


Figure 7h: Daily wind power variation over Marsabit

Figure 3.5: Graph of power against Days of the Year

3.1.2 CASE STUDY: MARSABIT REGION

Our objective is to design a wind turbine system that is capable of providing enough energy for a homestead from basic lighting to powering of electronic devices like the television and radio sets. The cumulative power output of this wind turbine system is to produce at least **6000 watts** of electrical power this is obtained from data of required home use of electricity in Kenya courtesy Kenya power and lighting company website. From the wind data available we observe that most frequent wind speed is 8m/s in this region

3.1.3 GENERATOR SPECIFICATIONS

The design speed of the generator for this wind turbine system is 220 RPM. Our chosen generator starts power production upon achieving a speed of 60 RPM

With such running speeds, then there is no drive train required. Gearbox is eliminated hence the cost is too reduced. Maintenance costs are cut and high speed shaft too eliminated from the design of the wind turbine design. The rotor shaft is connected to the generator via a coupling. Power losses are minimized. The tower holds the system at an appropriate height

above ground with the concrete foundation holding it firmly to the ground. An electrical system rests on the ground.

A lightning arrestor is installed to cushion the wind turbine system from the lightning strikes that may probably occur.

The proposed wind turbine is a three-bladed horizontal-axis wind turbine that operates at variable speeds. It features upwind design which is self-regulating by means of a passive yaw orientation system and gravity controlled furling system that controls the power output.

3.1.4 PERMANENT MAGNET GENERATORS

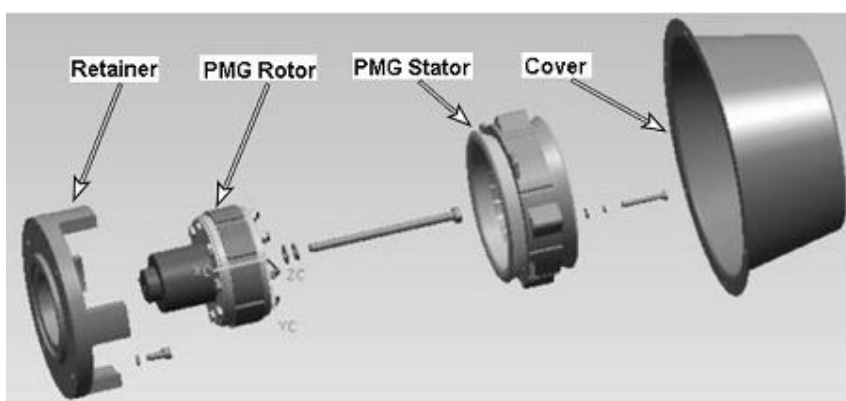


Figure 3.6: Major parts of a permanent magnet generator

The designed **6KW** turbine incorporates a permanent magnet generator. Permanent magnets are used in wind turbines producing power output of up to **10KW**. This kind of magnet provides the magnetic field hence no need of field windings or supply of current to the field.

Magnets are integrated directly into a cylindrical cast aluminium rotor. Power is taken from the stationary armature so there is no need of commutator, slip rings or brushes. Construction of machine is so simple; permanent magnets generators are quite rugged.

Principle of operating of permanent magnet generators is similar to that of synchronous machines. They are usually not connected to an AC network. Power produced by generator is a variable voltage and frequency. AC currents are rectified to Dc currents for connection to Dc loads or for battery charging. It is also converted to AC with a fixed frequency and voltage.



Figure 3.7: A 6KW permanent magnet generator (<http://www.taygol.en.alibaba.com>)

3.1.5 Generator weight

The mass of the generator is 200 kilograms. Therefore the weight of the generator is 1962 N. Such amount of weight can be supported on top of the tower without causing damage to the supporting structure which is the tower made of steel. This weight placed at the top of tower is set in a manner that it is well balanced like in the case of a crane.

3.1.6 Generator dimensions

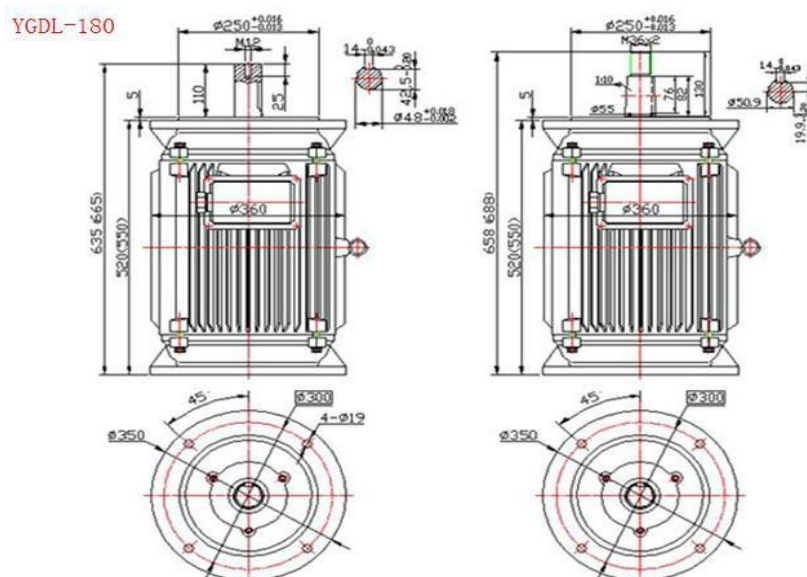


Figure 3.8: Dimensions of generator (<http://www.taygol.en.alibaba.com>)

Generator speed of revolution

The generator speed is 200 RPM and produces a power output of 6KW. There is no need of a gear box because the rotor speed exceeds the generator minimum working running speed.

3.1.7 Generator number of poles

The generator has got twenty four (24) poles. The rotor field is provided by permanent magnets; hence its efficiency is high because there is no power used to create electromagnet on the rotor.

3.1.8Power output

The power output ranges from 6KW to 8KW at maximum possible running speed of 900 RPM. In our design, the generator operates to a maximum speed of 250 RPM after which any speed increases causes it to stall therefore minimizing the speed of rotation of the blades to the prescribed one.

3.1.9Power storage

The generated power is stored in the lead acid batteries. A lead acid battery has capacity of 12V though sum have capacity of 24V. At least three lead acid batteries were connected parallel for power storage. Extra power produced can be used to heat water. The power bank should be able to supply power to the residential house to light all the rooms and power all the electronic devices when there is no power generation from the wind turbine generator.

3.2.0 Height of the tower

The meteorological department in Kenya from which the design data was collected does not specify the height and the environment it obtains the data from. In this design, it is assumed that the data is collected at the minimum hub height of hub recommended by most designers of 20 metres. To harvest the needed power the rotor height should be of similar height because above that height the wind speeds would be higher by wind shear phenomenon and could cause damage to the generator. We decided that the height of the tower is approximately 20 metres in height. The tower was constructed out of metal beams. The (L) beams were used. Tower of such height was best suited because it was well over height of all other surroundings. This implies there was no obstruction of the wind path towards the turbine.

3.2.1 Tower construction

The tower is constructed from streets struts of the (L) shape. Such struts were chosen because they are easily bolted. They were bolted together to achieve the design structure. It was later painted white and red colour.

3.2.2 The foundation of the tower

The foundation is prepared from concrete that is reinforced with steel struts.

The foundation is dug to 3M deep into ground and of square area of (4M by 4M).

Small sized rocks are introduced into the pit and a mixture of sand, cement and water is poured into the pit.

Steel struts are also introduced into the pit and sand mixture of small sized rocks, cement, sand and water is used to fill the pit up to surface.

CHAPTER FOUR: MECHANICS AND DYNAMICS OF WIND TURBINE

4.0 MECHANICS AND DYNAMICS OF WIND TURBINE

A wind turbine should withstand the loads it experiences. Principles of strength of materials are used in wind turbine design. Vibrations are another major course of dynamic loading of the turbines. In this section response of the wind turbine are observed to aerodynamic forces using progressively more detailed approaches. The second approach deals with effect of rotation of the wind turbine rotor in addition the operation environment. First approach uses a steady state approach that follows directly from discussions of the ideal rotor in chapter three. Second approach includes the effects of rotation of the wind turbine rotor. Second approach environment is not assumed to be uniform so effects of vehicle wind shear and yaw motion and turbines orientation are accounted for. Finally some more detailed approaches to investigating wind turbine dynamics are discussed in this section

4.0.1 WIND TURBINE LOADS

Are of five types

- Steady (static and rotating)
- Cyclic
- Transient (including impulsive)
- Stochastic
- Resonance induced

4.0.2 Steady loads

Do not vary relatively over long period of time. They can either be static or rotating. They are non time varying loads that impinge on a structure. E.g. steady wind

4.0.3 Cyclic loads

Vary with regular periodic manner are as a result such as the weight of blades, wind shear and yaw motion

4.0.4 Transient loads

Arise in response to some temporary external event example include loads in drive train resulting from application of brakes .One example of an impulsive load is that experienced by a blade of a downwind rotor when it passes behind the tower (through the tower shadow) The force on a teeter damper when normal range of teeter is exceeded is another example of impulsive load. Teetering is pinning rotors to the shaft allowing them to move around the pin.

4.0.5 Stochastic loads

These are time varying as cyclic transient and impulsive loads. Loading varies in a more apparently random manner. Mean value may be relatively constant but there may be significant fluctuations from the mean especially during turbulence of the wind.

4.0.6 Resonance induced load

Resonance results as the result from dynamics response of some part of the wind turbine being excited at one of its own natural frequencies.

They occur due to poor design of the wind turbine system.

4.0.7 Sources of loads

- Aerodynamics
- Gravity
- Dynamic interactions
- Mechanical control

4.0.8 Effects of loads

The loading experienced by a wind turbine are important and must be able to withstand those loads Varying loads can cause fatigue damage in machine components. Such that a given components may fail at much lower loads

4.0.9 TOWER LOADS

1. Aerodynamic tower loads:

Aerodynamic tower loads include thrust during normal operation the moment from the torque and extreme wind loads

2. Tower vibrations

The most important is to avoid natural frequencies near rotor frequencies

Soft tower is one which fundamental natural frequency is below blade passing frequency

3. Dynamic tower loads

Result from dynamic response of wind turbine itself

The moments for each blade (flap wise, lead-lag, and torsion) are transferred to tower co-ordinates.

4.1.0 TYPE OF COMMON BLADES

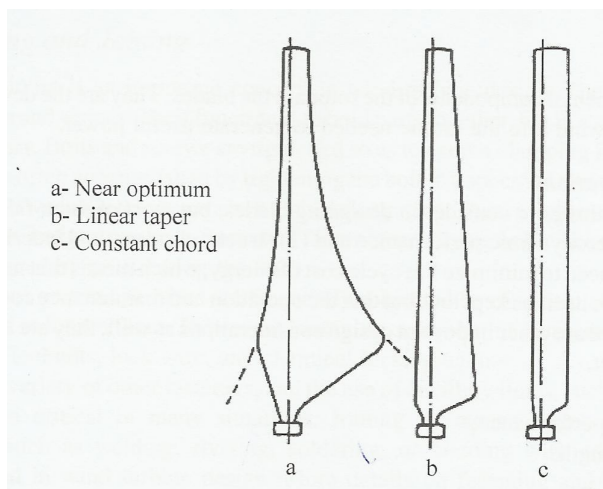


Figure 4.0: Types of blades

4.1.1 WIND TURBINE MATERIALS AND COMPONENTS

Considers materials and components commonly used in wind turbines

Wind turbine mainly made up of composite materials.

4.1.2 MATERIAL FATIGUE

This is failure to withstand load when it a load is applied removed and then applied again a numbers of times. This is called fatigue damage

4.1.3 FATIGUE IN WIND TURBINES

Is proportional to number of blade revolutions over a turbines lifetime

Total cycles n_L would be

$$n_L = 60Kn \text{ rotor } H_{op} Y \dots\dots\dots (4.01)$$

Number of cyclic events of a blade can be much more than once per revolution of the rotor

Higher frequencies i.e. about 5Hz would result in fluctuating stress which would contribute to fatigue damage and would need to be considered in wind turbine design

4.1.4 ASSESSMENT OF FATIGUE

Alternating stress does not have a mean zero

$$\Delta\sigma = \sigma_{max} - \sigma_{min}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

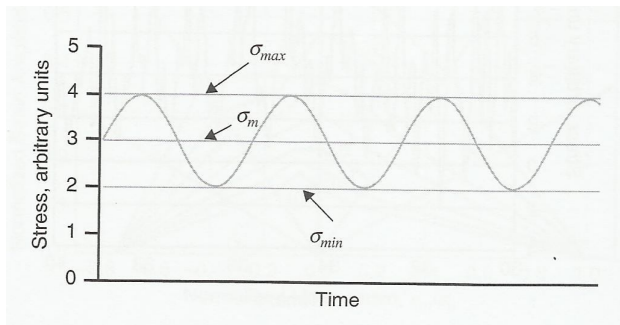


Figure 4.1: Variation of stress verses Time

The stress ratio

$$R = \sigma_{max} / \sigma_{min} \dots\dots\dots (4.02)$$

4.1.5 Goodman diagram

Is used to predict fatigue life of materials

The graphs are approximated by the use of Goodman rule

$$\sigma_{a1} = \sigma_e \left(1 - \sigma_m / \sigma_u\right)^c$$

For the zero mean stress and

$$\sigma_e = \sigma_{a1} / \left(1 - \sigma_m / \sigma_u\right)$$

For the non zero mean stress

There are Goodman diagrams available for different materials including fibreglass. (See book)

$$d = \frac{n}{N} = \frac{\text{Number of cycle applied}}{\text{Number of cycles to failure}}$$

D is defined by miners rule to be the sum of the damages due to each of the cycles at echo amplitude

Cumulative damage is given by

$$D = \sum_{i=1}^m n_i / N_i < 1 \dots\dots\dots (4.03)$$

E.g. randomly applied load cycles

4.1.6 EFFECT OF OPERATING CONDITIONS

The loads that affect a wind turbine include high winds, periodic loads, wind shear, yaw error, yaw motion and gravity and stochastic loads from turbulence, transient loads from events as gusts starts and stops

4.1.7 WIND TURBINE MATERIALS

Most important materials for wind turbines are steel and composites

Composites comprise of fibreglass, carbon fibres or wood together with a matrix of polyester and epoxy

4.1.8 REVIEW OF BASIC MECHANICAL PROPERTIES OF MATERIALS

1. Hooke's law
2. Modulus of elasticity
3. Yield strength, breaking strength
4. Ductility and brittleness
5. Hardness and machinability
6. Failure by yielding or fracture

1. Steel

Constructs tower hub mainframe shaft, gears and gear cases fasteners and reinforcing concrete.

2. Composites

Used in blade construction

3. Glass fibres

Formed by spinning glass fibres into long threads. There is *E* glass and *S* glass

4. Carbon fibre reinforcing

Carbon fibres are more expensive than glass fibres by a factor 8. They are stronger and stiffer

5. Copper

Design the electrical equipments because it has excellent conductivity

6. Concrete

Used for the construction of towers

4.1.9 MACHINE ELEMENTS

1. Shaft

Transmit torque. Shafts have resonant natural frequencies at critical speeds. Operation at such speeds should be avoided or large vibrations will occur

Materials

- For least severe condition hot rolled carbon steel is used
- For greater strength higher carbon content steel is used
- After machining shafts are heat treated to improve yields

2. Coupling

Couples are used in between generators and high speed shaft of gearbox

Teeth of the coupling carry the weights

3. Bearing

Bearings are used to reduce friction in rotating parts and holding shafts.

The above equation gives an approximation for the spring constants for two meshing gears assuming moduli of elasticity E_1 and E_2 for the two gears.

Dampers

They are types of dampers used in wind turbines include wind turbines, hydraulic pumping circuits and linear fluid dampers

Fastening and joining

Most important are bolts and screws

Fatigue is an important factor in specifying bolts. Effects of fatigue are reduced by pre-stressing the bolts

The big disadvantage of the optimum rotor is that it is very expensive to manufacture

4.2.1 PRINCIPAL COMPONENTS

Blades are the fundamental components of the rotor blades

Design considerations

- (i) Aerodynamic performance
- (ii) Structural strengths
- (iii) Blade materials
- (iv) Recyclability

- (v) Blade manufacturing
- (vi) Worker health and safety
- (vii) Noise reduction
- (viii) Condition/health and monitoring
- (ix) Blade root and hub attachment
- (x) Passive control or smart blade option
- (xi) Costs

Aerodynamic performance

- Design rated power and rated wind speed
- Design tip speed ratio
- Solidity
- Airfoil
- Number of blades
- Rotor power control
- Rotor orientation

Overall size of rotor swept area

Modern blades differ from optimum rotor because the optimum rotor is expensive to manufacture

Structural strength

It is sufficient to provide strength at the base of the blade.

Blade materials

Materials used are carbon fibres, fibre glass, aluminium, wood and composites of various fibres sandwiched with resin.

4.2.2 BLADE MANUFACTURING



Figure 4.2: Blade manufacturing industry (<http://www.google.com/3905714943.html>)

i) Wet lay up

Placing multiple layers plies of fibre glass cloth into a mould. Each layer is soaked in binder (polyester or epoxy resin plus hardener). Lay up is done by hand

ii) Pre preg

Involves a fibre glass cloth which has been impregnated with a resin, which remains firm at room temperature. Cloth is placed in a mould and the entire mould is heated so that resin will flow and harden permanently. Heating is done in low temperature oven or an autoclave

iii) Resin infusion

Dry fibreglass cloth is laid into the mould and then resin is impelled by some means of flow into the glass. One method is known as vacuum assisted resin transfer molding (VARTM). Air is removed from one side of the mould by a vacuum pump. A reservoir will be pushed through the cloth by atmospheric pressure into space evacuated by vacuum pump produce high quality

iv) Compression molding

Fiberglass and resin are placed in a two part mould which is then closed. Heat and pressure are applied until the product is ready.

v) Pultrusion

Is a method in which a material is pulled through a die to form an object of specific cross sectional shape but indefinite length. Object is cut to desire length afterwards

vi) Filament winding

Glass fibres are wound about a mandrel while resin is applied simultaneously. This is a process that can be automated in resulting in a consistent product. The best method of production is VARTM which stands for vacuum assisted resin transfer moulding.

Noise reduction

A wind turbine rotor can be a source of unwanted noise one way to reduce noise is through selection of suitable airfoils. Another method is to design the rotor to operate at a relatively low tip speed ratio all these methods are based on prevention turbulence around the blade.

Blade roots and hub attachment

Most convenient methods are the Huber design carbon fiber strands are attached to the end of the blades. The strands are wound round bushings. Bolts pass through the pressure plate, bush plate and base plate to complete hub connection. It is composed of a base plate and pressure plate

Blade properties

Blade properties are determined by dividing the into sections in a manner similar to the aerodynamic analysis. Properties for each section are found based on dimension and material distribution and the combine for values of entire blade

Hubs

Hub connects the blades to the main rotor shaft of the wind turbine. Hub transmits and withstands all loads .There are three types of hubs

- i. Rigid hubs
- ii. Teetering hubs
- iii. Hubs for hinged blades

Rigid hubs

Have all major part fixed. Appropriate for wind turbines with 3 or more blades. A hub on a pitch controlled turbine must provide for bearing and pitching mechanism. Pitching mechanism may use a pitch rod passing through the main shaft together with hub linkage. The pitch rod is driven by the motor mounted on the main part of the turbine. Another method is to mount electric gear motors on the hub. This can be done with slip rings or rotary

Teetering hub

Used in nearly all two bladed wind turbines. This is because a teetering hub can reduce loads due to aerodynamic imbalances or loads due to dynamic effects. The blades should not be collinear and the angle between hub and blade is known as the $\delta\beta$ angle as used in helicopter industry

When blades are collinear $\delta\beta = 0$. There is a benefit in a none zero $\delta\beta$ angles in that it can reduce bending stresses.

Hinged hub

A cross between rigid and a teetering hub

4.2.3 WIND TURBINE DESIGN AND TESTING

DESIGN PROCEDURE

- 1) Determine application
- 2) Review previous experiences
- 3) Select topology
- 4) Estimate preliminary loads
- 5) Develop tentative design
- 6) Predict performance
- 7) Evaluate design

- 8) Estimate costs and costs of energy
- 9) Retire design
- 10) Build prototype
- 11) Test prototype
- 12) Design production machine

4.2.4 SOME IMPORTANT PARAMETERS

The analysis in this chapter 2 assumes that the prevailing wind is uniform and aimed with the rotor axis and that the blades rotate in a plane perpendicular to the rotor axis. Assumptions are rarely the case because of wind shear. Wind turbines also operate with a transient steady state yaw error. Yaw error results in a flow component perpendicular to the rotor disc. The wind at the rotor may also have components especially at sites in complex terrain. The angular position of the blade varying with the blade azimuth angle. Blades are attached to the hub at the slight angle to the plane perpendicular to the rotor axis. This blade coning is done to reduce bending moments in the blades or to keep the blade from striking the tower. In rotor analysis each of these situations is usually handled with appropriate geometrical transformations.

Blade coning is handled by resolving aerodynamic force into components that are perpendicular and parallel to the rotor plane, analyzing them and developing a solution for the coning angle. The axial and plane components of the flow depend on the blade position, result in angles of attack and aerodynamic forces that fluctuate cyclically as the blades rotate. BEM equation with these considerations is analyzed in chapter 2. Further reading can be done in (wind energy explained chapter 4, 2010).

CHAPTER FIVE: THE 6KW WIND TURBINE ROTOR DESIGN PROCEDURE

5.0 THE 6KW ROTOR DESIGN PROCEDURE

5.0.1 ROTOR DESIGN:

The rotor design is the design of a fixed blade wind turbine. Using a design procedure developed by the University of Massachusetts this design was arrived after several evaluations that yielded a power coefficient which was within tolerance of our design power coefficient of 0.4 after evaluation the power coefficient was found to be 0.36 which is in the range of 0.4

Need is to produce 6000W enough for home use

$$\text{Step 1: Power} = C_p \Omega \left(\frac{1}{2}\right) \delta \pi R^2 U^3 \dots\dots\dots (5.00)$$

Needed

C_p =probable power coefficient

Ω =probable efficiency

δ = air density

R = radius of the rotor

U = wind velocity

Estimated efficiencies Gear = 0.95

Generator = 0.88

Rotor = 0.45

Shaft = 0.98-0.99

$$0.4 \times 0.95 \times 0.88 \times 0.99 = 0.3825$$

Coefficient of power obtained from aerofoil empirical curves $\cong 0.4$

$$6000 = (0.4)(0.3825)(0.5) \times 1 \times \Omega \times 8^3 \times R^2$$

Probable R of the rotor

$$R = 5.383 \dots (5.01)$$

Step 2: Choose a tip speed ratio where power coefficient C_p is highest from empirical curves we chose **8**.

Step 3: Chose the number of blades

They are chosen from a table developed by the University of Massachusetts by tests of prototypes in wind tunnels the table is denoted table 4.

TABLE 3.0: Tip speed ration verses number of blades

TIP SPEED RATIO	NUMBER OF BLADES
1	8-24
2	6-12
3	3-6
4	3-4
>4	1-3

So we decided to design with 3 blades to improve on torque because two blades have lesser torque than three blades.

1. The blade takes an aerodynamic shape. The other option would be curved plates. Curved plates are used in case of high torque and low tip speed ratios especially in water pumping by wind. Aerofoil's do higher speeds than curved plates. In electricity production high speed is needed.

Empirical aerofoil curves are drawn from experiments of aerofoils. The empirical curves used in this design are developed by NASA, US space agency by wind tunnel testing of aerofoils

The type of aerofoil used in this design is the 17% thick LS (1) 4/7 aerofoil. This is because it has a larger cross section which is easier to embed a structural without altering the profile of the aerofoil which could alter the lift coefficient and also has good aerodynamic characteristics compared to other aerofoils. Aerofoils are developed by aeronautical organisations like NASA or the European space agency and the naming is done by the organisations themselves. Further reading (wind energy explained ,chapter 3, 2010)

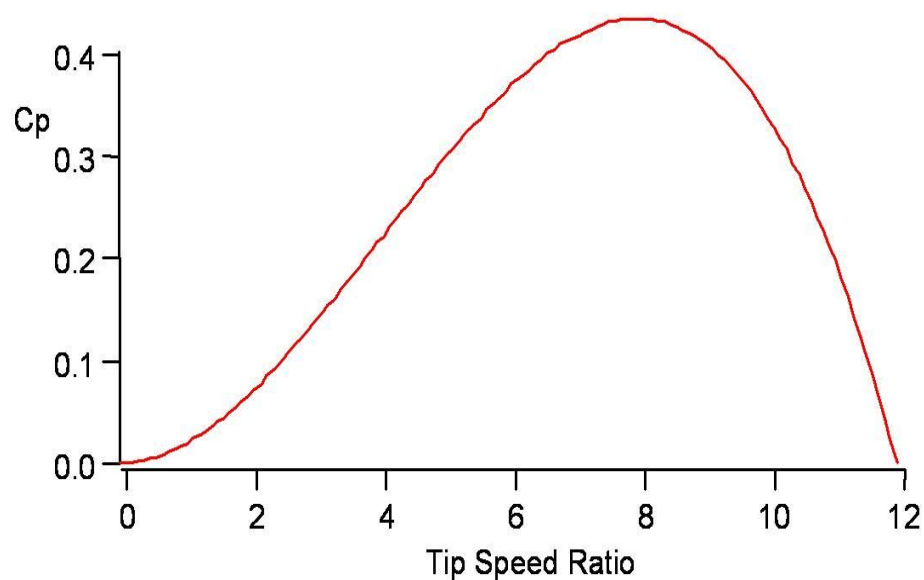


Figure 5.0: A graph of coefficient of performance verses Tip speed Ratio for 17% thick LS (1) 4/7 aerofoil (<http://www.google.com/876678669.html>)

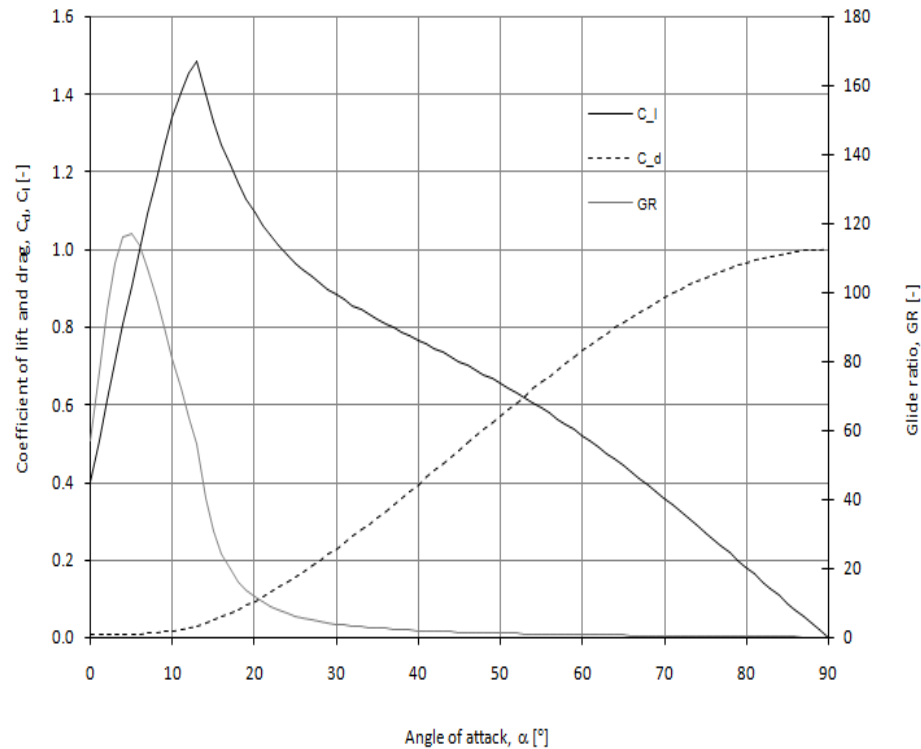


Figure 5.1: A graph of coefficient of lift, coefficient of drag versus Angle of attack for 17% thick LS (1) 4/7 aerofoil (<http://www.google.com/876678668.html>)

The blade is designed by using the angle of attack that would produce the maximum lift coefficient and minimum drag coefficient.

A check in the empirical curves that would be 6° and coefficient of lift = 1.5, coefficient of lift is chosen where C_l/C_d ratio is maximum

The angle of attack (design) is 6° and the coefficients of lift design is 1.5 at this point

The blade is divided into 10 blades according to the blade element theory that is discussed section

So if radius is 5.383. Width of each element is 0.53838

The blade shape is estimated with a midpoint radius r_i

$$\lambda_{rt} = \lambda \left(\frac{r_i}{R} \right) \text{ where } \lambda \text{ is the chosen tip speed ratio}$$

(Wind Energy Explained, pg 134)

$\lambda r_i = \text{local tip speed ratio}$

$R = \text{blade radius}$

$r_i = \text{blade shape estimated midpoint radius}$

1. Blade element 1

$$r_i = \frac{0.5383}{2} = 0.2692$$

$$\lambda r_i = 8 \left(\frac{0.2692}{5.383} \right) = 0.4$$

$$\varphi_i = \frac{2}{3} \tan^{-1} \left(\frac{1}{\lambda r_i} \right) = \frac{2}{3} \tan^{-1} \left(\frac{1}{0.4} \right) = 45.47 \dots \dots \dots (5.02)$$

Where φ_i is the angle of relative wind of the aerofoil

The cord length of the aerofoil is given by

$$C = \frac{8\pi r_i}{\beta(C_{id})} (1 - \cos \varphi_i) \dots \dots \dots (5.03)$$

Where C - chord length

β - number of blades

r_i - midpoint radius of blade

φ_i - angle of relative wind

C_{id} - coefficient of lift design

$$C_i = \frac{8\pi(0.2692)}{3(1.5)} (1 - \cos 45.47) = 0.449 \text{ m} \dots \dots \dots (5.04)$$

$$\varphi_i = \varphi_{pi} + \alpha \text{ design } i \dots \dots \dots (\text{Wind Energy Explained, pg 120})$$

Where φ_{pi} = pitch angle of blade element

α = angle of attack i.e. the design value

$$\varphi = \varphi_{pi} + \alpha \text{ design}$$

$$45.47 = \varphi_{pi} + \alpha \text{ design}$$

$$45 = \varphi_i + 6$$

$$45.47 - 6 = 39.47$$

$$\varphi_p = 39.47^\circ. \varphi_{pi} = \varphi_{p0}$$

Where φ_{p0} is the initial pitch angle

Blade element 2

$$\lambda r_i = \lambda \left(\frac{r_i}{R} \right)$$

$$\lambda r_i = 8 \left(\frac{0.8075}{5.383} \right) = 1.2$$

$$\varphi_i = \frac{2}{3} \tan^{-1} \left(\frac{1}{1.2} \right) = 26.537^\circ$$

$$Cl = \frac{8\pi(0.8075)}{3(1.5)} (1 - \cos 26.537) = 0.456 \text{ m}$$

$$\theta_T = \theta_{pi} - \theta_{po}$$

$$\theta_T = 20.537 - 39.42 = -18.933^\circ$$

θ_T Is the angle of twist relative to the first blade element

Blade element 3

$$\lambda r_i = \lambda \left(\frac{r_i}{R} \right)$$

$$8 \left(\frac{1.3458}{5.383} \right) = 2$$

$$\varphi_i = \frac{2}{3} \tan^{-1} \left(\frac{1}{2} \right) = 17.71^\circ.$$

$$C_i = \frac{8\pi(1.3458)}{3(1.5)} (1 - \cos 17.71) = 0.35621 \text{ m}$$

$$\varphi_i = \varphi_{pi} + \alpha_i \text{ design}$$

$$17.71 = \varphi_{pi} + 6$$

$$\varphi_{pi} = 11.71$$

$$\varphi_T = 11.71 - 39.47 = -27.76^\circ$$

Where

λr_i - Local tip speed

φ_{pi} - Local pitch angle

C_i - Length of chord

φ_i -relative angle of wind

α -angle of attack

φ_T -twist angle

Blade element 4

$$\lambda r_i = \lambda \left(\frac{r_i}{R} \right)$$

$$8 \left(\frac{1.8838}{5.383} \right) = 2.79963$$

$$\varphi_i = \frac{2}{3} \tan^{-1} \left(\frac{1}{2.79963} \right) = 13.10415^\circ$$

$$C_r = \frac{8\pi r_i}{\beta(C_{id})} (1 - \cos \varphi_i)$$

$$Ci = \frac{8\pi(1.8838)}{3(1.5)}(1 - \cos 13.10415) = 0.2739 \text{ m} \dots \dots \dots (5.09)$$

$$\varphi_i = \varphi_{pi} + \alpha_i \text{ design}$$

$$13.10415 = \varphi_{pi} + 6$$

$$\varphi_{pi} = 7.10415$$

$$\theta_T = \theta_{pi} - \theta_{po}$$

$$\varphi_T = 7.1045 - 39.47 = -32.3655^\circ$$

Where

λri - Local tip speed ratio

φ_{pi} - Local pitch angle

Ci - Length of chord

φ_i -relative angle of wind

α -angle of attack

φ_T -twist angle

Blade element 5

$$\lambda ri = \lambda \left(\frac{ri}{R} \right)$$

$$8 \left(\frac{2.4218}{5.383} \right) = 3.5992$$

$$\varphi_i = \frac{2}{3} \tan^{-1} \left(\frac{1}{3.5992} \right) = 10.351596^\circ$$

$$C = \frac{8\pi(2.4218)}{3(1.5)}(1 - \cos 10.352) = 0.22016 \text{ m}$$

$$\varphi_i = \varphi_{pi} + \alpha_i \text{ design}$$

$$10.3516 = \varphi_{pi} + 6$$

$$\varphi_{pi} = 4.3516$$

$$\varphi_T = \varphi_{pi} - \varphi_{po}$$

$$\varphi_T = 4.3516 - 39.47 = -35.1184^0$$

Where

λri - Local tip speed ratio

φ_{pi} - Local pitch angle

Ci - Length of chord

φ_i -relative angle of wind

α -angle of attack

φ_T -twist angle

Blade element 6

$$\lambda ri = \lambda \left(\frac{ri}{R} \right)$$

$$8 \left(\frac{2.9598}{5.383} \right) = 4.39$$

$$\varphi_i = \frac{2}{3} \tan^{-1} \left(\frac{1}{4.399} \right) = 8.5385^0$$

$$Ci = \frac{8\pi(2.9598)}{3(1.5)} (1 - \cos 8.5385) = 0.18322m$$

$$\varphi_i = \varphi_{pi} + \alpha_i \text{ design}$$

$$8.53685 = \varphi_{pi} + 6$$

$$\varphi_{pi} = 2.5385^0$$

$$\varphi_T = 2.5385 - 39.47 = -36.9315^0$$

Where

λri - Local tip speed ratio

φ_{pi} - Local pitch angle

Ci - Chord length

φ_i -Angle of relative wind

α -angle of attack

φ_T -twist angle

Blade element 7

$$\lambda ri = \lambda \left(\frac{ri}{R_1} \right)$$

$$8 \left(\frac{3.4981}{5.383} \right) = 5.1987$$

$$\varphi_i = \frac{2}{3} \tan^{-1} \left(\frac{1}{5.1987} \right) = 7.26^0$$

$$C = \frac{8\pi(3.4981)}{3(1.5)} (1 - \cos 7.26) = 0.1566m$$

$$\varphi_i = \varphi_{pi} + \alpha_{design}$$

$$7.26 = \varphi_{pi} + 6$$

$$\varphi_{pi} = 1.26^0$$

$$\varphi_T = 1.26 - 39.47 = -38.21^\circ$$

Blade element 8

$$\lambda r_i = \lambda \left(\frac{r_i}{R} \right)$$

$$8 \left(\frac{4.0364}{5.383} \right) = 5.9987$$

$$\varphi_i = \frac{2}{3} \tan^{-1} \left(\frac{1}{5.9987} \right) = 6.30395^\circ \dots\dots\dots (5.13)$$

$$C = \frac{8\pi(4.0364)}{3(1.5)} (1 - \cos 6.30395) = 0.13655m \dots\dots\dots (5.14)$$

$$\varphi_i = \varphi_{pi} + \alpha_{design}$$

$$6.30395 = \varphi_{pi} + 6$$

$$\varphi_{pi} = 0.30395^\circ$$

$$\varphi_T = \varphi_{pi} - \varphi_{po}$$

$$\varphi_T = 0.30395 - 39.47 = -39.1655^\circ$$

Where

λr_i - Local tip speed ratio

φ_{pi} - Local pitch angle

C_i - Chord length

φ_i -Angle of relative wind

α -angle of attack

φ_T -twist angle

Blade element 9

$$\lambda r_i = \lambda \left(\frac{r_i}{R} \right)$$

$$8 \left(\frac{4.5744}{5.383} \right) = 6.798$$

$$\varphi_i = \frac{2}{3} \tan^{-1} \left(\frac{1}{6.738} \right) = 5.58^\circ$$

$$C_i = \frac{8\pi(4.5744)}{3(1.5)} (1 - \cos 5.58) = 0.1211m \dots \dots \dots (5.15)$$

$$\varphi_i = \varphi_{pi} + \alpha \text{ design}$$

$$6.30395 = \varphi_{pi} + 6$$

$$\varphi_{pi} = -0.42^\circ$$

$$\varphi_T = \varphi_{pi} - \varphi_{po}$$

$$\varphi_T = -0.42 - 39.47 = -39.89^\circ$$

Where

λr_i - Local tip speed ratio

φ_{pi} - Local pitch angle

C_i - Chord length

φ_i -Angle of relative wind

α -angle of attack

φ_T -twist angle

Blade element 10

$$\lambda r_i = \lambda \left(\frac{r_i}{R} \right)$$

$$r_i = 4.8447$$

$$8 \left(\frac{4.8447}{5.383} \right) = 7.2$$

$$\varphi_i = \frac{2}{3} \tan^{-1} \left(\frac{1}{7.2} \right) = 5.29^\circ$$

$$C_i = \frac{8\pi(4.8447)}{4.5} (1 - \cos 5.27) = 0.1143m \dots \dots \dots (5.16)$$

$$\varphi_i = \varphi_{pi} + \alpha$$

$$7.75 = \varphi_{pi} + 6$$

$$\varphi_{pi} = -1.25^\circ$$

$$\varphi_T = \varphi_{pi} - \varphi_{po}$$

$$\varphi_T = -1.25 - 39.47 = -40.72^\circ$$

Where

λr_i - Local tip speed ratio

φ_{pi} - Local pitch angle

C_i - Chord length

φ_i -Angle of relative wind

α -angle of attack

φ_T -twist angle

ri -midpoint radius

R –Rotor radius

TABLE 4.0: Blade elements chord length, twist distribution and pitch (summary)

BLADE ELEMENT	CHORD LENGTH	TWIST DISTRIBUTION	PITCH
1.	0.449	0	39.47^0
2.	0.456	-18.933^0	20.537^0
	0.35621	-27.76^0	11.71^0
	0.2739	-32.3655^0	7.10415^0
	0.22016	-35.1184^0	4.3516^0
	0.18322	-36.9315^0	2.5385^0
	0.1566	-38.21^0	1.26^0
	0.13655	-39.16605^0	0.30395^0
	0.1211	-39.89^0	-0.42^0
	0.1143	-40.72^0	-1.25^0

The chord length and twist distribution and angle of attack are relevant when fabricating the blade

The pitch angle is relevant when fixing the blade to the hub

5.0.2 CALCULATION OF ROTOR PERFORMANCE AND MODIFY DESIGN

Our design is aimed to achieve design of an optimum rotor which is easy to fabricate and if the wind speed changes it ceases from being an optimum rotor

$$C_{li} = 4F_i \sin \varphi_i \frac{(\cos \varphi_i - \lambda ri (\sin \varphi_i))}{\sigma^1 (\sin \varphi_i + \lambda ri \cos \varphi_i)} \dots\dots\dots (5.17)$$

Where,

F - tip loss factor

φ_i -angle of relative wind

λr_i -local tip speed ratio

σ - Blade solidity given as $\frac{\beta C}{2\pi r}$ (5.18)

Where β the number of blades is, C is the chord length and r_i - is the midpoint radius

The tip loss can be estimated as

$$F_i = \left(\frac{2}{\pi}\right) \cos^{-1} \left[\exp \left(- \left\{ \frac{\beta/2(1-r/R)}{r_i/R \sin \varphi} \right\} \right) \right] \dots\dots\dots (5.19)$$

$$\varphi = \varphi_i = \frac{2}{3} \tan^{-1} \left(\frac{1}{\lambda r_i} \right) \dots\dots\dots (5.20)$$

After calculating the lift coefficient you can use the empirical curves for aerofoil models developed in laboratories to get the actual angle of attack from the curves that is suitable for the blade

The optimum analysis does not consider the following short comings

- i) Yaw error
- ii) Wind shear phenomenon
- iii) Blade coning
- iv) Assumes prevailing wind is uniform with rotor axis
- v) Tip losses

The model of using empirical curves is most appropriate but in case of turbulent wake other ways of evaluating blade design are used one of them being iterative solution for axial induction factor. Turbulent wake convenes if axial induction factor is more than 0.4.

Coefficient of lift can then be determined from air foil graphical data of coefficient of lift versus angle of attack where angle of attack is given as

$$\alpha_{ij} = \varphi_{ij} - \theta_{pi} \dots\dots\dots (5.21)$$

Where,

α_{ij} - Angle of attack

φ_{ij} -relative angle of wind

θ_{pi} - pitch angle

So evaluation of the design begins . The method of using empirical curves is valid only if the axial induction factor is less than 0.4 i.e. when the wake is not turbulent if the axial induction factor is greater than 0.4 iteration method is used to calculate the axial and angular induction factors that are used to calculate parameters used in rotor evaluation i.e. equation (2.68)

Hence blade element 1:

From the momentum theory

$$a_i = \frac{1}{\left[\frac{1+4 \sin^2 \varphi_i}{\sigma^1 C_{li} \cos \varphi_i} \right]}$$

$$C_{li} = 4F_i \sin \varphi_i \frac{(\cos \varphi_i - \lambda r i (\sin \varphi_i))}{\sigma^1 (\sin \varphi_i + \lambda r i \cos \varphi_i)} \dots \dots \dots (5.23)$$

$$C_i = 4 \sin 45.54 \frac{(\cos 45.42 - 0.4 (\sin 45.47))}{0.7964 (\sin 45.54 + 0.4 \cos 45.54)}$$

$$C_i = 1.4966$$

$$a = \frac{1}{\left[\frac{1+2.0377}{0.7964 (0.7004)} \right]} = 0.1836$$

Blade element 2

$$C_{li} = 4F_i \sin \varphi_i \frac{(\cos \varphi_i - \lambda r i (\sin \varphi_i))}{\sigma^1 (\sin \varphi_i + \lambda r i \cos \varphi_i)}$$

$$\varphi_i = 26.537^\circ$$

$$\lambda r i = 1.2$$

$$F = \left(\frac{2}{\pi}\right) \cos^{-1} \left[\exp \left(- \left\{ \frac{1.5(1 - 0.15)}{0.15 \sin 26.537} \right\} \right) \right] = 0.999935$$

$$F \cong 1$$

$$Cl = 4(0.447) \frac{(0.8946 - 1.2(0.447)) (0.3582)}{(0.447 + 1.2(0.8986)) - 1.5202}$$

$$\sigma^1 = \frac{3 \times 0.456}{2 \times \pi \times 0.8075} = 0.2696$$

$$\frac{0.64046}{0.4044} = 1.58$$

$$1.58 = Cl$$

$$a = \frac{1}{\left[\frac{1+4 \sin^2 \varphi_i}{\sigma^1 Cl i \cos \varphi_i} \right]} = \frac{1}{\left[\frac{1+0.7983}{0.2696(1.58)(0.8946)} \right]} = 0.2.$$

Blade element 3

$$\varphi_i = 17.71^\circ$$

$$\lambda r i = 2$$

$$C = 0.35621$$

$$\sigma^1 = 0.1264$$

$$F \cong 1$$

$$Cl = 4(0.3042) \frac{(0.9526 - 2(0.3042))}{0.1264(0.3042 + 2(0.9526))}$$

$$Cl = 1.4995$$

$$a_i = \frac{1}{\left[\frac{1+4 \sin^2 \varphi_i}{\sigma^1 Cl i \cos \varphi_i} \right]} = \frac{1}{\left[\frac{1+0.3701}{0.1264(1.4995)(0.9526)} \right]} = 0.13$$

Blade element 4

$$\varphi_i = 13.10415^\circ$$

$$r_i/R = 0.34905$$

$$\lambda r_i = 2.79$$

$$C_i = 0.2739$$

$$\sigma^1 = 0.06942$$

$$C_{li} = 4(0.2267) \frac{(0.974 - 2.79(0.2267))}{0.06942(0.2267 + 2.79(0.974))}$$

$$Cl = 1.5$$

$$F \cong 1$$

$$a = \frac{1}{\left[\frac{1 + 4 \sin^2 13.1045}{0.06942(1.5) \cos 13.1045} \right]} = 0.069465$$

Blade element 5

$$\varphi_i = 10.351596^\circ$$

$$\lambda r_i = 3.5992$$

$$C = 0.22016$$

$$F_i = \left(\frac{2}{\pi} \right) \cos^{-1} \left[\exp \left(- \left\{ \frac{1.5(1 - 0.4499)}{0.449(0.1797)} \right\} \right) \right]$$

$$F_i \cong 1$$

$$\sigma^1 = \frac{3 \times 0.22016}{2 \times \pi \times 2.4218} = 0.0217$$

$$Cl = 4(0.1797) \frac{(0.9837 - (0.5013))}{0.043(0.1797 + 2.74)}$$

$$a = \frac{1}{\left[\frac{1+4\sin^2 3.5992}{0.0217 (\cos 3.5992)} \right]} = 0.02.$$

Blade element 6

$$\varphi_i = 8.5385^\circ$$

$$r_i/R = 0.5498$$

$$\lambda r_i = 4.39$$

$$C = 0.18322$$

$$\sigma^1 = 0.029556$$

$$Fi = \left(\frac{2}{\pi} \right) \cos^{-1} \left[\exp \left(- \left\{ \frac{1.5(1 - 0.5498)}{0.5498(0.1485)} \right\} \right) \right]$$

$$Fi \cong 1$$

$$Cl = 1.5$$

$$a = \frac{1}{\left[\frac{1+4\sin^2 8.5385}{0.029556(1.5)0.9889} \right]}$$

$$a = 0.043$$

Blade element 7

$$\lambda r_i = 5.1987$$

$$\varphi_i = 7.26^\circ$$

$$r_i/R = 0.6498$$

$$C = 0.1566$$

$$\sigma^1 = 0.021$$

$$Fi = \left(\frac{2}{\pi}\right) \cos^{-1} \left[\exp \left(- \left\{ \frac{1.5(1 - 0.6498)}{0.6498 \sin 7.26} \right\} \right) \right]$$

$$Fi \cong 1$$

$$Cl = 1.5$$

$$a = \frac{1}{\left[\frac{1+4\sin^2 7.26}{0.021(0.992)} \right]}$$

$$a = 0.027$$

Blade element 8

$$\lambda = 5.9987$$

$$\varphi_i = 6.30395^\circ$$

$$r_i/R = 0.7498$$

$$Ci = 0.13655$$

$$Fi = \left(\frac{2}{\pi}\right) \cos^{-1} \left[\exp \left(- \left\{ \frac{1.5(1 - 0.7498)}{0.07498(0.1098)} \right\} \right) \right]$$

$$Fi = 0.9933$$

$$\sigma^1 = \frac{3 \times 0.13655}{2 \times \pi \times 4.0364} = 0.01615$$

$$Cl = 4(0.9933) \frac{(0.1098)(0.99395 - 0.6587)}{0.01615(0.1098 + 5.962)}$$

$$Cl = 1.49$$

$$a = \frac{1}{\left[\frac{1+4(0.1098)}{0.01615(0.994)} \right]} = \frac{1.4392}{0.0161}$$

$$a = 0.01154$$

Blade element 9

$$\lambda r_i = 6.798$$

$$\varphi_i = 5.58^\circ$$

$$r_i/R = 0.8498$$

$$C = 0.1211$$

$$\sigma^1 = 0.01264$$

$$F = 0.958$$

$$Cl = 1.4353$$

$$a = \frac{1}{\left[\frac{1+0.3889}{0.01264(0.9953)} \right]} = 0.00905$$

Blade element 10

$$\lambda r_i = 8$$

$$\varphi_i = 4.75^\circ$$

$$r_i/R = 0.8498$$

$$C = 0.1033$$

$$\sigma^1 = 9.163 \times 10^{-3}$$

$$Fi = 0.7475$$

$$Cl = 1.26$$

$$a = \frac{1}{\left[\frac{1+0.3312}{9.163(0.997)} \right]} = 0.00686$$

It is found that the actual coefficient of lift is similar to the design coefficient of lift except for the last blade element which is significantly reduced by tip losses. This evaluation of blade design also shows that there is no turbulent wake developed behind the turbine which would

affect lift by reducing lift since no axial induction factor is greater than 0.4 so it can be concluded that the blade will achieve its design target of 6 kilowatts.

STRUCTURAL DESIGN OF WIND TURBINES

5.0.3 MATERIAL SELECTION OF WIND TURBINE

The following are the materials commonly used in the fabrication of wind turbines. The table is generated from data available in the internet

Table 5.0: Material Selection Index (<http://www.materialselectionindex.php/properties>)

MATERIAL	DENSITY g/cm ³	TENSILE STRENGTH (MPA)	YOUNG'S MODULUS (GPA)	SHEAR STRENGTH (MPA)	PRICE. £ Per Kg.
E GLASS	2.55	2000	80	40	1-2
S GLASS	2.49	4750	89	45	12-20
ALUMINIUM	2.75	310	68.9	1.8	0.39
CARBON FIBRE (PLASTIC REINFORCED)	2.00	2900	525	70	20-30
WOOD	0.745	87	64	65.33	3
ALUMINIUM REINFORCED CARBON FIBRE	3.28	1950	297	167	10-20

Material selection index is applied to select the most appropriate material selection index is defined as

$$\frac{\text{Most desirable properties}}{\text{Most undesirable properties}} = \text{Material selection index}$$

This information is obtained from (Material Science by Calister 2000, Chapter 4)

Most desirable properties are

1. High tensile strength
2. High shear strength

3. High young's modulus

Most undesirable properties are

1. High price
2. High density

MATERIAL (1)

$$Wood. = \frac{87 \times 65.33 \times 64}{3 \times 0.745} = 162,755.007$$

MATERIAL (2)

$$Aluminium = \frac{68.9 \times 1.8 \times 310}{0.39 \times 2.75} = 35,847.27$$

MATERIAL (3)

$$S. Glass = \frac{89 \times 45 \times 4750}{2.49 \times 20} = 382,003.012$$

MATERIAL (4)

$$E. Glass = \frac{40 \times 80 \times 2000}{2.55 \times 2} = 1,254,901.961$$

MATERIAL (5)

$$Carbon Fibre = \frac{70 \times 525 \times 2900}{30 \times 2} = 1,776,520$$

MATERIAL (6)

$$Aluminium reinforced carbon fibre = \frac{1950 \times 297 \times 167}{3.28 \times 15} = 176,570.122$$

Table 6.0: Summary of material selection coefficients.

Wood	162,755.007
Aluminum	35,847.27
S. glass	382,003.012
E. glass	1,254,901.961
Carbon fibre reinforced plastic	1,776,520
Aluminum reinforced carbon fiber	176,570.122

Material with the highest material selection coefficient is carbon fibre. Other factors to be considered in the material selection are;

- (1) Corrosion resistance
- (2) Availability
- (3) Ease of fabrication
- (4) Replacement

- The above factors are not measurable and require someone to make a considerate decision
- Carbon fibre satisfies the above factors and has the highest material selection index

So material chosen is carbon fibre reinforced plastic.

5.0.4 MECHANICAL DESIGN OF THE BLADES

To design the blades appropriately one must know how to calculate some parameters of the aerofoil. These parameters are second moment of area of the aerofoil and the area

The following theory is generated from aeronautical engineering handbooks on how to calculate the second moment of area I and the area of an aerofoil

Area and Bending Inertia of Airfoil Sections

Calculation of the vertical deflection of a wing requires knowing the span wise bending stiffness distribution $EI(y)$ along the primary axis of loading. For a wing made of a uniform solid material, the modulus E is a simple scaling factor. The moment of inertia of the airfoil cross- sections about the bending axis x (called the *bending inertia*), is then related only to the airfoil shape given by the upper and lower surfaces $Z_u(x)$ and $Z_l(x)$. As shown in Figure 1, both the area A and the total bending inertia I are the integrated contributions of all the infinitesimal rectangular sections, each dx wide and $Z_u - Z_l$ tall. The inertia of each such section is appropriately taken about the *neutral surface* position \bar{z} defined for the entire cross section.

$$A = \int_0^c [Z_u - Z_l] dx \dots \dots \dots (3.01)$$

$$\bar{z} = \frac{1}{A} \int_0^c \frac{1}{2} [Z_u^2 - Z_l^2] dx \dots \dots \dots (3.0.2)$$

$$I = \int_0^c \frac{1}{3} [(Z_u - \bar{z})^3 - (Z_l - \bar{z})^3] dx \dots \dots \dots (3.0.3)$$

These relations assume that the bending deflection will occur in the z direction, which is a good assumption if the x axis is parallel to the airfoil's chord line.

For the purpose of approximating A and I , we first define the maximum thickness t , and maximum camber h , in terms of the upper and lower surface shapes. We also define the corresponding thickness and camber ratios T and E .

$$t = \max\{Z_u(x) - Z_l(x)\} \dots \dots \dots (3.0.4)$$

$$h = \max\{[Z_u(x) + Z_l(x)]/2\} \dots \dots \dots (3.0.5)$$

$$T \equiv t/c$$

$$E \equiv h/c$$

Examination of equation (3.0.3) indicates that A is proportional to tc , and examination of (5.26) indicates that I is proportional to $ct(t^2 - h^2)$. This suggests estimating A and I with the following approximations.

$$A \simeq K_A ct = K_A C^2 T \dots \dots \dots (3.0.6)$$

$$I \simeq K_I ct(t^2 + h^2) = K_I C^4 T (T^2 + E^2) \dots \dots (3.0.7)$$

The proportionality coefficient can be evaluated by equating the exact and approximate A and I expressions above, e.g.

$$K_A \leftarrow \frac{1}{C^2 T} \int_0^c [Z_u - Z_l] dx$$

$$K_I \leftarrow \frac{1}{C^4 T (T^2 + E^2)} \int_0^c \frac{1}{3} [(Z_u - \bar{Z})^3 - (Z_l - \bar{Z})^3] dx$$

Evaluating these expressions produces nearly the same K_A and K_I values for most common airfoils:

$$K_A \simeq 0.60 \dots \dots \dots (3.0.8)$$

$$K_I \simeq 0.036 \dots \dots \dots (3.0.9)$$

There are 3 types of aerofoil mainly used for wind turbines they are:

- (i) 12% thick NACA 0012 aerofoil
- 17% thick LS (1) 4/7 aerofoil
- 12% thick NACA 63-418 aerofoil

The thickest aerofoil is the best to use because it is easier to gain more structural strength from the thick aerofoil than the thin aerofoil because a larger beam can be embedded into the air foil the aerodynamic design is done with the same aerofoil as the structural design.

So for this design case the choice of aerofoil is the LS (1) 4/7 aerofoil. That is 17% thick 17% thick means that the thickness of aerofoil is 0.17 times the chord length of the aerofoil.

This was the aerofoil that was chosen for the aerodynamic design so there is no contrast between aerodynamic and structural design.

5.0.5 STRUCTURAL DESIGN OF THE WIND TURBINE BLADES

The common way of designing the structure of the wind turbine is to embed a beam into the aerofoil of the wind turbine. The beam is called the spar

The spar carries most of the load of the wind turbine blades while the aerofoil contributes to the lift of the blade. The aerofoil also carries a little or some structural load of the wind turbine.

The spar can be made of any shape, it can be an (I) beam, circular or any shape as long as it can carry the bending load.

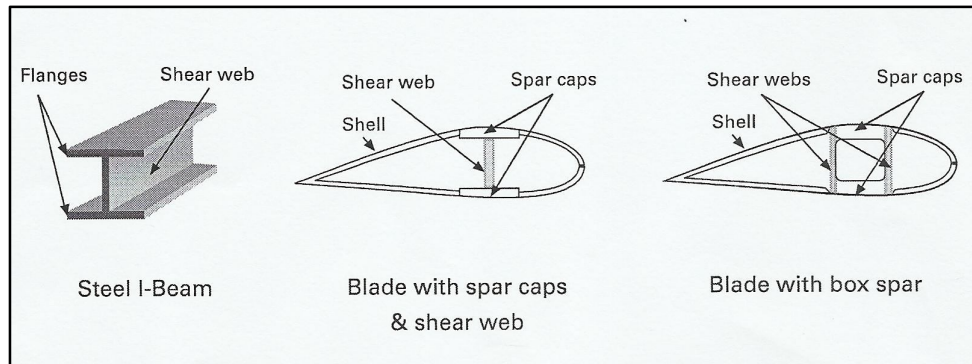


Figure 5.2: A cross section view of an aerofoil

In this particular case of design a circular beam is chosen as the shape of the spar

The cross section of the aerofoil shape would be as follows;

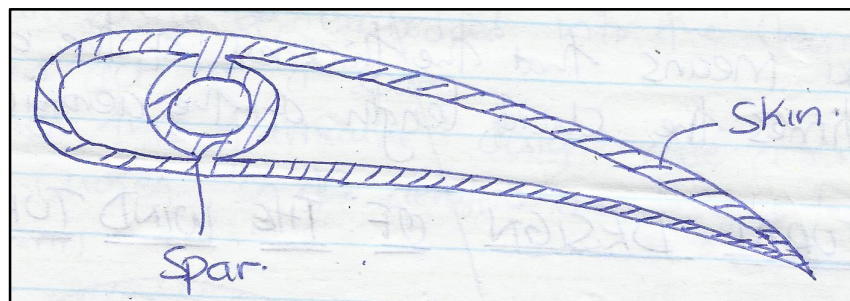


Figure5.3: A hand drawn cross section view of an aerofoil

The blade of the aerofoil is taken as a cantilevered beam. The lift force which drives the turbine round is distributed along the blade approximately in proportion to the local radius i.e. there is more force close to the root than the tip. Lift force makes the blade bend. If a section of the blade is subjected to lift at some point along its length all the force outboard at that point will have cumulative effect on the tendency to bend. This effect is called the bending moment.

The variation of the bending moment is as shown in fig 5.4

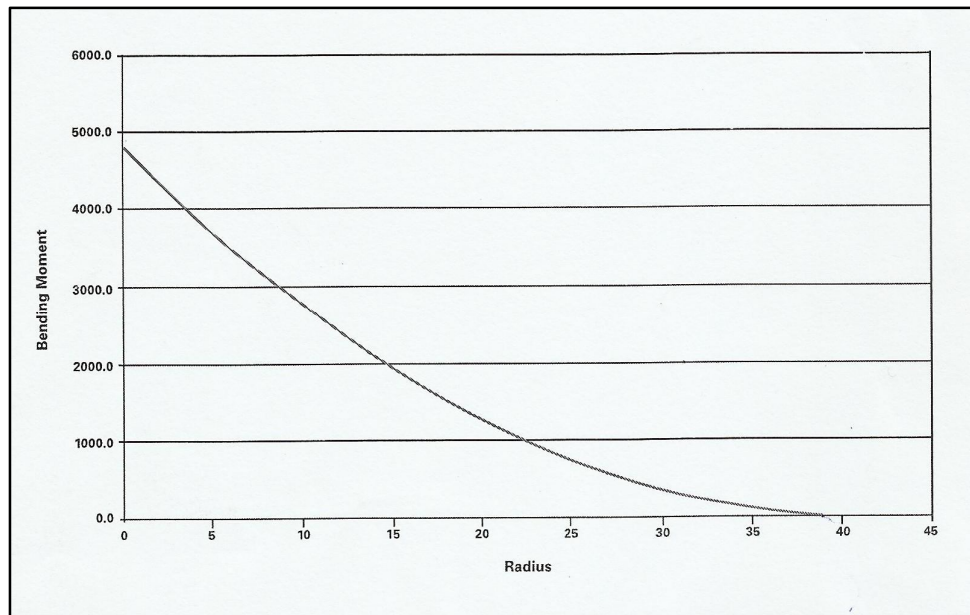


Figure5.4: variation of bending moment with radius

The bending moment of the flap wise direction of the wind turbine is given a

$$M = \frac{1}{B} \int_0^R r \left(\frac{1}{2} \cdot \delta \cdot \pi \cdot \frac{8}{9} \cdot U^2 \cdot 2r dr \right) \text{ (Wind energy explained pg173, 2010 edition)}$$

Integrating

$$M_B = \frac{T}{B} \cdot \frac{2}{3} \cdot R$$

Where

$$T = \text{torque}$$

$$B = \text{number of blades}$$

$$R = \text{radius}$$

The design torque can be evaluated as

$$T = \frac{P}{W} \text{ (Mechanics of machines by Hannah and Stephens, chapter1)}$$

Where

$$P = \text{power}$$

$$T = \text{torque}$$

$$W = \text{angular velocity}$$

W Can be evaluated as

$$W = \frac{\lambda \times u}{r}$$

Where

λ = tip speed ratio

u = wind speed

r = total radius of the blade

Design tip speed ratio $\lambda=8$, design radius = 5.38m

Wind speed = 8m/s

Substituting $W = 11.896 \text{ rads}^{-1}$

Design power is 6000W

So the torque can be evaluated as

$$T = \frac{P}{W}$$

Where power (P) is 6000 watts and angular speed (W) is 11.896 rads/sec

Evaluating the torque is found to be **504.41NM**

There are two bending moments in a wind turbine blade there is the bending moment in the flap wise direction and there is a moment on the edgewise direction i.e. the direction of flow of the wind

The flap wise moment is always greater for wind turbine that use aerofoil than the edgewise moment and both moments are perpendicular to each other so they cannot be added or summed up. The blades should be designed in such a way that it will not fail due to either flap wise or edgewise bending moment

The edgewise bending moment is given as

$$\frac{\text{Torque}}{\text{Number of blades}} = \text{edgewise bending moment}, \text{ (wind energy explained, pg 174)}$$

From structural theory of wind turbines (described in chapter 4, wind energy explained).

This is edgewise bending moment at the root of blade.

The design torque is 504.41NM

The numbers of blades are 3

$$\frac{504.41}{3} = 168.14 \text{ NM} \dots\dots\dots (5.33)$$

Edgewise moment = 168.14NM

The analysis of the flap wise bending moment for 10 sections denoted as blade elements of the blade is done in the next section.

5.0.6 THE FLAPWISE BENDING MOMENTS OF VARIOUS BLADE ELEMENTS

1. Blade element 1

Moment is given by

$$M_B = \frac{1}{B} \int_0^R r \left(\frac{1}{2} \cdot \delta \cdot \pi \cdot \frac{8}{9} \cdot U^2 \cdot 2r dr \right) \text{(wind energy explained chapter 4 pg 127)}$$

Integrating,

$$M_B = \frac{T}{B} \cdot \frac{2}{3} \cdot R$$

Where,

$T = \text{designed torque}$

$B = \text{number of blades}$

$R = \text{radius of blade}$

Torque = 504.41 NM

$B=3$

$R = 5.38$

$$M_B = \frac{504.41}{3} \cdot \frac{2}{3} \cdot 5.38 = 603.05 \text{ NM} \dots\dots\dots (5.34)$$

2. Blade element (2)

$$M_B = \frac{1}{B} \int_0^R r \left(\frac{1}{2} \cdot \delta \cdot \pi \cdot \frac{8}{9} \cdot U^2 \cdot 2r dr \right)$$

Integrating,

$$M_B = \frac{T}{B} \cdot \frac{2}{3} \cdot R$$

Where,

$T = \text{design torque}$

$B = \text{number of blades}$

$R = \text{radius of element}$

At element 2

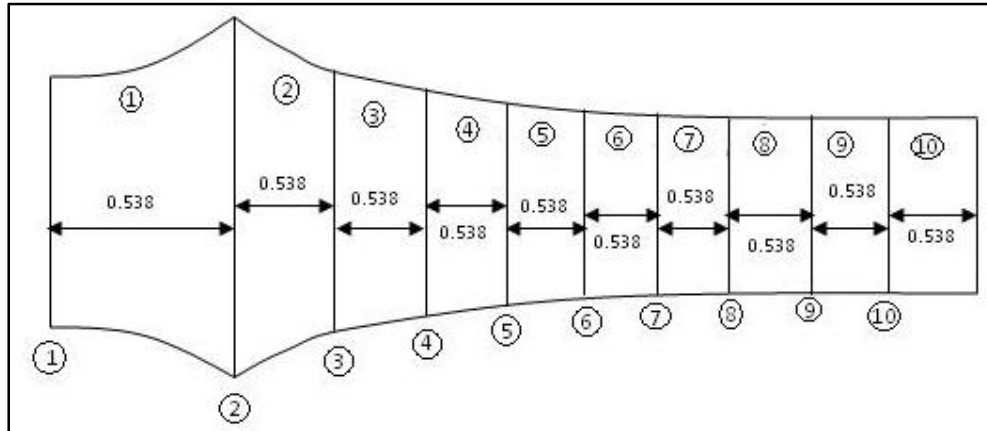


Figure 5.5: A sketch of blade elements sections arrangement.

At $2 R = 4.842M$

$$M_B = \frac{1}{B} \int_0^R r \left(\frac{1}{2} \cdot \delta \cdot \pi \cdot \frac{8}{9} \cdot U^2 \cdot 2r dr \right)$$

$$M_B = \frac{T}{B} \cdot \frac{2}{3} \cdot R \quad B = 3, T = 504.41 \text{ NM}$$

Torque = 504.41 NM

$$M_B = \frac{504.41}{3} \cdot \frac{2}{3} \cdot 4.842 = 542.75 \text{ NM}$$

$$M_B = 542.75 \text{ NM} \dots \dots \dots (5.35)$$

3. Bending moment at element 3

$$M_B = \frac{1}{B} \int_0^R r \left(\frac{1}{2} \cdot \delta \cdot \pi \cdot \frac{8}{9} \cdot U^2 \cdot 2r dr \right)$$

intergrating

$$M_B = \frac{T}{B} \cdot \frac{2}{3} \cdot R$$

R at 3 = 4.304

$T = \text{design toque}$

$B = \text{number of blades}$

$R = \text{radius to the element}$

$$M_B = \frac{504.41}{3} \cdot \frac{2}{3} \cdot 4.304$$

$$M_B = 482.44 \text{ NM} \dots \dots \dots (5.36)$$

4. Bending moment at element 4

$$M_B = \frac{1}{B} \int_0^R r \left(\frac{1}{2} \cdot \delta \cdot \pi \cdot \frac{8}{9} \cdot U^2 \cdot 2r dr \right)$$

Integrating,

$$M_B = \frac{T}{B} \cdot \frac{2}{3} \cdot R$$

Where,

$T = \text{designed torque}$

$B = \text{number of blades}$

$R = \text{radius to the element}$

$$R = 3.766$$

$$M_B = \frac{504.41}{3} \cdot \frac{2}{3} \cdot 3.766 = 422.135 \text{ NM} \dots\dots\dots (5.37)$$

5. Blade element 5 bending moment (flap wise)

$$M_B = \frac{1}{B} \int_0^R r \left(\frac{1}{2} \cdot \delta \cdot \pi \cdot \frac{8}{9} \cdot U^2 \cdot 2r dr \right)$$

$$M_B = \frac{T}{B} \cdot \frac{2}{3} \cdot R$$

Where,

$T = \text{designed torque}$

$B = \text{number of blades}$

$R = \text{radius to the blade element}$

$$R = 3.228 \text{ MT} = 504.41$$

$$M_B = \frac{504.41}{3} \cdot \frac{2}{3} \cdot 3.228 = 361.83 \text{ NM}$$

$$M_B = 361.83 \text{ NM} \dots\dots\dots (5.38)$$

6. Flap wise bending moment at element (6)

$$M_B = \frac{1}{B} \int_0^R r \left(\frac{1}{2} \cdot \delta \cdot \pi \cdot \frac{8}{9} \cdot U^2 \cdot 2r dr \right)$$

$$M_B = \frac{T}{B} \cdot \frac{2}{3} \cdot R$$

Where,

$T = \text{designed torque}$

$B = \text{number of blades}$

$R = \text{radius to the blade element}$

$$M_B = \frac{504.41}{3} \cdot \frac{2}{3} \cdot 2.69 = 301.53 \text{ NM}$$

$$M_B = 301.53 \text{ NM} \dots \dots \dots (5.39)$$

7. Flap wise bending moment at element 7

$$M_B = \frac{1}{B} \int_0^R r \left(\frac{1}{2} \cdot \delta \cdot \pi \cdot \frac{8}{9} \cdot U^2 \cdot 2r dr \right)$$

Integrating,

$$M_B = \frac{T}{B} \cdot \frac{2}{3} \cdot R$$

Where,

$T = \text{designed torque}$

$B = \text{number of blades}$

$R = \text{radius to the element}$

$R \text{ at } 7 = 2.152$

$$M_B = \frac{504.41}{3} \cdot \frac{2}{3} \cdot 2.152 = 241.22 \text{ NM}$$

$$M_B = 241.22 \text{ NM} \dots \dots \dots (5.40)$$

8. Bending moment at element 8

$$M_B = \frac{1}{B} \int_0^R r \left(\frac{1}{2} \cdot \delta \cdot \pi \cdot \frac{8}{9} \cdot U^2 \cdot 2r dr \right)$$

$$M_B = \frac{T}{B} \cdot \frac{2}{3} \cdot R$$

Where,

$T = \text{designed torque}$

$B = \text{number of blades}$

$R = \text{radius to the element}$

$R \text{ at } 8 = 1.614 \text{ M}$

$$M_B = \frac{504.41}{3} \cdot \frac{2}{3} \cdot 1.614 = 180.768 \text{ NM}$$

$$M_B = 241.22 \text{ NM} \dots \dots \dots (5.41)$$

9. Bending moment at element 9

$$M_B = \frac{1}{B} \int_0^R r \left(\frac{1}{2} \cdot \delta \cdot \pi \cdot \frac{8}{9} \cdot U^2 \cdot 2r dr \right)$$

$$M_B = \frac{T}{B} \cdot \frac{2}{3} \cdot R$$

Where,

$T = \text{designed torque}$

$B = \text{number of blades}$

$R = \text{radius to the element}$

$$M_B = \frac{504.41}{3} \cdot \frac{2}{3} \cdot 1.076 = 120.61 \text{ NM}$$

$$M_B = 120.61 \text{ NM} \dots\dots\dots (5.42)$$

10. Bending moment at element 10

$$M_B = \frac{1}{B} \int_0^R r \left(\frac{1}{2} \cdot \delta \cdot \pi \cdot \frac{8}{9} \cdot U^2 \cdot 2r dr \right)$$

$$M_B = \frac{T}{B} \cdot \frac{2}{3} \cdot R$$

Where,

$T = \text{designed torque}$

$B = \text{number of blades}$

$R = \text{radius to the element}$

$R \text{ at } 10 = 0.538$

$$M_B = \frac{504.41}{3} \cdot \frac{2}{3} \cdot 0.538 = 60.31 \text{ NM}$$

$$M_B = 60.31 \text{ NM} \dots\dots\dots (5.43)$$

Table 7.0: Blade elements chord length and thickness

BLADE ELEMENT	CHORD LENGTH (M)	THICKNESS (M)
(1)	0.449	0.07633
(2)	0.456	0.07752
(3)	0.35621	0.0606
(4)	0.2739	0.04656
(5)	0.220.16	0.03743
(6)	0.18322	0.03115
(7)	0.1566	0.026622
(8)	0.13655	0.02321
(9)	0.1211	0.021
(10)	0.1143	0.0194

THE SUMMARY OF FLAPWISE BENDING MOMENTS SUMMARIZED ON TABLE

Table 8.0: Blade element and corresponding bending moment

BLADE ELEMENT	BENDING MOMENT (NM)
(1)	603.05
(2)	542.75
(3)	482.44
(4)	422.135
(5)	361.83
(6)	301.53
(7)	241.22
(8)	180.768
(9)	120.61
(10)	60.31

With this parameters the stress analysis of the design can be shown below

The design factor of safety is 3 since a very high factor of safety would make the blades very heavy and reduce their efficiency .this is arrived at after strength, efficiency consideration and use of our own engineering intuition.

The tensile yield strength of plastic reinforced carbon fibre which is our design material is 2900MN/M^2

The design stress is

$$\text{design stress} = \frac{\text{yield strength}}{\text{factor of safety}} = \frac{2900}{3} = 966.67$$

$$\text{So the design stress} = 966.67\text{MN/M}^2$$

(Information obtained from strength of materials by Timoshenko)

]

$$\text{Bending stress} = \frac{M\bar{y}}{I}$$

Where M is the bending moment

I Is the bending inertia of the beam

5.0.7 Stress analysis of the blades

Stress analysis is done using parameters summarised in tables 8 and 9 .the bending stress of a material is given as

Blade element 1

Thickness of aerofoil = 0.07633M M = 603.05

$$966.67 \times 10^6 = \frac{M\bar{y}}{I}$$

\bar{y} For a circular beam is $\frac{\text{thickness}}{2} = \frac{0.07633}{2} = 0.038165M$

$$966.67 \times 10^6 = \frac{603.05 \times 0.038165}{I}$$

$$966.67 \times 10^6 = \frac{23.02}{I}$$

$$I_{\text{required}} = 2.381 \times 10^{-8} M^4 \dots\dots\dots (5.44)$$

For the carbon fibre beam to function appropriately the carbon fibre beam should have a bending inertia of $2.381 \times 10^{-8} M^4$

I For a circular spar is given as $\frac{\pi r^4}{4}$ when

$r = \text{radius}$

Since the spar thickness is 0.07633M

The approximate r would be 0.03817M

So the spar I would be

$$\frac{\pi 0.03817^4}{4} = 1.67 \times 10^{-6} M^4 \dots\dots\dots (5.45)$$

But the I needed is 2.381×10^{-8} so some portion of the spar will have to be subtracted since the I needed is much less than the (I) obtained at beam thickness in order to reduce weight of beam

So it is denoted

$I = \text{bending inertia needed to support structural load}$

$I_1 = \text{maximum bending inertia that can be obtained at that thickness of aerofoil}$

$I_2 = \text{bending inertia to be removed or subtracted}$

$$I_1 - I_2 = I$$

In blade element 1

$$I = 2.381 \times 10^{-8} M^4$$

$$I_1 = 1.67 \times 10^{-6} M^4$$

$$I_2 = ?$$

$$I_2 = I_1 - I$$

$$I_2 = 1.6462 \times 10^{-6} M^4$$

$$\text{Substituting for the radius } \frac{4 \times \pi r^4}{4} = 1.6462 \times 10^{-6} \times 4$$

$$r = 0.038 \dots \dots \dots (5.46)$$

So the cross section of the beam spar would be

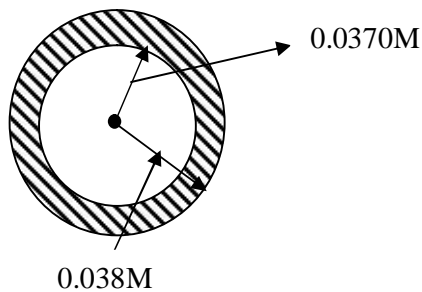


Figure 5.6: A sketch of inner and outer diameters.

Blade element 2

For the second blade element

Thickness of the aerofoil = 0.07752M

Bending moment = 542.75 NM

So the stress is given by

$$966.67 \times 10^6 = \frac{542.75 \times \bar{y}}{I}$$

$$\bar{y} \text{ For a circular beam is } \frac{\text{thickness}}{2} = \frac{0.07752}{2} = 0.03876M$$

$$966.67 \times 10^6 = \frac{542.72 \times 0.03876}{I}$$

$$I \text{ needed } 2.18 \times 10^{-8} M^4$$

I Available from the aerofoil geometry

$$I = \frac{\pi r^4}{4}$$

Where r the maximum radius of the beam is embedded which is equal to $\frac{\text{thickness}}{2} = 0.03876$

$$I = 1.7726582 \times 10^{-6} M^4 \dots\dots\dots (5.47)$$

Since the I available or I that can be achieved by that thickness is much higher than I needed some I can be taken away or subtracted amount of material need with the following formula

$$I_1 - I_2 = I$$

Where

I_1 is the available high that can be achieved at the available thickness at the available thickness of the aerofoil

I_2 is the amount of bending inertia to be removed to reduced to reduce weight of blade

I is the amount of bending inertia needed to prevent failure

$$I = 2.818 \times 10^{-8} M^4$$

$$I_1 = 1.772658 \times 10^{-6} M^4$$

$$I_2 = ?$$

$$I_2 = 1.7444 \times 10^{-6}$$

$$\therefore \frac{\pi r^4}{4} = 1.7444 \times 10^{-6} \times M^4$$

$$r = 0.03861 \dots\dots\dots (5.48)$$

So the circular beam at element 2

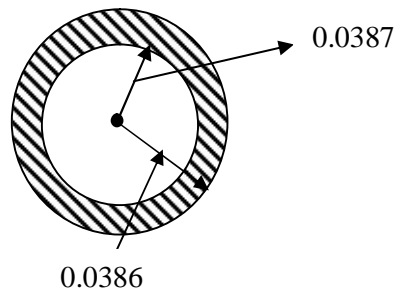


Figure 5.6: A sketch of inner and outer diameters.

The procedure is repeated for the remaining blade elements and the results are summarized in the table shown below for the profile of the circular beam spar to be embedded into the aerofoil

Table 9.0: Blade elements characteristics

BLADE ELEMENT	OUTER RADIUS	INNER RADIUS (M)	ARES M ²
(1)	0.03817	0.038	4.1×10^{-5}
(2)	0.03876	0.0386	4.86×10^{-5}
(3)	0.0303	0.030	45.68×10^{-5}
(4)	0.02328	0.023	4.07×10^{-5}
(5)	0.018715	0.0184	3.67×10^{-5}
(6)	0.01558	0.0152	3.67×10^{-5}
(7)	0.013311	0.0128	4.19×10^{-5}
(8)	0.011605	0.0096	1.33×10^{-4}
(9)	0.0105	0.001	3.43×10^{-4}
(10)	0.0097	0.0094	1.8×10^{-5}

$$MEAN AREA = 7.964 \times 10^{-5}$$

Since the beam spar is not the only component of the wind turbine blade. There is also the presence of skin on which the beam is embedded

The area of the skin can be assumed to be equal to the area of beam. So the total area of blade cross section can be assumed to be twice the mean area to give better accuracy of results.

$$Total\ area\ of\ blade\ cross\ section = 2 \times mean\ area$$

$$= 2 \times 7.964 \times 10^{-5} M^3$$

From the mean volume the mass of each blade can be calculated

$$Density = \frac{Mass}{Volume}$$

$$Mass = Density \times Volume$$

Density of each carbon fibre which is the design material is 2000Kg/M³

$$Mass = 2000 \times Mean\ Volume$$

$$Mean\ Volume = 8.569264 \times 10^{-4} M^2$$

$$Mass = 2000 \times 8.569264 \times 10^{-4} = 1.713858 Kg$$

$$Mass\ of\ the\ beam = 1.713858 Kgs$$

Due to the technician error the mass of the blade can be scaled up to 2Kilograms

The next analysis will be done with reference to a mass of 2 Kilograms

5.0.8 SHEAR STRENGTH ANALYSIS

The shear stress at the root of the blade of a wind turbine is given as

$$\frac{\text{Thrust}}{\text{Number of blades}} = \text{shear force}$$

The thrust in a wind turbine can be calculated as

$$T = C_T \frac{1}{2} \delta \pi R^2 U^2$$

Where U is the wind speed

R is the blade radius

C_T Is the coefficient of thrust

δ Is the density of wind

$$T = \frac{8}{9} \times \frac{1}{2} \times \delta \times \pi \times R^2 \times U^2$$

For the region of design i. e. Marsabit $U = 8\text{m/s}$

Rotor radius = 5.38

$\delta = 1.0$

$C_T = \frac{8}{9}$ for an ideal case

$$T = \frac{8}{9} \times \frac{1}{2} \times 1 \times \pi \times 5.38^2 \times 8^2 = 2586.5\text{N}$$

$$\text{Shear force} = \frac{T}{B}$$

$B = 3$ for the design case

$$\frac{2586.5}{3} = 862.16\text{N}$$

$$\text{Shear stress at the root of blade} = \frac{\text{Shear force}}{2 \times \text{Area at root of blade}}$$

$$= \frac{862.167}{2(\text{Area at root of blade i.e cross sectional area})}$$

$$\text{Area at root of blade} = 4.1 \times 10^{-5} M^2$$

$$\frac{862.167}{2(4.1 \times 10^{-5})} = 10,514,231.71 N/M^2$$

$$\simeq 11 MPa \dots\dots\dots$$

But the shear strength of carbon fiber reinforced plastic is $70 MPa$

So the blade would not fail by shear

5.0.9 TEST OF WHETHER THE BLADE WOULD ROTATE

For an aerodynamic body to move vertically upwards the lift force must be greater than the weight of the body

The lift force is given by

$$L = CL \times \frac{1}{2} \times \delta \times R^2 \times U^2 \times A$$

Where

$CL = \text{coefficient of lift}$

$\delta = \text{density of fluid in this case air}$

$A = \text{surface area of the body}$

$U = \text{speed of the fluid}$

The design coefficient of lift is 1.5

Area of the blades can be approximated as mean chord lengths times' blade length

The mean chord length of the blades is $0.2745M$

$$0.2745 \times 5.38 = 1.47687 M^2$$

$$\text{design wind speed} = 8 M/S$$

$\delta =$ can be taken as 1 since it varies between day and night

So the lift force is,

$$L = 1.5 \times \frac{1}{2} \times 1 \times 8^2 \times 1.4768 = 70.88N$$

The blade mass is 2 Kgs so weight of blades

$$= 2 \times g = 2 \times 9.81 = 19.62N \dots\dots\dots (5.49)$$

Where g is the acceleration due to gravitational force.

So that blade would rotate since lift is greater than weight

5.1.0 SHAFT DESIGN

Shaft design is done by the use of a formula created by Goodman. The formula is states

$$d = \left\{ \frac{16n}{\pi} \left[\frac{2(K_f M_a)}{S_e} + \left(\frac{(\sqrt{3} K_{f(s)} T)^2}{S_{ut}} \right)^2 \right]^{\frac{1}{2}} \right\}^{\frac{1}{3}} \text{ (Shigley's Mechanical engineering Design, pg 527)}$$

Where,

n is the safety factor

K_f = fatigue concentration factor due to bending

$K_{f(s)}$ = fatigue concentration factor due to torsion

S_e = endurance limit at the critical location of a machine

in the geometry and condition of use

S_{ut} = ultimate yield strength

M_a = moment applied on shaft

T = torque applied

Any material can be used in shaft design if it is corrosion resistant and rust free

The above criterion take care of fatigue, bending and torsion

In this case an easy to get shaft making material is heat treated steel shaft whose properties are, *yield strength* = 82 KPsi

$$S_{ut} = 105 \text{ KPsi}$$

Converted to MPa they are

$$\text{yield strength} = 131.2 \text{ MPa}$$

$$S_{Wtimate} = 168 \text{ MPa}$$

Taking a safety factor of 2

Since there are keys and notches

K_f can be taken as 2

$K_{f(s)}$ can be taken as 2.2

Solving

$$d = \frac{16(n)}{\pi} \left\{ \frac{2(K_f M_a)}{S_e} + \left[\frac{(\sqrt{3} K_{f(s)} T)^2}{S_{ut}} \right]^{\frac{1}{2}} \right\}^{\frac{1}{3}}$$

$$M_a = \text{Blade weight} \times \text{length}$$

Allowable length can be about 0.3M

$$\text{Total blade weight} \times 3 = 6 \text{ Kg}$$

$$\text{Moment} = 0.3 \times 6 \times 9.81 = 17.658 \text{ NM}$$

Torque previously calculated is 504.41NM.

$$S_e \simeq 0.6 \times \text{yield strength} = 0.6 \times 131.2$$

$$S_e = 78.72 \text{ MPa}$$

$$n = 0.2$$

$$d = \frac{16(2)}{\pi} \left\{ \frac{2(2)(17.658)}{78.72 \times 10^6} + \left[\frac{(\sqrt{3}(2.2) \cdot 41)}{168 \times 10^6} \right]^{\frac{1}{2}} \right\}^{\frac{1}{3}}$$

$$d = \frac{16(2)}{\pi} \{ [8.97 \times 10^{-7}] + [2.61 \times 10^{-7}] \}^{\frac{1}{3}}$$

$$d = 0.107M \text{ of shaft}$$

$$\text{So shaft diameter} = 0.107M \dots \dots \dots (5.50)$$

5.1.1 THE 6KW WIND TURBINE DRAWINGS

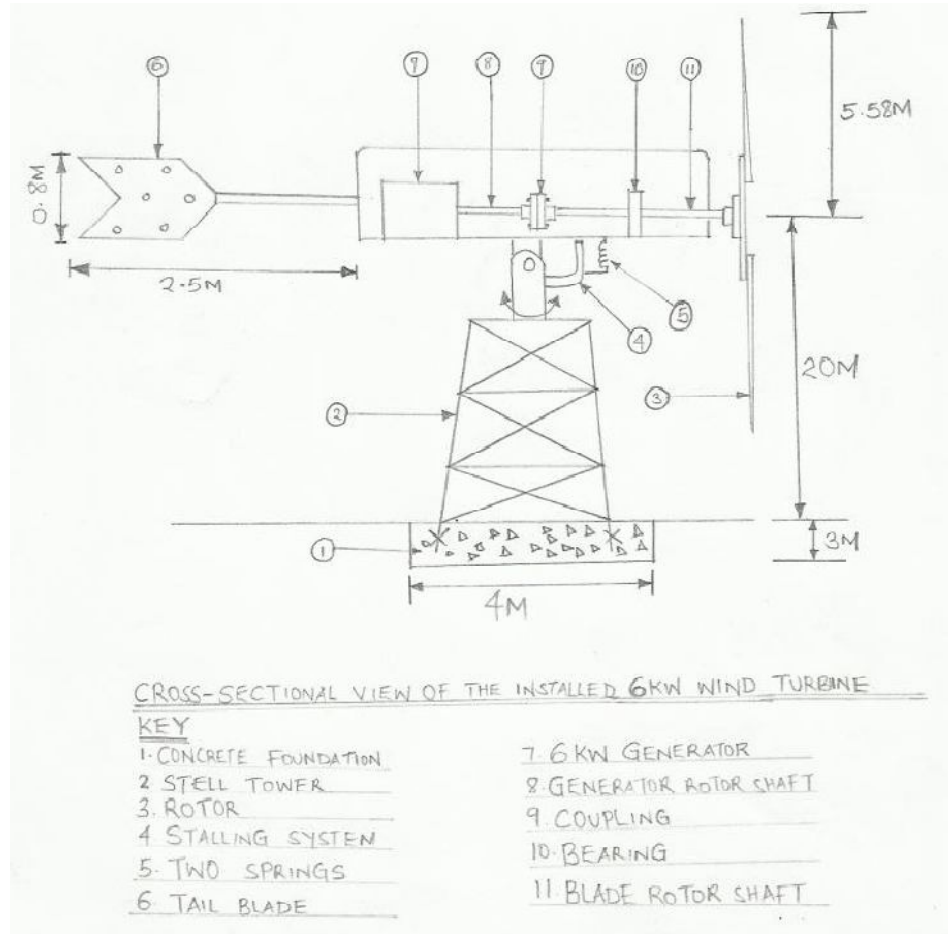


Figure 5.7: parts of the 6KW wind turbine

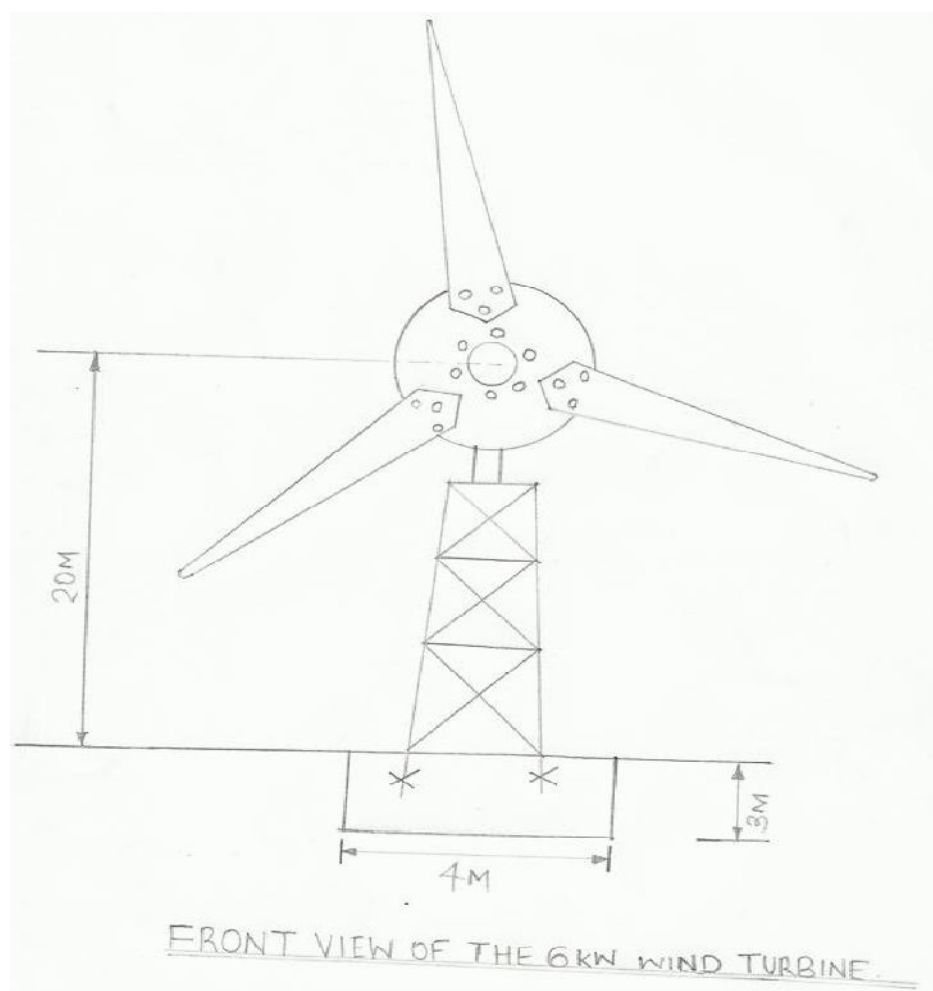


Figure 5.8: Front view of the 6KW wind turbine.

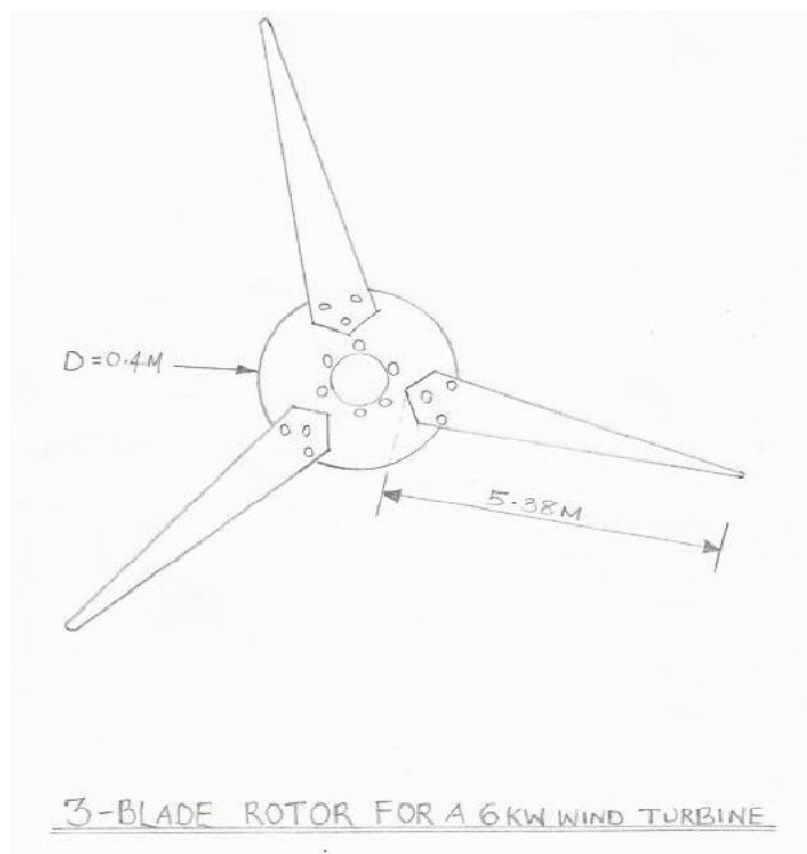


Figure 5.9: 3-blade rotor of the 6KW wind turbine.

CONCLUSION

There is an ever growing demand for energy globally due to increase in population and growth of industries globally. Non-renewable source of energy looks to be slowly losing its dominance. Through innovative ideas in technology, renewable sources of energy have been tapped to provide this clean energy.

By using this clean source of renewable source of energy, not only will it reduce the money spent on electricity bills but also help our planet recover from the effects of pollution and therefore reduce emission of greenhouse gases to the ozone layer.

Electric power generated from the wind power can be variable at several different time scales such as hourly, daily or even seasonally. Because of such instantaneous electrical generation and consumption must remain in balance to maintain grid stability, therefore this variability presents a substantial challenges to incorporating large amounts of wind power to the national grid.

Wind power is predictable and output from a single turbine can vary greatly and rapidly as local winds wind speeds vary. Therefore several turbines are connected over larger and larger areas and the average power output becomes less variable and more predictable.

Wind speeds can be accurately forecast over large areas and hence the total wind power generated connected to the national grid.

It is apparent that most parts in Kenya have good wind speed capable of driving large wind energy machines for purposes of wind energy generation. Generating electricity using energy is one way of achieving the Millennium Development Goals.

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