1. Introduction

It is often necessary to know the moment of inertia of some machine components. If that component is available, it is often simpler to determine its moment of inertia through measurement rather than calculation. The common methods available for the determination of moment of inertia are the following:

- Torsional (trifilar) pendulum (see Fig. 1, on page 2)
- Compound pendulum (see Fig. 2, on page 3)
- Calculation (see Fig. 3 on page 4).

2. Goals

1. To determine the moment of inertia of an object by use of the trifilar pendulum method.
2. To determine the moment of inertia of the same object by use of the compound pendulum method.
3. To determine the moment of inertia of the same object through calculation and to compare the value so obtained with the values obtain from the two experimental methods.

3. Procedure

3.1 The Torsional Pendulum Method

It can be shown that a test piece placed on the platform of a trifilar pendulum and given a small initial angular displacement about a vertical axis will oscillate with simple harmonic motion whose period is given by the following equation:

\[ T = 2\pi \sqrt[4]{\frac{I_2 L}{gmr^2}} \]  

where:

- \( T \) is the periodic time of the oscillation in seconds,
- \( I_2 \) is the moment of inertia, in kgm\(^2\), of the combined test piece and platform, about the centre of gravity of the combination,
- \( L \) is the length of wires suspending the platform in metres,
- \( g \) is the acceleration due to gravity, approximately 9.81 m/s\(^2\),
- \( m \) is the mass of the combination of platform and test piece, in kilograms,
- \( r \) radial distance from the centre of the platform to the points at which the wires suspending the platform are fastened to the platform, in metres. This distance is assumed to be equal for all the three wires.

Equation (1) above assumes that the centre of gravity of the combination of test piece and platform coincides with the vertical axis of oscillation of the pendulum, which also passes through the centre point of the platform.
Fig. 1 – The Trifilar Pendulum

Set the platform into oscillations and measure the duration of say twenty oscillations by use of a suitable instrument such as a stop watch. Remember to keep the oscillations small. Next, place the test piece onto the platform with its centre of gravity over the centre of gravity of the platform. Again, set the combination so obtained into oscillation and measure the duration of twenty oscillations. Record your results in neat tables. Repeat the experiment with the centre of gravity of the test piece placed 0.1 metres away from the centre of gravity of the platform. Check the effect of placing the test piece radially and tangential to the axis of the platform.

3.2 The Compound Pendulum Method

It can be shown that the period of simple harmonic oscillations of a compound pendulum is given by the following equation:

\[ T = 2\pi \sqrt{\frac{I_x + mL^2}{mgL}} \]  \hspace{1cm} (2)

where:
Moment of Inertia of an Irregular Object

- $T$, $m$ and $g$ are as defined earlier in equation (1),
- $L$ is the distance from the centre of gravity of the specimen, which is the pendulum, to the point of suspension of the pendulum, in metres.

Fig. 2 – The Compound Pendulum

Suspend the test piece on a knife-edge from the bore of the small end and measure the duration of twenty small oscillations. Repeat the experiment with the test piece suspended on the knife-edge from the bore of the big end. Record your results in neat tables.

3.3 Calculation of the Moment of Inertia of the Test Piece

In these experiments, the geometry of the laboratory test pieces was deliberately made simple enough for their moments of inertia to be readily calculated by known methods.
Fig. 3 – Geometry of Test Piece

The moment of inertia of the test piece is calculated by considering each individual component shape of the test piece, calculating the moments of inertia of each of these component shapes about their own centres of gravity and then using the parallel-axis theorem to calculate the moments of inertia of the component shapes about the centre of gravity of the test piece as a whole. Finally, the moment of inertia of the whole test piece, about its own centre of gravity is obtained by adding the moments of inertia of the component shapes about the centre of gravity of the test piece.
Moment of Inertia of an Irregular Object

**Fig. 4 — Components of the Test Piece**

With the notation of Fig. 3 and reference to Fig. 4 above, for the cylindrical component with a bore, the moment of inertia about the centre of gravity of this component is obtained as follows:

\[ I_{O1} = \frac{m_1}{8} \left( r_o^2 - r_i^2 \right) \]  

(3)

For the circular rod, the moment of inertia about the centre of gravity of this component is obtained as follows:

\[ I_{O2} = m_2 \left( \frac{d^2}{16} + \frac{l^2}{12} \right) \]  

(4)

For the cuboid component with a bore, the moment of inertia about the centre of gravity of this component is obtained as follows:

\[ I_{O3} = m_3 \left( \frac{A^2}{12} + \frac{B^2}{12} - \frac{D^2}{8} \right) \]  

(4)
Moment of Inertia of an Irregular Object

The parallel axis theorem can now be applied to calculate the component moments of inertia about the centre of gravity of the test piece, as follows:

\[
\begin{align*}
I_{G1} &= I_{O1} + m_1r_1^2 \\
I_{G2} &= I_{O2} + m_2r_2^2 \\
I_{G3} &= I_{O3} + m_3r_3^2
\end{align*}
\]  

Finally, the total moment of inertia of the test piece about its own centre of gravity is obtained as follows:

\[
I_G = I_{G1} + I_{G2} + I_{G3}
\]

4. Reporting

Write a report in the usual format of laboratory reports. Compare experimental results with the calculated result and comment upon the relative ease and accuracy of the various methods. Identify possible sources of error.